Calculation of Vehicle Following/Unfollowing Relationship and Its Application in The Automatic Train Operation Control

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Abstract: Whether the following vehicle’s behavior is bounded in safety, efficiency, and comfort by the position, velocity and control strategy of the preceding vehicle depends on whether vehicle following relationship exists between the two successive trains or not. The mathematical models is established to calculate the critical values of judging vehicle following/unfollowing relationship under absolute and relative braking modes. Based on the real-time calculation of vehicle following/unfollowing relationship and dynamic safe following distance, the states of automatic train following system, such as unfollowing state (i.e. free driving state), safe following state, and unsafe following state under the absolute and relative braking modes, are given and can be recognized by automatic detection systems to create conditions for the following vehicle to travel in safety, efficiency, and (comfort).

Keywords: Vehicle following/unfollowing relationship; critical value; safe following distance; vehicle following state; automatic train control

1. Introduction

Vehicle following operation is quite common in many traffic fields such as railway and highway, even aviation or space. We usually think on a subconscious level that vehicle following relationship exists between two successive vehicles moving at the same direction on the same line no matter how far the actual following distance is. In fact, vehicle following control would make no sense if the actual following distance is so long that there is no any safety problem even if the following vehicle moves at the maximum speed. On the other hand, the following vehicle would have to reduce its speed even take the emergency stop for safety if the actual following distance is too short. Clearly, there is two boundary curves made up of the countless critical points when the actual following distance monotonically increases from 0 to +∞, one can be used to judge whether vehicle following system is safe and efficient or not at the present moment, another can be used to judge whether vehicle following relationship exists between two successive vehicles, only under vehicle following relationship need the control laws to be implemented so that the following vehicle can move in safety, efficiency, and comfort. If the critical values of vehicle following distance were not used to recognize the safety and efficiency of vehicle following operation and whether vehicle following relationship exists between two successive vehicles, the blindness of vehicle following control might not be avoided and result in that safety, efficiency, and comfort may not be well reached by the behavioral adjustment of the following vehicle even if vehicle following system under control is asymptotically stable in mathematics.

For example, the high acceleration and the unrealistic deceleration may occur in the optimal velocity model (OVM) presented by Bando, et al.[1]. In order to eliminate it, the velocity difference term was introduced by Helbing and Tilch [2] to establish the generalized force model (GFM). Jiang, et al. [3] presented the full velocity difference model (FVDM) to overcome the shortage of the GFM model in describing time delay, phase transition, and congestion evolution. In 2005, the acceleration difference was introduced into the FVDM model by Zhao and Gao [4] to construct the full velocity and acceleration difference model (FVADM), which can describe the driver’s behaviors in emergency and avoid the collision and the unrealistic deceleration. Based on the study results of Okumura and Tadaki [5], Gong, et al. [6] proposed an asymmetric full velocity difference car-following model to reduce the safety risks in the GFM and FVDM models, which would probably lead to an unfavorable situation: the following vehicle may not slow down even if the actual following distance is extremely short. Peng, et
al. [7] established a new optimal velocity difference model to eliminate the negative velocity problem caused by the smaller sensitivity coefficients $\lambda$ in the FVDM model. However, the above models did not refer to the determination of vehicle following or unfollowing relationship so that the real-time calibration of the model parameters corresponding to different vehicle following situation become very complicated. Somda and Cormerais [8] used the relative braking mode to calculate safe inter-distance as a reference value for the following vehicle to adjust its own behavior more efficient than the absolute braking mode. Kesting, et al. [9] used the intelligent driver model [10] to represent adaptive cruise control (ACC) vehicles, proposed an ACC-based traffic assistance system for the improvement of road capacity, the elimination of traffic congestion, and the comfort of vehicle behavior change. There is a lack of clarity about how to calibrate the safe and efficient time gap(TG) scientifically. Considering the importance of time gap to vehicle organization, Lin [11] gave a valuable discussion about its effects on the driving performance of vehicles equipped with adaptive cruise control system. No doubt the larger time-gap can lend more safety margin to vehicle following system, but the efficiency of vehicle organization would be another important aspect needed to be considered. Lu and Madanat [12] presented the recommended following distance for truck on the basis of their investigation of vehicle braking distance and consideration of threat assessment under variable conditions.

