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Hohmann Transfer Orbiting Applying into the Space Traveling

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ABSTRACT. *There are three main steps in the process: First to Launch spacecraft into circular orbit around the Earth. And then accelerate the spacecraft at the appropriate point in its orbit around the Earth to transfer the spacecraft orbit into an elliptical orbit around the Sun such that the aphelion of the spacecraft orbit is at Mars's orbit and perihelion is at the earth's orbit. When the spacecraft approaches Mars, slow the spacecraft the correct amount so that it enters orbit around Mars. After that, we will discuss sensitivity and uncertainty during the whole trip. Finally, we will conclude all the data and approaches.*

KEYWORDS: *Hohmann transfer orbit, Kepler's third law, Kepler's third law, Newton's law of gravitation, Newton's second law*

1. Introduction

Space traveling had been an unreachable subject before Kepler's law was proposed. In 1925, Germany physician Walter Hohmann first proposed the concept "Hohmann transfer orbit" based on Kepler's laws and several gravitational principles, suggesting to apply this orbit into visual construction by illustrating the orbit transferring from Earth to Mars.

A Hohmann transfer orbit can take a spacecraft from Earth to Mars. The orbit is an elliptical one, where the periapsis is at Earth's distance from the Sun and the apoapsis is at Mars' distance from the Sun. The transfer orbit has to be timed so that when the spacecraft departs Earth, it will arrive at its orbit apoapsis when Mars is at the same position in its orbit. Earth and Mars align properly for a Hohmann transfer once every 26 months.[1]

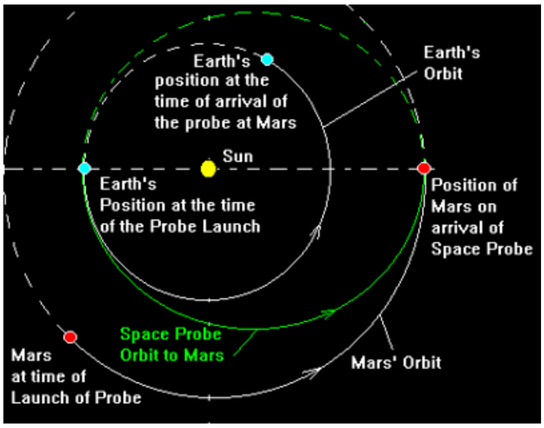

Fig.1 Illustration of Hohmann Transfer Orbit

Nowadays, several problems still concern engineers a lot. In thrust (fuel) assumptions and the launch time calculating, celestial bodies 'engagements still pose obstacles in reaching the final goal. Quite a lot research units are trying to solve the problems with their mechanic knowledge and precisely calculating or modeling.

Problems are being solved by scientists from different countries. A new Tethered Satellite System (TSS)[2] has been proposed to theoretically build a new system basing on Hohmann transfer orbit. However, the author

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choose to represent this process in an essentially explanation. In this model, objects can transfer their orbit by accelerating or decelerating their velocity after precisely calculating the energy assumption. In this article, the author will focus on recurring the process, in which the Hohmann transfer orbit is used, by using the fundamental principles of Gravitational Mechanics and modeling in Python.

2. Experimental Setup

$$G = 1.976e - 44 Newton's constant in AU^3/kgs^2$$

$$M = 2e30 Mass of sun in kg$$

$$m(earth) = 5.97e24 Mass of earth in kg$$

$$m(mars) = 6.39e23 Mass of mars in kg$$

$$AUperm = 1/1.496e11$$

$$Rearth = 6.4e6 Radius of the earth in AUperm$$

$$Horbit = 3.6e5 Height of satellite above the earth surface in AUperm$$

$$Rmars = 3.397e6 Radius of the mars in AUperm$$

$$ae = 1 distance from earth to sun in AU$$

$$am = 1.524 distance from earth to sun in AU$$

3. Analysis

3.1 Setting Up

In this stimulation, factors which are contained in lines are as below: the mass of the celestial bodies in the system, the phase differentiations between the Earth and Mars and the velocity vector changes during the process.

Due to the gravitational effects on the celestial bodies, the sun has both acceleration and force on the Earth and Mars. The planets rotate around the solar and have different velocity and run cycles. By setting up a coordination, people can stimulate the process in which two planets have interactional effects and finally find both the intersection and the right time to launch the spacecraft, which will have significant contribution in reality.

