# Nonlinear Programming Model Based on Monte Carlo Simulation

## Yudong Wang[1,*], Xiaochen Shi[2], Ruitao Dong[2], Mingzi Li[2]

*[1]Department of Science, Northeast Forestry University, Harbin, 150040, China*
*[2]Department of Statistics, Chengdu University of Information Technology, Chengdu, 610103, China*
*\*Corresponding author: 2106680057@qq.com*

***Abstract:*** *This paper focuses on planning the choice of raw material ordering and transportation solutions for enterprises, and gives the most economical ordering and forwarding solutions for enterprises through quantitative analysis of supplier supply characteristics; and predicts the future production capacity of enterprises through time series analysis models, and finally gives the optimal ordering and transportation strategies. Among them, according to the existing optimal forwarder transshipment scheme to supplement, on the basis of the original consideration of the cost required for the production of enterprises, the model is improved, by increasing the target letter material type purchase limits, the establishment of multi-objective planning model based on Monte Carlo simulation to solve the optimal procurement scheme, the development of transshipment scheme. On this basis, the supplier supply data given in the article is divided in 240 weeks, 24 weeks as a cycle, and its long-term trend fluctuation over time is judged by drawing a time series diagram through SPSS, and the enterprise capacity prediction is made in cycles. In this paper, we find out the single-week capacity by selecting the capacity factor prediction, optimize the model, and substitute it into the improved multi-objective planning model to get the optimal strategy for purchasing and forwarding.*

***Keywords:*** *MATLAB; Multi-objective optimization; Optimal transit scheme; Monte Carlo simulation*

## 1. Introduction

In the process of producing raw materials for a company, a 24-week raw material ordering and transportation plan is needed to determine the most suitable suppliers and the order quantity according to the capacity required, and to identify the corresponding forwarders. However, during the production process, the actual supply quantity that the supplier can provide for the company may be different from the ordered quantity. Therefore, in order to ensure the production of raw materials in the enterprise, the raw material inventory should be as large as possible to meet the needs of two weeks of production. In the actual transfer process, there are losses in the transfer of raw materials, and the procurement cost of raw materials directly affects the production efficiency of the enterprise.

In this paper, we will investigate and analyze the development of a new ordering and trans-shipment plan under the constraints of purchasing more raw materials of category A and less raw materials of category C, while keeping the production costs as low as possible and reducing the trans-shipment loss rate of the trans-shipment provider. On this basis, if the company's potential to increase production capacity. We forecast the company's weekly production capacity increase for the next 24 weeks and decide on the ordering and forwarding plan for the next 24 weeks.

## 2. Assumptions and notations

### 2.1 Assumptions

We use the following assumptions.

1) Assume that the entrepreneur only considers the cost of raw materials when choosing a decision, and does not consider other factors such as personal preferences and corporate preferences

2) It is assumed that there is no breach of contract or violation in the supply and demand cooperation between the entrepreneur, supplier and forwarder, and that each cooperation can achieve the required forwarding and delivery.

3) the data in Annex I and II are all commercial transactions between the enterprise, the supplier and the forwarder, and there is no false data or omission of records.

4) The completion of purchase orders by suppliers and the forwarding of forwarders can fully reflect the real business strength of the partner.

### *2.2 Notations*

The primary notations used in this paper are listed as Table 1.

*Table 1: Notations*

| Symbols | Symbols indicate the meaning |
|---|---|
| K | Corporate strength indicators |
| $S_{ij}$ | The quantity supplied by the ith supplier in week j |
| $O_{ij}$ | The quantity ordered by the firm from the i-th supplier in week j |
| $\rho_{wij}$ | The single-week supply-to-demand ratio of the order quantity to the ith supplier in the jth week in the wth week |
| e | e is the number of times the company ordered |
| $n_s$ | Number of continuous supply and demand stabilization |
| $\rho_s$ | Supplier's total supply strength capacity in 240 weeks |
| $Q_i$ | The enterprise chooses or not to choose the ith supplier |
| m | Number of suppliers |
| $\lambda$ | Raw Material Category |
| $P_w$ | Total business cycle capacity |
| U | Enterprise capacity factor |
| $X_{\lambda\_}\max$ | The maximum quantity that a supplier can supply for a material |

