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Research and Analysis of Complex Network Models

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Abstract: *There are various kinds of networks in nature and human society, such as neural networks, social networks, road networks and the Internet. Researching the regularity of complex networks and their characteristics helps us to build suitable networks for different problems and to use complex networks to solve practical problems according to the characteristics of different networks. In this paper, we mainly analyze the properties and algorithmic implementation of random network models and small-world network models, which will provide important guidance for studying the internal mechanisms of other complex networks.*

Keywords: *Complex networks, stochastic network models, small-world network models*

1. Introduction

There are all sorts of simple or complex networks in nature, human society, social networks, food chain networks, neural networks in the human body. We usually say that the network is composed of many nodes and the edges connecting these nodes, which are used to represent the relationship between nodes in the network. Only nodes with a certain relationship can be connected by edges. Nodes directly connected by one edge are called adjacent nodes. Nodes without relationship cannot be connected by edges. The connected edges between nodes are divided into directed and undirected edges, which depend on the needs of specific problem analysis. The distance of edges reflects the intimacy between nodes in some cases. Through these simple elements, we can roughly judge the importance of a node or an edge, so as to solve some practical problems.

As a result, people have begun to explore how to derive valuable data from the connectivity of complex networks. In recent years, scientists have developed the concept of complex networks. Among the various models of complex networks, the most representative ones are the small-world model and the scale-free network model, which more accurately portray the real network. The question of what kind of network models can accurately portray realistic complex networks has been studied until now, and scientists have come up with network models that are more and more similar to real complex networks. Up to now, there is still no network model that can completely accurately portray all the static and dynamic properties of complex networks.

2. Current status of research

Research in network science is divided into three main phases: (1) the regular network [1] phase, when network research was in its infancy and people's knowledge of networks was limited to very regular networks; (2) the random network phase [2]: there was already a lot of literature and writings on network research, and people had realized that real-world networks were not only regular networks constructed in laboratories, but have a random nature. (3) Complex networks stage [3-5]: the concept of complex networks was introduced, and due to the massive increase in computer computing performance, humans had the ability to understand and study the properties of super-complex networks.

2.1. Rules network phase

The first known application of network science was Euler’s solution to the Konigsberg Seven Bridges problem. Through this problem, the graph theory is established, and it is powerfully proved that the abstraction of the actual problem does help to solve the actual problem. It is at this stage that regular networks have received a lot of research, and these studies have laid a solid foundation for more complex network research in the future.

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2.2. Random network stage

The proposal of randomness in the network has a milestone significance in network science research. It is the first time that network science research has begun to move from the network in the laboratory to the complex network that exists in the real world. In the transitional stage of complex network science, complex network models are based on the randomness of the network and then further assumptions and inferences are made.

2.3. Complex network stage

After the 21st century, with the substantial increase in computer computing performance and the large scale growth of internet connections, the research on complex networks is both very necessary and sufficient. The concept of complex networks is put forward, the static and dynamic properties of the network are simultaneously valued, the scale-free network model and the BA network model are all showing that human research on complex networks has entered a new stage. According to the characteristics of different network models, its anti-attack ability, effective suppression strategies for virus transmission, and efficient organizational structure are used in all aspects, which indicate that network research has considerable value in the future.

3. Analysis of classical network models

Since the emergence of graph theory, scientists have been trying to build a complex network model that is closest to reality. The most famous and representative of these is the random network model, and the small world network model[6-7]. The proposal of the random network model marked the beginning of mankind. Trying to use the network model to study some common characteristics of the complex networks that exist in reality. The proposal of the small-world network model indicates that humans have been able to discover and abstract some of the networks that exist in reality.

3.1. *Stochastic network model*

The random network model is a relatively simple network model. It is based on the belief that most of the networks in the real world are random in the early days of the research of the network model, and the nodes in the network are like beans scattered all over the ground, and they are connected. It uses dice to string these beans together to form a realistic network. At the same time, scientists discovered that as long as each bean can be connected to other beans by a wire, other beans can be lifted by one bean, and no beans are left on the ground, which means that as long as the degree of each node in the network is at least 1. Therefore, this network can be regard as a complete network. Furthermore, scientists found that as this random connection process continues to repeat, there are more and more links in the network. At the same time, when the average degree of node is greater than or equal to 1, there will be quite few islands in the network. And most of the networks that exist in reality are indeed complete networks, and their average degrees are far greater than 1, just as the person in human social networks knows far more than 1. The schematic diagram of the random network model is shown in Figure 1.