1) $F = G \cdot Mm/r^2$ *Newton's law of universal gravitation*

$$2) Force(centripetal) = m \cdot v^2/r$$

Through equation transformation, velocity can be represented as

$$3) Velocity = \sqrt{G} \cdot M/r$$

In which the M is the mass of solar and the r is the distance between the observed celestial body and the Sun.

Accordingly:

$$Vearthinitial = \sqrt{G} \cdot M/ae$$

$$Vmarsinitial = \sqrt{G} \cdot M/am$$

$$Vsatelliteinitial = \sqrt{G} \cdot M/ae + Ho$$

Include the $initial phase angle of the Mars = 63.934162977071665 in degree$ into consideration:

(The angle defining is through the modeling process and finally this figure was determined. This figure can be quite precise in time step of 10)

$$Initial phase of mars = (\pi/180) \times initial phase angle of mars$$

ISSN 2616-7433 Vol. 2, Issue 12: 09-15, DOI: 10.25236/FSST.2020.021203

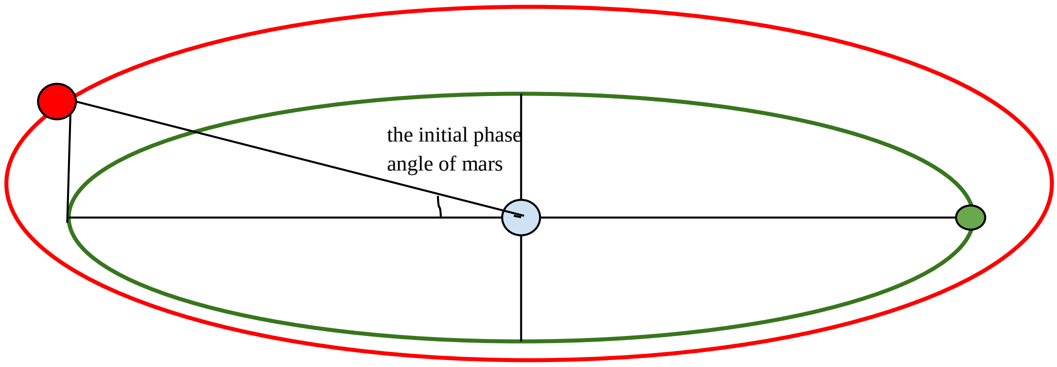

Fig.2 The Illustration of Mars' Phase

As figure 3 shows, the position of each of celestial bodies can be represented as below:

$$initialEarth(x, y) = (0, ae)$$
$$initialMars(x, y) = (am \cdot Cos(Pimars), am \cdot Sin(Pimars))$$
$$initialsatellite(x, y) = (ae + Re + Ho, 0)$$

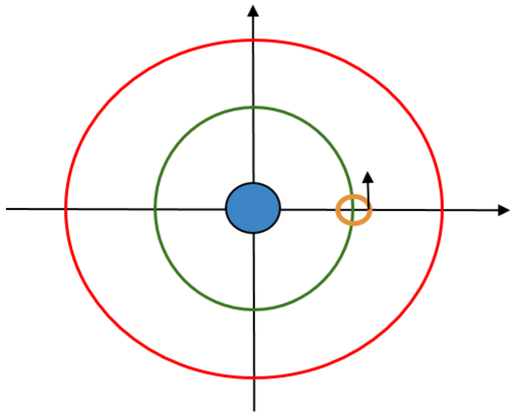

Fig.3 The Coordination Illustration, Trajectory in Red, Green and Yellow is of Mars, Earth and Satellite.

3.2 Orbit Transferring Process

As the Pythagorean theorem shows, the distance between of two different objects can be calculated as:

4)$D = \sqrt{(x^2 + y^2)}$*distance between two different celestial bodies*

After formula 1) transformation, a new formula is generated to represent acceleration of object:

$$5)a = G \cdot M/D$$

Accordingly:

$$Xa = -(G \cdot M \cdot Xinitial)/D^2$$
$$Ya = -(G \cdot M \cdot YInitial)/D^2$$

After a passage of time Tpassing by, the final position of object can be represented as below by using formula 6),in which all the position can be represented as formula 7)

