Purely Axial Torsional—translational Coupling Model Research of Helical Planetary Gear Sets

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ABSTRACT. *In this paper, a purely axial torsional—translational model of helical planetary gear set is proposed. The model is acceptable for the research of planetary gear transmission whose ratio of radial support stiffness to mesh stiffness is greater than 10. The gear-shaft bodies were modeled as rigid bodies and all of the planets were uniformly distributed. Compared with previous planetary gear models, presented model greatly reduced the number of degrees of freedom. All vibration modes were classified into one of two types: overall modes and planet modes. The properties of these mode types were presented.*

KEYWORDS: *planetary gear, simplified model, vibration mode*

1. Introduction

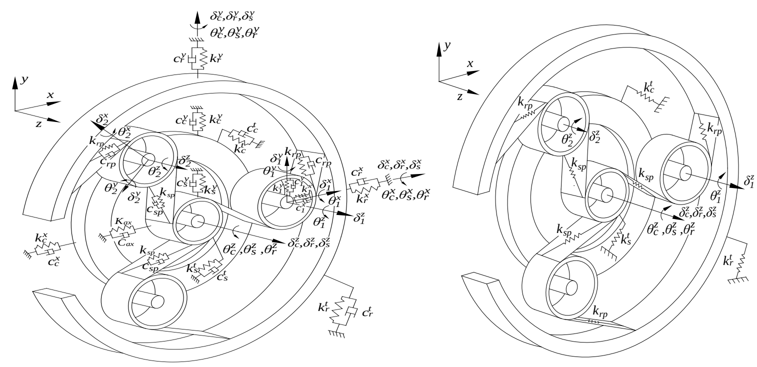
Planetary gear sets are used commonly by automotive and aerospace industries. Typical applications include jet propulsion systems, rotorcraft transmissions, passenger vehicle automatic transmissions and transfer cases and off-highway vehicle gearboxes. The internal helical have the advantages such as good meshing performance, smooth transmission, low noise, high contact ratio and low single tooth load. So, it is often used as the preferable choice of counter-shaft gear reduction systems.

Despite their long history and wide use, planetary gears are still noise and vibration problems. Over the past twenty years, a large number of studies were conducted on dynamic performance of planetary gear in order to decrease the vibration and noise [1-5]. The modal property structure of high-speed planetary gears with gyroscopic effects was investigated in Ref. [6, 7]. In addition, the model and vibration modes of planetary gears were researched by some scholars [8-11]. The complex nonlinear dynamic behavior of spur planetary gears was examined [12-13]. On the one hand, a nonlinear time-varying dynamic model was proposed to predict modulation sidebands of planetary gear sets. On the other, the theoretical and experimental investigation were conducted [14].

As we all know, dynamic model is the foundation of dynamic behavior analysis and simulation, the rationality of the model has direct influence on the accuracy of the analysis and simulation. Three-dimensional helical planetary gear models had been developed to investigate out-of-plane vibration [11]. Their the DOF is 6(N+3). It means that the DOF reaches 36 when the number of the planets gets 3. It is too much complicated and unnecessary, especially in the case of the key research is axial vibration (torsional or translational). So, a simplified purely axial torsion—translation model for helical planetary gear is proposed in this paper.

2. Purely axial torsional—translational model

The three-dimensional model of helical planetary gear set is shown in Figure. 1(a). While in this paper, the ratio of radial support stiffness to mesh stiffness is supposed to be greater than 10. The sun gear *s*, the ring gear *r*, planets *p* (uniform distributed) and the carrier *c* are assumed to only rotate around the axis and translate along the axis (the radial translational-twisted vibration is ignored). The damping is also ignored to simplify the mathematical model. Then the simplified purely axial torsional-translational model of 2K-H helical planetary gear transmission is shown as Figure. 1(b). Hence, the dynamic model has 2(n+3) degrees of freedom. Gears and the carrier are connected to the housing through linear springs k_s^t, k_r^t and k_c^t respectively. The spring k_{rp} and k_{sp}, which act along the lines of action of the ring-planet(r-p) and sun-planet(s-p) meshes, represent the average gear mesh stiffness.



(a) (b)

Figure.1 Dynamic model of a planetary gear set (all the axial support stiffness k_i^z and damping c_i^z are not be marked) : (a) 6(n+3)-degree-of-freedom model; (b) 2(n+3) -degree-of-freedom model, all the components are assumed to rotate only in the θ direction and translate in the δ direction.

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In Figure. 1(b), δ_i^z represents the mass center's linear displacement of component i deviated from its theoretical position along z-axis and θ_i^z represents the angular displacement of component i along z-axis ($i = s, r, c, 1, 2, \cdots, N$).