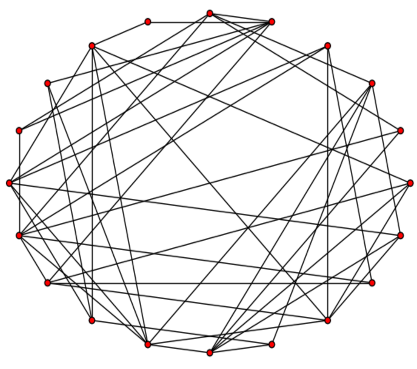

Figure 1: Schematic diagram of the stochastic network model

3.1.1. Construction of random network algorithm

Currently, there are two models for constructing ER random graph model. One construction model is

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the ER random graph G(N,M), which have a fixed number of edges. The construction algorithm is based on the following idea: Firstly, given the number of nodes N and the number of connections M in the network to be constructed, then randomly connect the node pairs. The process of randomly connecting the node pairs is to randomly select the node pairs without edge connection, and then add an edge between the randomly selected node pairs. Finally, repeat this process until an edge is added between M pairs of different nodes.

Another construction model is the ER random graph G(N,p) with a fixed probability of connecting edges. The construction algorithm is based on the following idea: firstly, considering N nodes and the probability p of connecting edges between each pair of nodes. Then, the node pairs are connected randomly. If this random number r is less than the given edge probability, an edge is added between the nodes just selected, otherwise this step is skipped and the process is repeated until all pairs of nodes in the network have been selected once.

3.1.2. Topological properties

For a given number of network nodes N and the probability of connecting edges p, the probability that the generated random graph has exactly M edges is a standard binomial distribution.

$$P(M)=\begin{pmatrix}\binom{N}{2}\\M\end{pmatrix}p^{M}(1-p)^{\binom{N}{2}-M} \tag{1}$$

where $\begin{pmatrix}\binom{N}{2}\\M\end{pmatrix}$ denotes the total number of networks that can be constructed with M edges and N nodes. $p^{M}(1-p)^{\binom{N}{2}-M}$ indicates that an edge has been added between M pairs of nodes， $\binom{N}{2}^{-M}$ represent the number of nodes pairs without edge connections.

Mean of the marginal distribution：

$$<M>=\sum_{M=0}^{\binom{N}{2}}MP(M)=\binom{N}{2}p=pN(N-1)/2 \tag{2}$$

When the probability of connecting edges is 1, a total of N(N-1) edges can be connected between N nodes, and according to the generative algorithm, the probability of performing the operation of connecting edges between each node pair is p.

Degree distribution:For any given node in the network, the probability that it is connected to other different k nodes by edges is $p^{k}(1-p)^{N-1-k}$. Since there are $\binom{N-1}{k}$ total of ways to select these k other nodes. Therefore, the probability of any given node in the network with degree k also obeys the binomial distribution::

$$P(k)=\binom{N-1}{k}p^{k}(1-p)^{N-1-k} \tag{3}$$

3.2. Small World Network Model

The small world model is the first network model established by mankind that can truly portray the characteristics of the actual network. Its appearance is not only of great significance to the research of network science, but also a milestone in sociology. To put it simply, the small world network means that although we are in a big world with billions of people, if we want to get in touch with anyone in the world who is even remotely close to us, we need to go through very few intermediaries, and the relationship between us and the person we want to get in touch with and everyone who gets in touch with us can be understood as a small world network. The small world network model is shown in Figure 2.

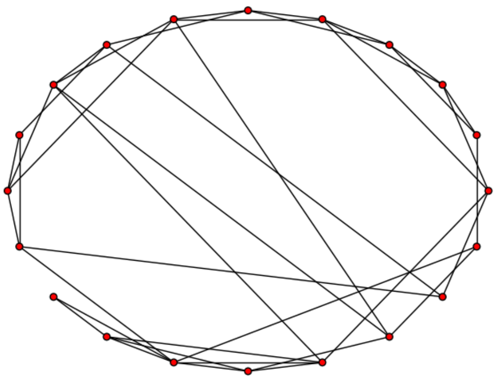

Figure 2: Diagram of the Small World Network Model

3.2.1. Construction of small world network algorithm

Small world network models are also importantly divided into two types in simulation, one is the NW small world model construction algorithm and the other is the WS small world model construction algorithm.