In theory, the dynamic safe following distance or time gap can be regarded as a reference value of the actual following distance control, but little attention were paid to the practical effects of the actual following distance acting on the following vehicle’s behavioral adjustment. If the actual following distance is far shorter or longer than safe following distance at any time, the behavioral adjustment of the following vehicle for safety or efficiency would probably lose its comfort, as a result, the passengers would feel uncomfortable and the property would be damaged. From the research results mentioned above, we can see that the safety and efficiency of vehicle following system can be judged by the real-time comparison between dynamic safe following distance and the actual following distance. However, little attention was paid to the judgement of vehicle following/unfollowing relationship.

This article is organized as follows: In Section 2, we first introduce the calculation of safe following distance under different braking modes. In Section 3, the mathematical models are established to calculate the critical value of vehicle following distance for the determination of vehicle following and unfollowing relationship. In Section 4, we discuss the engineering application of vehicle following/unfollowing relationship in the automatic train following(ATO) control. The conclusion is given in Section 5.

2. Calculation of Safe Following Distance under Different Braking Modes

For collision avoidance, safe following distance in engineering is often defined as a shortest inter-vehicle distance kept by the following vehicle away from the preceding vehicle at any time. Moreover, if the actual following distance is too much longer than the safe following distance, it means that the following vehicle should speed up to improve its own following efficiency, i.e. the utilization efficiency of line traffic capacity. Safe following distance is one of importance indicators to evaluate the behavioral quality of the following vehicle, also one of the optimization goals of the following vehicle’s behavioral adjustment. Obviously, the critical value of vehicle following distance for different following/unfollowing relationship must be greater than safe following distance which varies with dynamic vehicle following situation in the process of train following movement, so does the critical value of vehicle following distance for the judgment of vehicle following/unfollowing relationship.
As shown in Figure 1, the preceding vehicle and the following are denoted by Vehicle$_p$ and Vehicle$_f$ respectively. Vehicle following relationship exists between them and their body lengths are equal.

Safe following distance $L_{\text{Safe}}$ can be calculated as follows [13, 14]:

$$L_{\text{Safe}} = L_f - L_p + \Delta L$$  (1)

where $L_{\text{Safe}}$ is safe following distance, $L_p$ is the braking distance of the preceding vehicle, $L_f$ is the braking distance of the following vehicle, $\Delta L$ is an additional safety distance. If $L_p = 0$, the safe following distance would be calculated under the absolute braking mode; If $L_p \neq 0$, we can get the safe following distance under the relative braking mode.

As far as the safety and efficiency of vehicle following operation are concerned, the actual following distance should be all the time a little greater than or equal to dynamic safe following distance no matter what braking mode is adopted.

Assuming that $L_{\text{Absolut Safe}}$ is the absolute safety following distance and $L_{\text{Relative Safe}}$ is the relative safety following distance, we have

$$L_{\text{Relative Safe}} = L_{\text{Absolut Safe}} - L_p$$  (2)

where

$$L_{\text{Absolut Safe}} = L_f + \Delta L$$  (3)

Clearly, $L_p$ and $L_f$ are closely related to the initial velocities and the braking strategies (decelerations) of the preceding and following vehicles. No matter what braking mode is adopted, the real-time calculation of safe following distance and its values should strictly abide to the following principles [15]: (1) the following vehicle can adjust its own behavior in safety, efficiency, and comfort under the common and worst cases. (2) The worst case refers to the emergencies of the preceding vehicle braking immediately to the best of its braking ability for safety and at the same time the following vehicle can adjust its own behavior quickly and comfortably to avoid collision. Thus, $L_p$ and $L_f$ can be calculated according to the below equations:

$$\begin{cases} 
L_p = -\frac{v_p^2(0)}{2a_{p,\text{emergency}}} \\
L_f = -\frac{v_f^2(0)}{2\text{minsmoothness}(a_{f,\text{braking}})}
\end{cases}$$  (4)

where $v_p(0)$ and $v_f(0)$ are the initial velocities of the preceding and following vehicles respectively. $a_{p,\text{emergency}}$ is the emergency braking acceleration of the preceding vehicle and less than 0 m/s$^2$, $a_{f,\text{braking}}$ is the braking acceleration of the following vehicle and less than 0 m/s$^2$, $\text{minsmoothness}(a_{f,\text{braking}})$ is the absolute minimum value of $a_{f,\text{braking}}$ meeting the comfort requirements.

According to ISO2631, $\text{minsmoothness}(a_{f,\text{braking}})$ is equal to $\vert-0.63\vert$ m/s$^2$. By (4), we can get

$$L_f = \frac{v_f^2(0)}{1.26}$$  (5)

It deserves to be specially noted that the traditional calculation method of safe following distance regards the velocity of the preceding vehicle as the initial velocity of the following vehicle braking to
stop. In practice, as shown in Figure 1, the velocity of the following vehicle is not always equal to that of the preceding vehicle under the fast-changing following situation. Obviously, the precondition reduces the complexity of calculating safe following distance, but it is not consistent with the real situation of vehicle following operation. In order to solve this problem, we first calculate the absolute safe following distance \( L_{\text{Absolut Safe}} \) for vehicle following control under the absolute braking mode according to (3) and (5), whereas the relative safe following distance \( L_{\text{Relative Safe}} \) would be calculated by substituting \( L_{\text{Absolut Safe}} \) and \( L_p \) into (2) if the relative safe following distance is needed by vehicle following control.

3. Calculation of Vehicle Following and Unfollowing Relationship

Suppose that \( L_{\text{Vfur Absolute}} \) and \( L_{\text{Vfur Relative}} \) are the critical values of vehicle following distance to respectively differentiate vehicle following and unfollowing relationship under the absolute and relative braking modes. As mentioned above, \( L_{\text{Absolut Safe}} < L_{\text{Vfur Absolute}} \) and \( L_{\text{Relative Safe}} < L_{\text{Vfur Relative}} \).

Similarly to the calculation of safe following distance under different braking modes, the determination of \( L_{\text{Vfur Absolute}} \) and \( L_{\text{Vfur Relative}} \) should also ensure that the following vehicle can move in safety and efficiency through its own comfortable behavioral adjustment when the actual following distance \( L_{\text{Actual}} \) is equal to or a little less than \( L_{\text{Vfur Absolute}} \) or \( L_{\text{Vfur Relative}} \).

3.1. Theorem 1

If vehicle following relationship does not exist between the preceding and following vehicles under the absolute braking mode, it would not exist between the preceding and following vehicles under the relative braking mode. If vehicle following relationship exists between the preceding and following vehicles under the absolute braking mode, it would not surely exist between the preceding and following vehicles under the relative braking mode.

3.2. Proof

As mentioned above, the calculation of the relative safety following distance \( L_{\text{Relative Safe}} \) must consider the velocity and control strategy of the preceding vehicle, which does not need to be considered in the calculation of the absolute safety following distance \( L_{\text{Absolut Safe}} \). By (2), we can get

\[
L_{\text{Absolut Safe}} \geq L_{\text{Relative Safe}}
\]  

Similarly, we can also have \( L_{\text{Vfur Absolute}} \geq L_{\text{Vfur Relative}} \). Under the absolute braking mode, vehicle following relationship would not exist between the preceding and following vehicles if \( L_{\text{Actual}} > L_{\text{Vfur Absolute}} \) where \( L_{\text{Actual}} \) is the actual following distance between two successive vehicles at any time. So we can get \( L_{\text{Actual}} > L_{\text{Vfur Relative}} \), which means that under the relative braking mode vehicle following relationship would not exist between the preceding and following vehicles if \( L_{\text{Actual}} > L_{\text{Vfur Absolute}} \).