ISSN 2616-7433 Vol. 2, Issue 12: 09-15, DOI: 10.25236/FSST.2020.021203

$$6) Vfinal = aT$$

$$7) Final = XInitial - (G \cdot M \cdot XInitial)/D^2 \cdot T^2$$

$$Yfinal = Yinitial - (G \cdot M \cdot Yinitial)/D^2 \cdot T^2$$

Import formula 7), accordingly:

The Earth $Xfe = Xie - (G \cdot M \cdot Xie)/D^2e \cdot T^2 Yfe = Yie - (G \cdot M \cdot Yie)/D^2e \cdot T^2$

The Mars $Xfm = Xim - (G \cdot M \cdot Xim)/D^2m \cdot T^2 Yfm = Yim - (G \cdot M \cdot Yim)/D^2m \cdot T^2$

The satellite $Xfs = Xis - (G \cdot M \cdot Xis)/D^2s \cdot T^2 Yfs = Yis - (G \cdot M \cdot Yis)/D^2s \cdot T^2$

Now make an assumption on the interaction process between the satellite, the Earth and Mars. Apply the gravitational field of different celestial bodies into the stimulation and consider the distance changes during the process.

Include the acceleration due to earth into the magnitude of acceleration of satellite in formula 5)

$aes = G \cdot Me/(Xs - Xe)^2 + (Ys - Ye)^2$ *acceleration in earth gravitational field*

$$Xa = -(G \cdot M \cdot Xinitial)/D^2 Initialaccelerationin X$$

$$Ya = -(G \cdot M \cdot Yinitial)/D^2 Initialaccelerationin Y$$

$$Xae = Xa - aes \cdot (Xs - Xe)/(Xs - Xe)^2 + (Ys - Ye)^2$$

$$Yae = Ya - aes \cdot (Ys - Ye)/(Xs - Xe)^2 + (Ys - Ye)^2$$

Include the acceleration due to Mars into magnitude of acceleration of satellite based on the earth gravitational field:

$ams = G \cdot Mm/(Xs - Xm)^2 + (Ys - Ym)^2$ *acceleration in mars gravitational field*

$$Xam = Xae - ams \cdot (Xs - Xm)/(Xs - Xm)^2 + (Ys - Ym)^2$$

$$Yam = Yae - ams \cdot (Ys - Ym)/(Xs - Xm)^2 + (Ys - Ym)^2$$

$$Vxfs = Xa \cdot T + Xam \cdot T$$

$$Vyfs = Ya \cdot T + Yam \cdot T$$

To conclude all formula beyond, the velocity of satellite can be

$$8) Vsatellite = (V^2xfs + V^2yfs)$$

velocity of satellite which includes two planets gravitational field

The final position of satellite are $(Xs + Vxfs \cdot T, Ys + Vyfs \cdot T)$

$Ds{\sim}m = \sqrt{(Xfs - Xfm)^2 + (Yfs - Yfm)^2}$ *final distance between satellite and mars*

$Ds{\sim}sun = \sqrt{(X^2fs + Y^2fs)}$ *final distance between solar and satellite*

$Dm{\sim}sun = \sqrt{(X^2fm + Y^2fm)}$ *final distance between solar and mars*

According to cosine theorem in geometry:

$$Angleinneed = arccos(D^2s{\sim}m - D^2s{\sim}sun - D^2m{\sim}sun)/((-2) \cdot Ds{\sim}sun \cdot Dm{\sim}sun)/\pi \cdot 180$$

$$Vchange = \sqrt{(Vxfs - Vxfm)^2 + (Vyfs - Vyfm)^2}$$

3.3 After Entering the Orbit of Mars

After a passage of time $+\Delta t$, reconsider the position of celestial bodies:

The Earth $Xfinal = Xfe - (G \cdot M \cdot Xfe)/D^2 \cdot T + \Delta t Yfinal = Yfe - (G \cdot M \cdot Yfe)/D^2 \cdot T + \Delta t$

The Mars $Xfinal = Xfm - (G \cdot M \cdot Xfm)/D^2 \cdot T + \Delta t Yfinal = Xfm - (G \cdot M \cdot Xfm)/D^2 \cdot T + \Delta t$

The satellite

$$Xfinal = Xfs - (G \cdot M \cdot Xfs)/D^2 \cdot T + \Delta t Yfinal = Yfs - (G \cdot M \cdot Yfs)/D^2 \cdot T + \Delta t$$