The NW network model construction algorithm as follows:

(1) Start from the constructed circular short-distance connection network: Given a circular short-distance connection network containing N nodes, each node is connected to K/2 adjacent nodes on the left and right neighbours, K must be an even number.

(2) Randomize connecting edges: Connect edges between randomly selected K/2 pairs of nodes according to probability p, and stipulate that no reconnecting edges and self-connecting edges are allowed.

The WS network model construction algorithm as follows:

(1) Start from the constructed ring-shaped short-distance connection network: given a ring-shaped close-distance connection network containing N nodes, each node is connected to K/2 nodes adjacent to it on the left and right, K must Is an even number.

(2) Randomize connected edges: randomly reconnect each original edge in the network with probability p, that is, keep one end point of each edge unchanged, and at the same time change the other end point to a node randomly selected from the network, It also stipulates that there shall be no reconnection edges and self-connection edges.

3.2.2. Topological properties

Considering the randomness in the small-world network model, when the reconnection probability is *p*, since each node has k neighbor nodes, it can be inferred that the number of edges connected between these *k* neighbor nodes is $\mathrm{M}_0 = 3K(K-2)/8$. Assuming that when p=0, there is an edge connection between the two neighbors *j* and *k* of node *i*. Then when p>0, the probability that the original three edges connected between the three nodes remain unchanged is $(1-p)^3$. In addition, even if the original edge between nodes i and j is removed, it is possible that the other edge with node j as the end point happens to choose node i as its other end point when reconnecting in the second step. This results in another edge being added between nodes *i* and *j*. The probability that this may happen is 1/(N-1). Therefore, the probability that nodes j and k are still neighbors of node i while still being neighbors of each other is defined as $(1-p)^3 + O(1/N)$, and the calculation expression of the probability average between the neighbors of a node after reconnection is $M_0(1-p)^3 + O(1/N)$. It can be concluded that the value of the clustering coefficient of the small world model is roughly as follows:

$$\tilde{C}_{WS} \overset{\Delta}{=} \frac{M_0(1-P)^3 + O(1/N)}{K(K-1)/2} = C_{nc}(1-p)^3 + O(1/N) \tag{4}$$

The analysis of the mean path length of the small-world model is still difficult and no analytical expression for the mean path length L of this model has yet been obtained. However, it has been shown that the mean path length of a small-world model should have the following form:

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$$L = \frac{\ln(NK\mathrm{p})}{K^2 p}, \mathrm{NKp} >> 1 \tag{5}$$

When the small-world model has a reconnection probability p=0, there will be no reconnection. The degree of each node is K, that is, each node is connected to k edges; when p>0, according to the random reconnection algorithm implementation of the small-world model, each node still remains connected to at least K/2 of the original edges in the clockwise direction, i.e., each node has a degree of at least K/2. To this end, it is useful to note that the degree of node i is $K_i = s_i + K/2, s_i \geq 0$.

Furthermore, s_i can be divided into two parts: $s_i = s_i^1 + s_i^2, s_i^1$ represents the number of K/2 edges in the counterclockwise direction connected to the original node *i* that remain unchanged, and the probability that each edge remains unchanged is 1-p; s_i^2 represents the remote edge connected to node i through the random reconnection algorithm, and the probability of each such edge exists is p/N. Therefore, the calculation expression of $P_1(S_i^1)$ is as follows:

$$P_1(s_i^1) = \binom{K/2}{s_i^1}(1-p)^{s_i^1} p^{k/2-s_i^1} \tag{6}$$

4. Conclusions

Many complex networks in the real world are basically characterized by stochastic or small-world networks, from brain structures in living organisms to various metabolic networks, from the Internet to the World Wide Web, from large power networks to global transportation networks, from scientific cooperation networks to various political, economic and social relationship networks. In this paper, we mainly focused on the development of some complex network models, algorithms to simulation them and their important topological properties. Therefore, an in-depth study of models of small-world networks and stochastic networks can help us create appropriate network depend on different requirements, also we can use them better depend on their features and solve some practical problems.

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