Under the absolute braking mode, vehicle following relationship would exist between the preceding and following vehicles if \( L_{\text{Actual}} \leq L_{\text{Vfur Absolute}} \). However, \( L_{\text{Vfur Relative}} \leq L_{\text{Actual}} \leq L_{\text{Vfur Absolute}} \) and \( L_{\text{Actual}} \leq L_{\text{Vfur Relative}} \) are two situations of vehicle following operation, we cannot be sure that under the relative braking mode whether vehicle following relationship exist between the preceding and following vehicles or not when \( L_{\text{Actual}} \leq L_{\text{Vfur Absolute}} \).
There are some enlightenments for vehicle following control in Theorem 1.

3.3. Under the Absolute Braking Modes

Due to the fact that the velocity and control strategy of the preceding vehicle are not considered under the absolute braking mode, we may assume that \( L_{\text{accelerate}_f} \) is the acceleration traveling distance of the following vehicle for efficiency improvement and the actual following distance can be changed from \( L_{\text{Absolut Safe}} \) to \( L_{\text{Absolut Safe}} \). Thus, we have

\[
L = L_{\text{Absolut Safe}} + L_{\text{accelerate}_f}
\]

where

\[
L_{\text{accelerate}_f} = \frac{v_f'(t) - v_f(0)}{2a_f}
\]

\( t \) is a time parameter, \( v_f(0) \) is the initial velocity of the following vehicle before its behavioral adjustment, \( v_f(t) \) is the final velocity of the following vehicle at time \( t \), and \( a_f \) is the average acceleration of the following vehicle’s speeding up.

If \( t \) is the time the following vehicle spends to ensure that the actual following distance can be changed from \( L_{\text{fur Absolute}} \) to \( L_{\text{Absolut Safe}} \), we can get an equation as follows by (3), (5), (7), and (8) to calculate the critical value of vehicle following distance for the judgment of vehicle following/unfollowing relationship.

\[
L = \frac{v_f'(0)}{1.26} + \Delta L + \frac{v_f'(t) - v_f'(0)}{2a_f} \leq \frac{v_f'(t)}{1.26}
\]

Similarly, a new safe and efficient steady-following state is expected to be established quickly by the comfortable behavioral adjustment of the following vehicle. According to ISO2631, \( a_f = 0.63\text{m/s}^2 \) is an optimal control strategy for the following vehicle’s rapidly and comfortably behavioral adjustment.

By (9), we have

\[
L = \frac{v_f'(t)}{1.26} + \Delta L \leq \max(\frac{v_f'(t)}{1.26}) + \Delta L
\]

Thus, \( L_{\text{fur Absolute}} \) can be calculated according to the below equation.

\[
L_{\text{fur Absolute}} = \frac{(\max(v_f(t)))^2}{1.26} + \Delta L
\]

where

\[
\max(v_f(t)) = \min(v_{\text{line max}}, v_{\text{vehicle max}})
\]

\( v_{\text{line max}} \) is the permitted maximum velocity of a line, and \( v_{\text{vehicle max}} \) is the permitted maximum velocity of a vehicle.

In fact, the following vehicle should move at the permitted maximum velocity to reduce the inter-vehicle distance for the improvement of vehicle following efficiency if the inter-vehicle distance is longer than \( L_{\text{fur Absolute}} \). However, only the preceding vehicle being slower than the following vehicle can reduce the inter-vehicle distance. During the actual following distance is reduced to
$L_{\text{Vfur\_Absolute}}$, the following vehicle can move freely at any velocity not greater than the permitted maximum velocity. Therefore, $L_{\text{Vfur\_Absolute}}$ can be regarded as the critical value of vehicle following distance to judge vehicle following or unfollowing relationship under the absolute braking mode.