### 3. Model construction and solving

### *3.1 Optimal sourcing and forwarding solutions*

The Monte Carlo method is one of the effective methods to simulate the optimal solution, which is based on the theory of statistical sampling, where the corresponding random variables are sampled and tested or random numbers are simulated, and then the reliability index of the component is calculated by statistical analysis of the sampling results[1]. The Monte Carlo method was first proposed by Polish mathematician Stannis uram and American scientist von Neumann during the implementation of the "Manhattan Project" to develop the atomic bomb. The method of using computer to generate random numbers instead of actual data was used. According to this method, random numbers were successfully used to simulate and calculate the random walk path of neutrons[2].

In this case, the choice is made under the decision of whether to choose a supplier or not, and it is a "either/or" scenario, so we choose to use 0-1 variables to represent the final choice of the company. When $Q_i =0$, it means that no supplier is selected; when $Q_i =1$, it means that the company selects the i-th supplier.

At the same time, two random numbers in this array are used to numerically simulate fifty suppliers, and one million Monte Carlo numerical simulations are performed in Matlab to select the suppliers whose results all meet the constraints of the planning model, and the number of suppliers satisfying the conditions will tend to be constant as the number of simulations increases, so that the minimum number of suppliers required for the enterprise can be determined.

This section will solve the optimal transit solution based purely on the quantity of raw materials purchased Class A and Class C. The appropriate ordering and transit solution is found under this restriction. Therefore, the present problem is still solved by Monte Carlo simulation as well as by building a mathematical planning model. Since the restrictions of the requested variables are added to the base model, it can be improved to a multi-objective planning model for solving.

### 3.2 Forecast of future capacity based on time series model

1) Determination of the objective function

The rest of the conditions are based on the fact that only the restriction on the purchase quantity of material types is added. Let $\max\_A(j)$ be the maximum value of the sum of the supply quantities of all suppliers of type $Q_i=1$ selected after using the Monte Carlo simulation method for product A in week j. Similarly, $\min\_C(j)$ is the sum of the supply quantities of all suppliers of type $Q_i=1$ selected after using the Monte Carlo simulation method for product A in week j. Similarly, mi is the minimum value of the sum of the supply quantities of all suppliers of type $Q_i=1$ selected after using the Monte Carlo simulation method for product C in week j. The minimum value is the sum of the supply quantities of all suppliers of type $Q_i=1$ selected after using the Monte Carlo simulation method for product C in week j.

From this, the objective function can be determined.

$$\max\_A(j) \tag{1}$$

$$\min\_C(j) \tag{2}$$

$$\min \sum_{j=1}^{n}\left(\sum_{i=1}^{m} Q_{ij} \cdot X_{\lambda ij} \cdot d_{\lambda}\right) \tag{3}$$

2) Establishment of Constraints

$$\text{s.t.}\begin{cases} SUM\_\lambda(j) = \sum_{i=1}^{m} Q_{ij} \cdot X_{\lambda ij} \\ \dfrac{SUM\_\lambda(j)}{\upsilon} \geq t \\ \dfrac{SUM\_\lambda(j)}{\upsilon} \geq \dfrac{1}{3}t \\ X_{\lambda ij} < X_{\lambda\_}\max \end{cases} \tag{4}$$

3) Model establishment

After determining a number of functions as the objective function of this model, establish the corresponding supply and other indicators of constraints, and build the following model

$$\begin{cases} \max\_A(j) \\ \min\_C(j) \\ \min \sum_{j=1}^{n}\left(\sum_{i=1}^{m} Q_{ij} \cdot X_{ij} \cdot d_{\lambda}\right) \end{cases} \tag{5}$$

$$\text{s.t.}\begin{cases} SUM\_\lambda(j) = \sum_{i=1}^{m} Q_{ij} \cdot X_{\lambda ij} \\ \dfrac{SUM\_\lambda(j)}{\upsilon} \geq t \\ \dfrac{SUM\_\lambda(j)}{\upsilon} \geq \dfrac{1}{3}t \\ X_{\lambda ij} < X_{\lambda\_}\max \end{cases} \tag{6}$$

4) Model results

The transshipment scheme is solved iteratively using a greedy strategy. The optimal sourcing solution

and the transshipment solution are derived separately by solving the planning model by MATLAB.