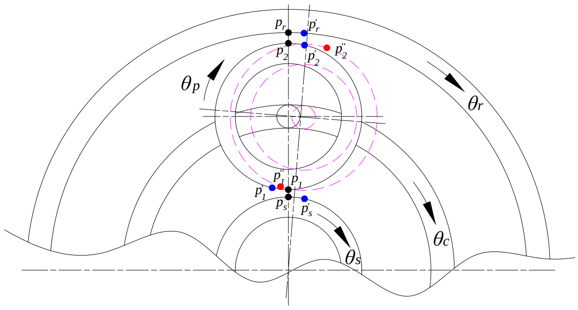

Figure.2 the circumferential transmission error model

Here, only the torsional and translational motions of the z-axis are discussed. In Figure. 2, the motions of each component are assumed to act by two steps. Step 1, the carrier is fixed and the sun, planets, ring are rotating. Step 2, the sun, planets, ring are fixed and the carrier is rotating. Points p_s , p_1 , p_2 and p_r are the initial acting positions. Points p'_s , p'_1 , p'_2 and p'_r are the positions acting after step 1

The circumferential and axial components of transmission error and mesh stiffness for both r-p and s-p meshes can be represented as

$$\delta_{rn}^t = r_r\theta_c^z + r_n\theta_n^z - r_r\theta_r^z + e_{rn}^t \tag{1}$$

$$\delta_{rn}^z = \delta_r^z - \delta_n^z + e_{rn}^z \tag{2}$$

$$k_{rp}^t = k_{rp}\cos\alpha_n\cos\beta \tag{3}$$

$$k_{rp}^z = k_{rp}\cos\alpha_n\sin\beta \tag{4}$$

$$\delta_{sn}^t = r_s\theta_s^z + r_n\theta_n^z - r_s\theta_c^z + e_{sn}^t \tag{5}$$

$$\delta_{sn}^z = \delta_s^z - \delta_n^z + e_{sn}^z \tag{6}$$

$$k_{sp}^t = k_{sp}\cos\alpha_n\cos\beta \tag{7}$$

$$k_{sp}^z = k_{sp}\cos\alpha_n\sin\beta \tag{8}$$

where r_i is the pitch radius of the gear component or the radius of circle which passing through the planet center for the carrier. n equals $1, 2, \cdots, N$. α_n is normal

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pressure angle and β is helix angle. e_1^t and e_2^t are the static transmission errors. The Z-direction displacement of the carrier relative to the planet is

$$\delta_{cn}^z = \delta_c^z - \delta_n^z \tag{9}$$

The axial support stiffness of the planets is

$$k_{cn}^z = \begin{cases} 0, | \delta_c^z < \delta_n^z \\ k_p^z, | \delta_c^z \geq \delta_n^z \end{cases} \tag{10}$$

3. Equations of motion

In terms of the usual situation, the sun gear is the input component. The carrier is the output component. The internal gear ring is fixed. Then,

$$\left(k_s^t, k_r^t, k_r^z, k_c^t\right) = \left(0, \infty, \infty, 0\right) \tag{11}$$

The differential equation of motion of 2K-H helical planetary gear transmission could be derived as:

$$\begin{cases} \sum_{n=1}^{N}(k_{rp}^t \delta_{rn}^t + k_{sp}^t \delta_{sp}^t) r_c + (J_c + N m_p r_c^2)\ddot{\theta}_c^z - k_c^t \theta_c^z r_c^2 = T \\ m_c \ddot{\delta}_c^z - k_c^z \delta_c^z - \sum_{n=1}^{N} k_p^z \delta_{cn}^z = 0 \\ J_r \ddot{\theta}_r^z - k_r^t \theta_r^z r_r^2 - \sum_{n=1}^{N} k_{rp}^t \delta_{rn}^t r_r = 0 \\ m_r \ddot{\delta}_r^z - k_r^z \delta_r^z - \sum_{n=1}^{N} k_{rp}^z \delta_{rn}^z = 0 \\ J_s \ddot{\theta}_s^z - k_s^t \theta_s^z r_s^2 - \sum_{n=1}^{N} k_{sp}^t \delta_{sn}^t r_s = -T \\ m_s \ddot{\delta}_s^z - k_s^z \delta_s^z + \sum_{n=1}^{N} k_{sp}^z \delta_{sn}^z = 0 \\ J_p \ddot{\theta}_n^z + (k_{rp}^t \delta_{rn}^t - k_{sp}^t \delta_{sn}^t) r_p = 0 \\ m_p \ddot{\delta}_n^z + (k_{rp}^z \delta_{rn}^z - k_{sp}^z \delta_{sn}^z) + k_p^z \delta_{cn}^z = 0 \end{cases} \tag{12}$$

