

Iterative Reconstruction Simulation of Incomplete Projection Image under Finite Viewing Angle

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Abstract: The current image reconstruction methods have the problems of poor accuracy and low efficiency. An iterative reconstruction method for incomplete projection images based on FOPD-POCS is proposed in this paper. The scene visible object image is collected and the partial incomplete data is obtained by using the adaptive median filter method to process the image. Based on the information entropy, the weights of the sampling points are obtained by using different weighting schemes, and the sampling angle is determined to realize the object information scanning. Combined with the preliminary data and information scanning results, the FOPD-POCS algorithm is used to complete the image iterative reconstruction. According to the concept of convex optimization, the corresponding FOPD iteration formula is given. Image reconstruction is regarded as a convex set optimization problem. The first order dual algorithm FOPD and the convex set projection algorithm POCS are used to iterate alternately to obtain the target image. The experimental results show that this method is a feasible method for image reconstruction because of its high fitting degree and short time consuming.

Keywords: Limited viewing angle; Incomplete projection; Image; Reconstruction

1. Introduction

In different forms of image reconstruction examples, the problem of image reconstruction under limited view angle is relatively common, that is, how to use the incomplete projection information data obtained by scanning to restore high-resolution images [1-2]. There are many reasons for obtaining incomplete information data, including hardware constraints of imaging system and specific projection model constraints, among which projection angle is the main reason. In view of the problem of image reconstruction under such incomplete information data, the reconstruction methods proposed by relevant scholars are mainly interpolation reconstruction and iterative reconstruction. The former has poor universality, while the latter does not depend on imaging geometry characteristics and detailed sampling mode, so it has strong reliability^[3]. With the development of image reconstruction technology, there are many feasible schemes.

Sijie Wang^[4] and others proposed an image reconstruction method based on structured scene. In the process, RANSAC algorithm and minimum distance method are used to calculate vanishing point line and vanishing point; parallel plane, random plane with parallel information and random plane model with vertical information are used to calculate 3D coordinates. Taking a university library as the experimental object, the experimental results show that the method is simple and easy to implement, but there is a problem of low reconstruction accuracy. HaoPeng Zhang^[5] and others proposed a scene image reconstruction method based on motion information recovery for the key role of target 3D reconstruction in spatial situation awareness. In the process, the imaging time sequence of the target image is regarded as the prior information, and the image is added according to a certain order to start the iteration, so as to prevent the reconstruction error caused by the symmetry of the target structure and texture repetition. In order to solve the problem of less space target imaging information data, the simulation of space target image is implemented, and the ground imaging test of space target is carried out. The experimental results show that this method has a certain accuracy, but it has the problem of long reconstruction time. Han min^[6] and others proposed a three-dimensional reconstruction algorithm of CT image based on wavelet transform for medical image reconstruction with different resolution given by different application conditions. In the process, the corresponding scale wavelet transform is implemented for the projection image to obtain the wavelet decomposition coefficients of each scale, and the corresponding scale wavelet coefficients are selected to complete the FDK reconstruction. Thus, the corresponding low resolution 3D image information data can be obtained, and the sectional image can be selected along the

radial direction according to the actual needs. At the same time, the corresponding inverse wavelet transform is carried out to obtain high-resolution 3D image information data. The experimental results show that the algorithm has good reconstruction stability, but the reconstruction accuracy is poor.

Image reconstruction is the research focus and innovation key in many fields, and the reliability of the above research results on image reconstruction needs to be improved. An iterative reconstruction method of incomplete projection image based on FOPD-POCS is proposed. The experimental results show that the method has good reconstruction effect, high fitting degree with the actual situation, and greatly reduces the time-consuming, which can effectively solve the problems in the above results.

2. Iterative reconstruction of incomplete projection image under limited view angle

2.1 Incomplete projection image filtering

In order to improve the efficiency of image iterative reconstruction, field object images are collected, and the image with incomplete information is processed by median filtering algorithm.

In the process of image processing, the pre judgment operator is introduced to classify the incomplete projection image area in detail, so as to achieve the purpose of noise suppression and image detail protection, and has good noise adaptability.