### 3.3 Time Series Analysis Model

1) Data preparation

Since the forecast of the procurement program for the next 24 weeks as well as the transshipment program is to be made, that is, the forecast of the capacity of the company after the increase. First, the matrix $I$ - $S_i$ of suppliers' availability in 240 weeks given in the annex, supplier I with the supply in 240 weeks. Since it is necessary to give a scenario for the next 24 weeks, we redivide this matrix into ten cycles with each 24 weeks as a period. In each cycle, the supply quantity of each supplier is summed to obtain the total supply quantity $S_w$ of each supplier in each cycle, and construct the matrix $I$ - $S_i$ of the total supply quantity $S_w$ of supplier I with its corresponding cycle, which is a 50×10 matrix[3].

After building the new matrix, it is necessary to sum the supply of different materials from each supplier in each cycle to obtain $S_{w\_}(A)$, $S_{w\_}(B)$, $S_{w\_}(C)$. The capacity of the enterprise in each cycle is calculated based on the consumption of different materials by the enterprise capacity $P_w$, and the formula is as follows.

$$P_w = \frac{S_{w\_}(A)}{0.6} + \frac{S_{w\_}(B)}{0.66} + \frac{S_{w\_}(C)}{0.72} \tag{7}$$

2) Establishing a time series analysis model for forecasting cycle capacity

①Drawing time series diagram

Time series analysis method is mainly used to solve the problem of time series with randomness, seasonality and stationarity. It was discovered by boxjenkins. The most basic model is the autoregressive moving average summation model ARIMA (p.d.q). When p=0, d=0, it is the moving average model MA (q). When q=0, d=0, it is the autoregressive model AR (P). When only d=0, the model becomes the autoregressive moving average hybrid model ARMA (P, Q), so the last three models are a special form of ARIMA model[4]. By using the traditional model created by the SPSS software according to the time forecast, the time series diagram of each supplier's supply quantity over the cycle is drawn. As shown in Figure 1.

Statistical prediction method is based on rigorous mathematical theory. It has the characteristics of simple structure, fast prediction speed and convenient operation. Compared with other time series analysis and prediction methods, it is more suitable for practical application[5]. It is clear from the graph that the supplier's supply has regular fluctuations over the time cycle and there is a long-term trend over time. Whereas the existence of capacity potential in a future cycle of the company, the total capacity in a future cycle can be predicted by using time series analysis.
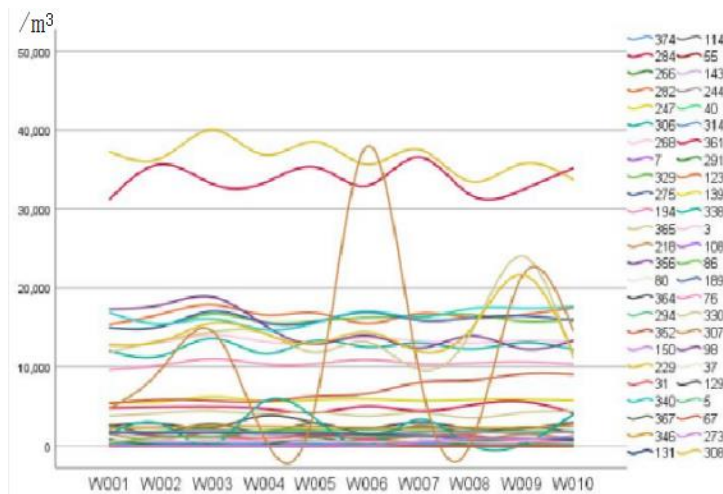