where J_i is rotational inertia, m_i is mass. The undamped non-homogeneous matrix equation is given by $M\ddot{q} + Kq = F$ with the formula (1)-(11) substitute into formula (12), where K is a high order multi parameter stiffness matrix. The displacement vector q, the diagonal mass matrix M and the excitation vector F are

$$q = \begin{bmatrix} \theta_c^z & \theta_r^z & \theta_s^z & \theta_1^z & \theta_2^z & \cdots & \theta_N^z & \delta_c^z & \delta_r^z & \delta_s^z & \delta_1^z & \delta_2^z & \cdots & \delta_N^z \end{bmatrix}^T \tag{13}$$

$$M = diag\begin{bmatrix} J_c + Nm_p r_c & J_r & J_s & J_1 & J_2 & \cdots & J_N & m_c & m_r & m_s & m_1 & m_2 & \cdots & m_N \end{bmatrix} \tag{14}$$

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$$F=\begin{bmatrix} T-\sum_{n=1}^{N}(k_{rp}^{t}e_{rn}^{t}+k_{sp}^{t}e_{sp}^{t})r_c \\ \sum_{n=1}^{N}k_{rp}^{t}e_{rn}^{t}r_r \\ \sum_{n=1}^{N}k_{sp}^{t}e_{sn}^{t}r_s-T \\ k_{sp}^{t}r_n e_{s1}^{t}-k_{rp}^{t}r_n e_{r1}^{t} \\ \vdots \\ k_{sp}^{t}r_n e_{sn}^{t}-k_{rp}^{t}r_n e_{rn}^{t} \\ 0 \\ \sum_{n=1}^{N}k_{rp}^{z}e_{rn}^{z} \\ -\sum_{n=1}^{N}k_{sp}^{z}e_{sn}^{z} \\ k_{sp}^{z}e_{s1}^{z}-k_{rp}^{z}e_{r1}^{z} \\ \vdots \\ k_{sp}^{z}e_{sn}^{z}-k_{rp}^{z}e_{rn}^{z} \end{bmatrix} \quad (15)$$

4. Characteristics of natural frequencies and vibration modes

In this paper, 4AT planetary transmission system is the research object. The natural frequency and the dynamic response will be studied. The parameters of planetary gear system are shown in Table 1.

Table 1 The parameters of planetary gear system in Figure.2

Basic parameters	Carrier	Ring gear	Sun gear	Planet gear
Tooth number	/	54	18	18
Normal module	/	2		
Press angle/°	/	20		
Tooth width/mm	/	20		
Mass/kg	0.306	0.493	0.16	0.16
Rotational inertia /kg.m2	362.039e-6	2155e-6	43.474e-6	43.474e-6
Pitch diameter/ m	0.0412	0.125	0.0412	0.0412
Tangential stiffness/(N/m)	0	1e15	0	0
Axial stiffness/(N/m)	5e7			
Mesh stiffness/(N/m)	$k_{an}=1.2e8$ $k_{bn}=1.5e8$			
Helix angle/°	27			

4.1 Natural frequencies

The free vibration equation can be obtained by the equation of section 3. Regardless of the internal and external excitation, the undamped homogeneous vibration equation as shown follows

$$M\ddot{q}+Kq=0 \quad (16)$$

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The average mesh stiffness is instead of the time-varying mesh stiffness in order to ignore the influence of the parameter excitation in calculation. The corresponding eigenvalue problem of free vibration equation is

$$\omega_i^2 M\varphi_i = K\varphi_i \qquad \left(i = 1, 2, \cdots, 2N + 6\right) \tag{17}$$

where ω_i and φ_i respectively are the *n*th order natural frequency and vector of mode shape of the system, namely the square root of characteristic value of $[M]^{-1}K$ and corresponding eigenvectors. The natural frequencies solved in MATLAB are shown as Table 2.

Table 2 Natural frequency of lumped parameter model of the gear system

Mode	1	2	3	4	5	6
Natural frequency/HZ	3203.2	3860.9	4436.6	4436.6	5463.5	6042.8
Mode	7	8	9	10	11	12
Natural frequency/HZ	7317.4	7780.6	9397.9	9397.9	11324.1	12013.5

4.2 Vibration Mode Analysis

The vector of vibration mode of (17) is

$$\varphi_i = \left[\varnothing_{h\theta}\, \varnothing_{b\theta}\, \varnothing_{a\theta}\, \varnothing_{1\theta} \varnothing_{2\theta} \cdots \varnothing_{N\theta}\ \ \varnothing_{hz}\, \varnothing_{bz} \varnothing_{az}\, \varnothing_{1z}\, \varnothing_{2z} \cdots \varnothing_{Nz}\right]^T \pm \tag{18}$$

The vectors of vibration mode can be obtained by solving the lumped parameter model. The vibration mode of the planetary system are shown in Table 3.