3.4. Under the Relative Braking Mode

The calculation of $L_{\text{Vfur\_Relative}}$ needs to subtract the distance $L_p$ of the preceding vehicle moving at an initial velocity under the braking strategy to be taken from $L_{\text{Vfur\_Absolute}}$, as shown in (13).

$$L_{\text{Vfur\_Relative}} = L_{\text{Vfur\_Absolute}} - L_p$$

(13)

In consideration with the worst case of the preceding vehicle braking abruptly in emergency, by (4) and (13) we can get

$$L_{\text{Vfur\_Relative}} = L_{\text{Vfur\_Absolute}} + \frac{v_r^2(0)}{2 * a_{p\_emergency}}$$

(14)

Clearly, $L_{\text{Vfur\_Relative}}$ changes with the initial velocity $v_r(0)$ and the braking strategy $a_{p\_emergency}$ of the preceding vehicle besides $L_{\text{Vfur\_Absolute}}$. In the process of vehicle following operation, $L_{\text{Vfur\_Relative}}$ can be in real time calculated by (14).

3.5. Discussion on Vehicle Following/Unfollowing Relationship

Based on (2), (3), (11), and (14), Figure 2 describes how to judge vehicle following/unfollowing relationship.

Under the absolute braking mode, we can think that no vehicle following relationship exists between two successive vehicles when the actual following distance $L_{\text{Actual}}$ is greater than $L_{\text{Vfur\_Absolute}}$, the following vehicle can travel freely (see “Unfollowing state (Free driving state)” in Figure 2 a). Similarly, under the relative braking mode, the following vehicle can move freely when the actual following distance $L_{\text{Actual}}$ is greater than $L_{\text{Vfur\_Relative}}$ (see “Unfollowing state (Free driving state)” in Figure 2 b), we also think that no vehicle following relationship exists between the preceding and following vehicles.
Additionally, vehicle following situation is divided into safe following states and unsafe following states under different braking modes shown in Figure 2.

From Figure 2, we also can see that the four boundary lines of $L_{\text{Absolute\_Safe}}$, $L_{\text{Relative\_Safe}}$, $L_{\text{Vfur\_Absolute}}$, $L_{\text{Vfur\_Relative}}$ can be used to judge the current state of vehicle following system. By comparing the actual following distance $L_{\text{Actual}}$ with the critical values of these four boundary lines, the current state of vehicle following system will be recognized so that the corresponding control strategy can be in real time adopted for the following vehicle to adjust its own behavior in safety, efficiency, and comfort.

Here, it needs to be emphasized that the curve of $L_{\text{Vfur\_Relative}}$ in Figure 2 b appears a parabola only when $v_p(0)$ is a constant, which can be also known from (14). In the process of vehicle following process, the velocity of the preceding vehicle often changes with road condition, weather and traffic information. The simplified description in Figure 2 b can help to express our idea more clearly, but the value of $L_{\text{Vfur\_Relative}}$ for vehicle following control under the relative braking mode should be in real time calculated strictly according to (14).

4. Application of Vehicle Following/Unfollowing Relationship in the Automatic Train Operation Control

Based on rational assumptions, we should accept the fact that the “safety first” principle is always implemented by the following vehicle at the initial time so that the initial following state of the following vehicle can be safe. In other words, $L_{\text{Actual}}$ should be not less than $L_{\text{Absolute\_Safe}}$ or $L_{\text{Relative\_Safe}}$ at the initial time. Figure 3 describes the application of vehicle following/unfollowing relationship in the automatic train operation control.

As shown in Figure 3, vehicle following situation can be divided into six states: (1) un-following state under the absolute braking mode, (2) un-following state under the relative braking mode, (3) safe following state under the absolute braking mode, (4) safe following state under the relative braking mode, (5) unfollowing state under the absolute braking mode, (6) unfollowing state under the relative braking mode.

Figure 3: Application of vehicle following/unfollowing relationship in ATO control
following state under the absolute braking mode, (4) safe following state under the relative braking mode, (5) unsafe following state under the absolute braking mode, (6) and unsafe following state under the relative braking mode.