*Figure 1: Supplier's cycle time chart*

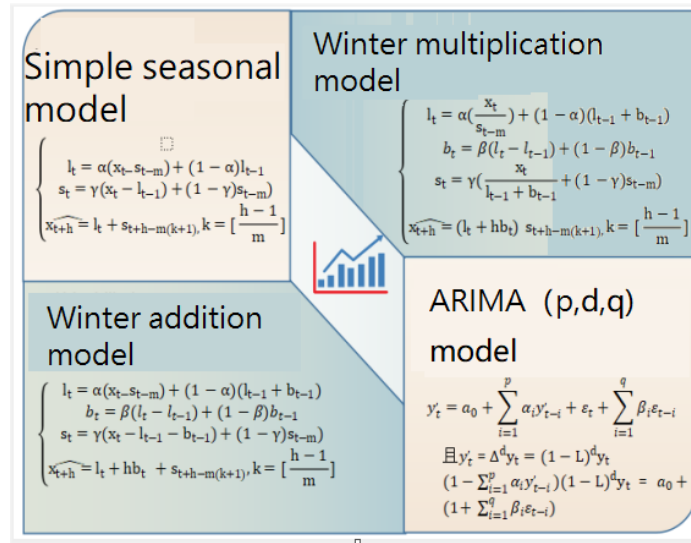②Build a time series analysis model as shown in Figure 2.



*Figure 2: Time Series Analysis*

The parameters related to each model are complicated, and only some of them are shown in this paper.

Winters additivity model parameters and ARIMA model parameters are shown in Table 2 and Table 2.

*Table 2: Winters additivity model parameters*

| Models | Parameters | Estimate |
|---|---|---|
| | Alpha(Level) | 0.199 |
| 306 | Gamma(Trend) | 3.14E-06 |
| | Delta(Season) | 9.64E-05 |
| | Alpha(Level) | 0.193 |
| 150 | Gamma(Trend) | 4.05E-07 |
| | Delta(Season) | 8.97E-06 |
| | Alpha(Level) | 0.199 |
| 356 | Gamma(Trend) | 5.27E-06 |
| | Delta(Season) | 0.001 |

*Table 3: ARIMA model parameters*

| Models | Estimation $a_0$ | t | Significance |
|---|---|---|---|
| 374 | 315.667 | 17.887 | 0 |
| 284 | 4783.8 | 34.25 | 0 |
| 266 | 666.5 | 672.125 | 0 |
| 282 | 16586.6 | 68.041 | 0 |

③Analyze the image

After solving, the total supply quantity of each supplier for the enterprise in the eleventh cycle is derived. Due to space limitations, only the estimated data of the eleventh week of the three groups of suppliers in the table are listed in this paper as shown in Table 4.

*Table 4: Forecasted total supply for each supplier*

| Supplier ID | Category | W011 |
|---|---|---|
| 352 | 3 | 9488 |
| 218 | 1 | 1516 |
| 306 | 3 | 13832 |

By using SPSS software, the predicted data from the three sets of models randomly selected in the table were plotted as shown in Figure 3.
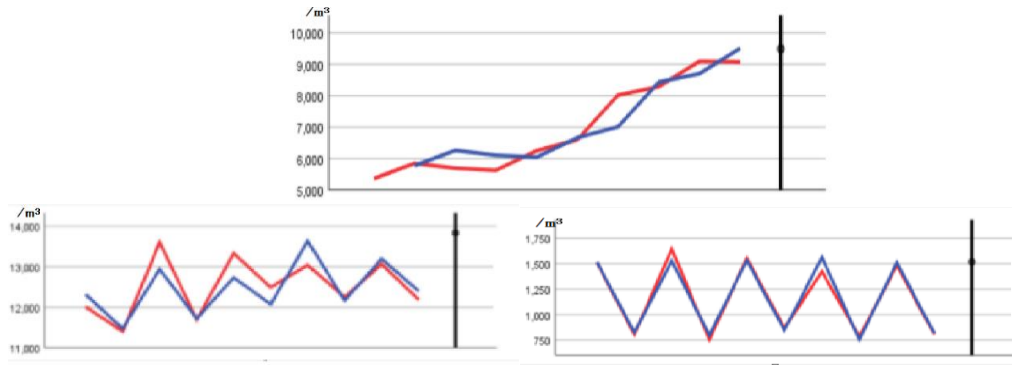
*Figure 3: Three sets of model prediction data*

The red color is the actual supply quantity, and the blue color is the predicted supply quantity. It can be seen from the graph that the direction as well as the change trend of the two are basically the same, and the actual error obtained is smaller.