ISSN 2616-7433 Vol. 2, Issue 12: 09-15, DOI: 10.25236/FSST.2020.021203

$asatsun = G \cdot M/(X^2final + Y^2final) acceleration\ between\ sun\ and\ satellite$

Accordingly:

$$asatelliteinX = -asatsun \cdot Xfinal/\sqrt{}(X^2final + Y^2final)$$

$$asatelliteinY = -asatsun \cdot Yfinal/\sqrt{}(X^2final + Y^2final)$$

Include the earth gravitational field

$$asatear = G \cdot Me/(Xs - Xe)^2 + (Ys - Ye)^2$$

$$aearth - satelliteinX = VsatelliteinX - asatear \cdot (Xs - Xe)/\sqrt{}(Xs - Xe)^2 + (Ys - Ye)^2$$

$$aearth - satelliteinY = VsatelliteinY - asatear \cdot (Ys - Ye)/\sqrt{}(Xs - Xe)^2 + (Ys - Ye)^2$$

Include the mars gravitational field

$$asatmar = G \cdot Mm/(Xs - Xm)^2 + (Ys - Ym)^2$$

$$asatellitefinalinX = aearth - satelliteinX - asatmar \cdot (Xs - Xm)/\sqrt{}(Xs - Xm)^2 + (Ys - Ym)^2$$

$$asatellitefinalinY = aearth - satelliteinY - asatmar \cdot (Ys - Ym)/\sqrt{}(Xs - Xm)^2 + (Ys - Ym)^2$$

using velocity in formula 8)

$$VfinalsatelliteinX = (V^2xfs + V^2yfs) + asatellitefinalinX \cdot (T + \Delta t) - \Delta vinX$$

$$VfinalsatelliteinY = (V^2xfs + V^2yfs) + asatellitefinalinY \cdot (T + \Delta t) - \Delta vinY$$

The velocity of satellite after entering the orbit of mars including two different gravitational field:

$$\Leftrightarrow 9) Vsatellite = (V^2finalsatelliteinX + V^2finalsatelliteinY)$$

$$\Leftrightarrow 10) Sfinal = Sinitial + v \cdot (T + \Delta t)$$

The final position of the satellite can be figured out in 10).

And the process of Hohmann transfer can be shown as figure 4. And the velocity can be figured out by formula 9)

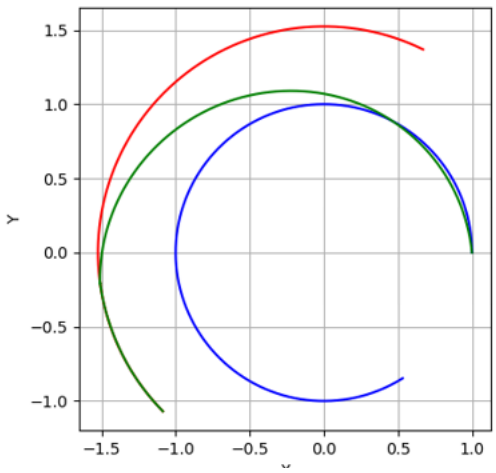

Fig.4 Final Figure Conducted by Program in Best Velocity of 4003 m/s

4. Uncertainty

To discuss the uncertainty in this process, we choose two different aspects to show how result can change when different factors are in the process, in which we change the timestep (the accuracy) and the initial velocity for satellite to launch.

When the $Vsatellitelaunch = 2002inm/s$,the satellite didn’t even escape from the earth gravitational field.

ISSN 2616-7433 Vol. 2, Issue 12: 09-15, DOI: 10.25236/FSST.2020.021203

When the $Vsatellitelaunch = 8006inm/s$, the satellite reached at mars's orbit and because the extra thrust worked on the satellite, it finally is out of both the two celestial bodies 'orbit due to the interactional gravitational field.