Firstly, we judge vehicle following/unfollowing relationship before the corresponding control measures are adopted by the following vehicle to adjust its own behavior. The values of $L_{Absolute, Safe}$, $L_{Relative, Safe}$, $L_{Vfur, Absolute}$, $L_{Vfur, Relative}$, and the actual following distance $L_{Actual}$ can be in real time calculated to judge the current state of vehicle following system, thereby the corresponding control laws can be determined for the following vehicle to adjust its own behavior comfortably in safety and efficiency.

Under $L_{Actual} > L_{Vfur, Absolute}$, we can think that vehicle following relationship does not exist between the preceding and following vehicles. If $L_{Actual} = L_{Vfur, Absolute}$ and the following vehicle moves slower than the preceding vehicle (i.e. $V_f < V_p$ ), the length of the actual following distance $L_{Actual}$ would be increasing, so we also think that no vehicle following relationship exists between two successive vehicles under this situation. If $L_{Actual} = L_{Vfur, Absolute}$ and the following vehicle moves faster than the preceding vehicle (i.e. $V_f > V_p$ ), the length of the actual following distance $L_{Actual}$ would be decreasing, so we also think that vehicle following relationship exists between two successive vehicles under this situation. When the unfollowing state exists between two successive vehicles, the following vehicle can get rid of the constraints from the preceding vehicle’s position and behavior to travel freely, and the following vehicle should travel at the permitted maximum velocity to improve the efficiency of vehicle following operation. If $L_{Absolute, Safe} < L_{Actual} < L_{Vfur, Absolute}$ is true, the following vehicle would adopt the control measures based on the absolute braking mode to adjust its own behavior.

Similarly, when $L_{Actual} > L_{Vfur, Relative}$, $L_{Actual} = L_{Vfur, Relative}$ or $L_{Relative, Safe} < L_{Actual} < L_{Vfur, Relative}$, the following vehicle would be able to adopt the corresponding control measures based on the relative braking mode to adjust its own behavior if the following vehicle can in real time get the information of the preceding vehicle’s behavior and the dynamic actual following distance value, etc. Regardless of $L_{Absolute, Safe} < L_{Actual} < L_{Vfur, Absolute}$ or $L_{Relative, Safe} < L_{Actual} < L_{Vfur, Relative}$, vehicle following control based on any one of two braking modes would make efforts to ensure that $L_{Actual}$ can enter into the right neighborhood of $L_{Absolute, Safe}$ or $L_{Relative, Safe}$ so that vehicle following efficiency can be improved under the safety-first principle. However, under unsafe following state the emergency braking should be taken by the following vehicle no matter what braking mode is adopted. The emergency braking can help vehicle following system reduce potential safety risk until it slows down gradually to be at rest or enters into the safe following state in which $L_{Actual}$ is greater than $L_{Absolute, Safe}$ or $L_{Relative, Safe}$ but less than $L_{Vfur, Absolute}$ or $L_{Vfur, Relative}$. Especially under $L_{Actual} = L_{Absolute, Safe}$ or $L_{Actual} = L_{Relative, Safe}$, even if $V_f < V_p$ the actual following distance would become larger and larger, the emergency braking will be also taken so that the “safety first” principle can be followed.

Every sampling period vehicle following state is tested and judged to determine the corresponding control measures.

5. Conclusion

The critical values of vehicle following distance to determine vehicle following relationship or safety and efficiency in real time can help to avoid the blindness of vehicle following control and strengthen the pertinence and actual effect of the adopted control law. We focus on the calculation of the boundary lines for judging vehicle following relationship, by which the correspondingly critical values under the absolute and relative braking modes can be used to judge whether vehicle following relationship exists between two vehicles or not. And then, combining the judgement of vehicle following safety and efficiency based on the calculation of dynamic safe following distance, some states of vehicle following system are presented and the corresponding targeted control measures can
be adopted for the following vehicle to adjust its own behavior in safety, efficiency, and comfort according to the current state of vehicle following system. As for future work, it will be interesting to focus on the control measures of vehicle following system and platoons.

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