④Solving time series analysis model results

After the above model and MATLAB solution results can be derived from the 50 suppliers in the 11th cycle of supply, and again to establish the supplier I and the total supply matrix in the 11 cycles of the cycle by using the calculation of the enterprise cycle capacity formula 7 can further obtain the total capacity of the enterprise in the 11 cycles of each cycle $P_W$.

1). Calculation of capacity factor

In order to measure the firm's capacity and forecast the single-week capacity in the 11th cycle, let the firm's cycle capacity ratio be, calculated as follows

$$R_{w} = \frac{P_{w+1}}{P_{w}} \tag{8}$$

The change in the supply volume of the enterprise cycle fluctuates and can be regarded as a small upward trend. In order to make the data error smaller and accurately determine the magnitude of the enterprise's weekly capacity increase. In this model, the enterprise capacity factor U is introduced and the formula is as follows.

$$U = \frac{\sum_{w}^{w=10} R_{w}}{W}, \left( w = 10 \right) \tag{9}$$

By calculating the capacity factor, we can find the weekly order quantity of the enterprise for the next 24 weeks. The single weekly supply quantity in the 24th week of the 10th cycle of the supplier is the weekly capacity of the enterprise in this week, and the formula for calculating the weekly capacity in the next 24 weeks is as follows.

$$P_{w\_j+1} = P_{w\_j} \times U \tag{10}$$

After finding the weekly capacity for the first week in the 11th cycle, using the iterative method, the weekly capacity for each of the next 24 weeks can be found in turn.

2). Multi-objective planning model-based sourcing solution selection

With the known weekly capacity in the next 24 weeks, a multi-objective planning model is used to solve for the material procurement solution. Since other conditions remain unchanged in this problem, the model is the same as in 3.2.

The future weekly production capacity of the enterprise is known and this condition imposes certain restrictions on the supplier's supply. Therefore, in this problem, a constraint should be established on the supply quantity of the sample suppliers after Monte Carlo simulation.

$$\sum_{i=1}^{m} Q_{ij} \cdot X_{\lambda ij} \geq P_{w\_j} \tag{11}$$

The constructed multi-objective planning model is specified as follows.

The following functions are solved by MATLAB for the final answer, and for the selection of the best transit solution, the greedy strategy chosen above should still be iteratively solved.

$$
\begin{cases}
\max\_A(j) \\
\min\_C(j) \\
\min \sum_{j=1}^{n}\left(\sum_{i=1}^{m} Q_{ij} \cdot X_{ij} \cdot d_{\lambda}\right)
\end{cases} \tag{12}
$$

$$
\text{s.t.}
\begin{cases}
SUM\_\lambda(j) = \sum_{i=1}^{m} Q_{ij} \cdot X_{\lambda ij} \\
\dfrac{SUM\_\lambda(j)}{\upsilon} \geq t \\
\dfrac{SUM\_\lambda(j)}{\upsilon} \geq \dfrac{1}{3}t \\
X_{\lambda ij} < X_{\lambda}\_\max \\
\sum_{i=1}^{m} Q_{ij} \cdot X_{\lambda ij} \geq P_{w\_j}
\end{cases} \tag{13}
$$

## 4. Conclusion

In this paper, in establishing the model of supplier importance analysis, a large amount of known data is screened and processed, and reliable and valid indicators are selected to measure a large number of indicators and involve a wide range of aspects, which can quantify the importance of suppliers in an all-round and effective way and achieve a final more accurate rating. In the subsequent establishment of ordering scheme and forwarding scheme, an effective and reasonable decision model is established through a variety of nonlinear programming, and the constraints have sufficient theoretical basis to solve the practical problems. In addition, the time series analysis predicts the availability of the new cycle, so that the product improves its potential and performs perfectly, and improves the main basis for the subsequent arrangement scheme. However, this paper does not sufficiently consider the influence of the relationship between the company and the supplier on the decision to order when establishing the optimal ordering scheme. In the time series analysis, only qualitative analysis of cyclical changes was performed, and there is room for improvement.

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