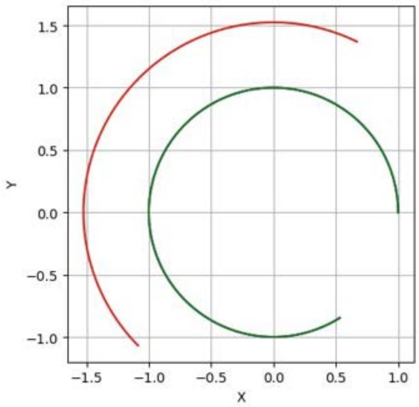

Fig.5 Speed of 2002 in m/s

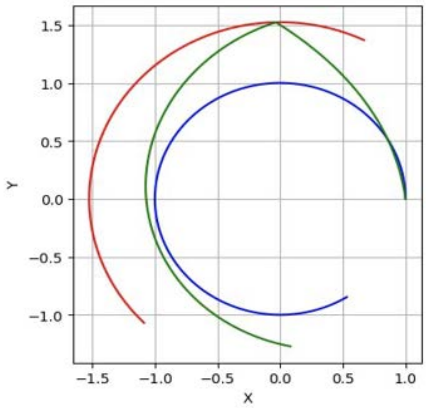

Fig.6 Speed of 8006 m/s

Consider the situation in analysis, the $Vsatellitelaunch = 4003inm/s$ is the best.

And to get a high accuracy graph, we adjusted the value of the step size to improve program accuracy, using step sizes of 70, 50 and 10 to generate graphs for the different accuracies as below.

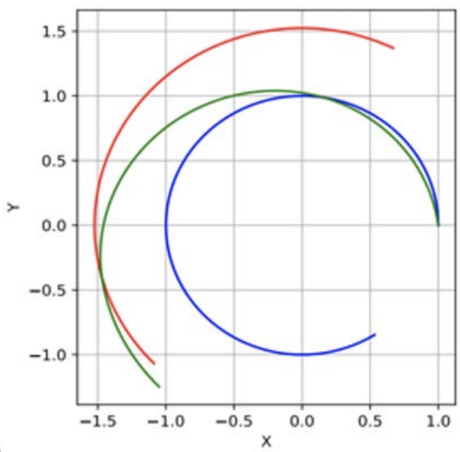

Fig.7 Step Size of 70 is used in Graph 1

ISSN 2616-7433 Vol. 2, Issue 12: 09-15, DOI: 10.25236/FSST.2020.021203

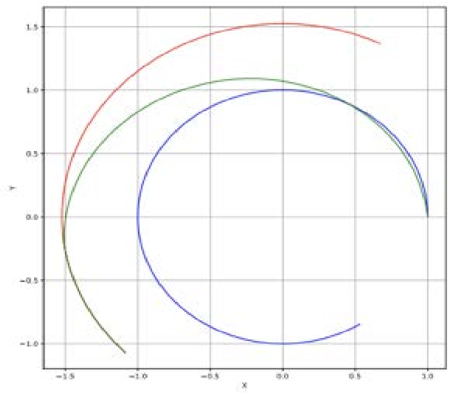

Fig.8 Step Size of 50 is used in Graph 2

5. Conclusion

Travel from earth to Mars is a complex process. The simulation begins at angle between Mars 'and earth's orbits are rounded down to 63.934162977071665 , with initial speed of satellite 0.03m/s in step size. The eccentricity equals to 0.208. After launching the satellite, the earth's gravity is considered when the satellite is close enough near to earth. Likewise, Mars' gravity is also included when the satellite approaches Mars to promote accuracy of the simulation. Based on interaction with the earth, Mars and the sun, timestep is divided to 0.1, as the satellites are close to earth or Mars with velocity under a fixed value, and 10 at rest of the time. Finally, the satellite arrives at Mars within 72720 seconds, and the cosine theorem is used to calculate angle in need to fire the rocket again to get into Mars's orbit. The summarized formula of velocity of satellite is $Vsatellite = (V^2xfs + V^2yfs)$. The final position of the satellite can be drawn as $Sfinal = Sinitial + v \cdot (T + \Delta t)$

The changes in initial velocity of the satellite can make a big difference. When v is increased, the satellite will rush out of Mars orbit, otherwise the trajectory will intersect Mars 'orbit when Mars has gone to another position.

The changes in initial velocity of the satellite can make a big difference to the entire process, so as the step size. When v is increased to 8006 m/s, the satellite will rush out of Mars 'orbit, whereas the satellite will not even escape from earth's gravitational field when decreased to 2002m/s. The dissimilarity in step size,70, 50 and 10, also highly affects the accuracy.

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