Table 3 Vibration mode of the planetary system

Element	7780.6Hz	3949.2Hz	9397.9Hz	4436.6Hz
$\varnothing_{h\theta}$	-0.5076	0.0000	0.0000	0.0000
$\varnothing_{b\theta}$	-0.0960	0.0000	-0.0006	0.0000
$\varnothing_{a\theta}$	0.3369	0.0000	-0.0005	0.0000
$\varnothing_{1\theta}$	-0.5407	0.0000	-0.7229	0.0000
$\varnothing_{2\theta}$	-0.5407	0.0000	0.6874	0.0000
$\varnothing_{3\theta}$	-0.5407	0.0000	0.0698	0.0000
$\varnothing_{hz}$	0.0001	-0.4708	0.0000	0.0000
$\varnothing_{bz}$	0.0004	0.0146	0.0000	0.0000
$\varnothing_{az}$	0.0007	0.0130	0.0000	0.0000
$\varnothing_{1z}$	-0.0008	-0.5094	-0.0006	-0.7588
$\varnothing_{2z}$	-0.0008	-0.5094	0.0005	0.6404
$\varnothing_{3z}$	-0.0008	-0.5094	0.0001	0.1184
	Rotational modes	*Translational modes*	*Planet rotational modes*	*Planet translational modes*

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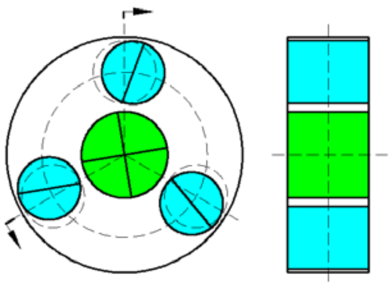
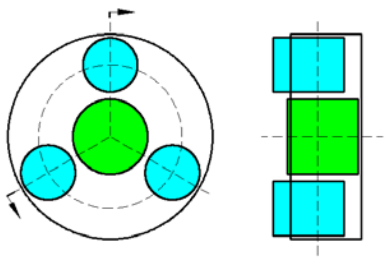
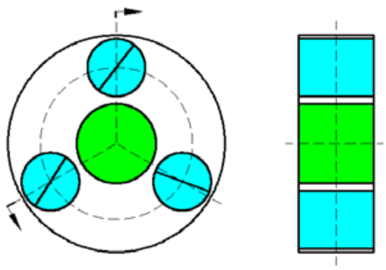
The vibration modes of the system fall into the categories of overall and planet modes. Overall modes include rotational modes (Figure. 3 (a)) and translational modes (Figure. 3 (b)). Planet modes include planet rotational modes (Figure. 3 (c)) and Planet translational modes (Figure. 3 (d)). Rotational modes in which the sun, ring, planets and carrier have only rotational motion and no translational motion, and all the planets have the same state. Translational modes in which the sun, ring, planets and carrier have only translational motion and no rotational motion, and all the planets have the same state. Planet rotational modes in which the planets have only rotational motion and the ring, sun and carrier have no motion. Planet translational modes in which the planets have only translational motion and the ring, sun and carrier have no motion.



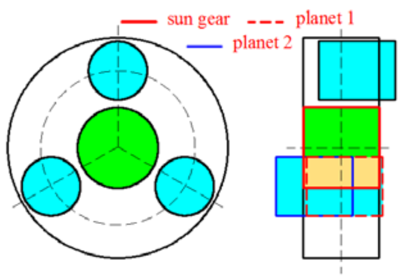
(a) Associated with 7780.6 Hz



(b) Associated with 3949.2 Hz



(c) Associated with 9397.9 Hz



(d) Associated with 4436.6Hz

Figure. 3 The vibration modes of the example system in Figure. 2 and Table 1. The deflections of carriers are not shown

5. Conclusion

A purely axial torsion—translation model for helical planetary gear is proposed in this paper. The presented model is suitable for the research of planetary gear transmission whose ratio of radial support stiffness to mesh stiffness is relatively great. By providing the modeling details and comparisons with previous planetary gear models, this model greatly reduces the number of degrees of freedom. The vibration modes of the system are divided into the overall and planet modes. The properties of these mode types are presented. The simplified model abandons the unnecessary and complicated part of the current model of planetary gear transmission. This is supposed to lay a good foundation for the subsequent nonlinear research and the dynamics research of more complex helical planetary gear system.

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