Because the detection threshold has a great impact on image noise detection, it must be reasonably configured^[7]. Fixed threshold can't meet the requirement of threshold selection for image noise change, so adaptive threshold is used to process the image. In order to consider the filtering speed, the amount of calculation should not be too large in the process of determining the adaptive threshold (i, j) . The adaptive median filtering threshold expression of the pixel is as follows:

$$th(i, j) = \frac{1}{W} \left\{ \sum_{n=-(W-1)/2}^{(W-1)/2} \sum_{m=-(W-1)/2}^{(W-1)/2} [f(i+n, j+m) - med] \right\} \quad (1)$$

Among them, $f(i, j)$ Representative point (i, j) Pixel value, med On behalf of (i, j) Is the median value of the window in the center, W Represents the window size. Set the maximum window size to $W_{max} \times W_{max}$ The pixel value of the filter output is $u(i, j)$.

To sum up, the implementation process of adaptive median filtering method is as follows:

- (1) Initialization window size, setting $W = 3$;
- (2) Gets the maximum value in the window f_{max} , median f_{med} And minimum f_{min} ;
- (3) The threshold value is calculated according to formula (1);
- (4) Hypothesis $f_{min} < f_{med} < f_{max}$ Go to step (5), otherwise increase the window $W = W + 2$, assuming $W < W_{max}$ Then go to step (2);
- (5) Hypothesis $|f(i, j) - med| < th(i, j)$, $0 < f(i, j) < 255$ This means that the point is not a noise point and the output remains unchanged. On the contrary, if the point is noise and replaced by the median value, there will be $u(i, j) = f_{med}$.

According to the processed image, part of the scene object information is obtained, which lays the foundation for image iterative reconstruction. Here, the information data set obtained in 2.1 is defined as U .

2.2 Information scanning

Using limited information for image reconstruction, the most important is scanning object

information. In the reconstruction of incomplete projection image with limited view angle, it is necessary to consider how to make the incomplete projection information data bring more information under the limited sampling environment, that is, how to use the existing sampling conditions^[8-9]. Based on the results of 2.1, the information entropy is used to scan the object information, and the data needed for reconstruction are obtained.

Set the radius to be $0 < r_{n'}$ On the circle of, $\beta_{n',i}$ Represents arc $C_{n',i}C_{n',i+1}$ Center angle, and so on $\beta_{n',2M}$ Represents arc $C_{n',2M}C_{n',1}$ Center angle. Therefore, the following questions are $\beta_{n',i}$ Related calculation.

When the radius is $r_{n'}$ In the circle of, there are two sampling points $A_{n',m'}$ 、 $B_{n',m'}$. It is assumed that u' The positive axis rotates counterclockwise to $\overrightarrow{OA_{n',m'}}$ The angle is $\theta_{n',m'}$, rotate to $\overrightarrow{OB_{n',m'}}$ The angle is $\phi_{n',m'}$. Suppose the incident angle of a beam of sound wave $\phi_0 = 0$ According to the combination model, there are:

$$\left. \begin{aligned} \theta_{n',m'} &= \theta_{n',0} + \phi_{m'} \\ \varphi_{n',m'} &= \varphi_{n',0} + \phi_{m'} \end{aligned} \right\} \text{for } 1, \dots, N \quad m = 1, \dots, M \tag{2}$$

Then:

$$\theta_{n',0} = \arcsin\left(\frac{\omega_{n'}}{r_{n'}}\right) + \pi \tag{3}$$

$$\varphi_{n',0} = 2\pi - \theta_{n',0} \tag{4}$$

Replace equation (4) into equation (2), and consider the angle range from 0 to 2π Then there are:

$$\left. \begin{aligned} \theta_{n',m'} &= \arcsin\left(\frac{\omega_{n'}}{r_{n'}}\right) + \pi + \phi_{m'} \bmod(2\pi) \\ \varphi_{n',m'} &= -\arcsin\left(\frac{\omega_{n'}}{r_{n'}}\right) + \pi + \phi_{m'} \bmod(2\pi) \end{aligned} \right\} \tag{5}$$

To sum up, we can get $\beta_{n',i}$ Value. For a range of scanning angles $\phi_{m'}$ In terms of formula (5), it is possible to obtain the $\theta_{n',m'}$ And $\varphi_{n',m'}$ Constitutive $2M$ Angle. According to the above calculation, these angles are arranged according to the order from large to small to get the sequence $\alpha_{n'} = [\alpha_{n',1}, \alpha_{n',2}, \dots, \alpha_{n',2M}]$ Then there are:

$$\left\{ \begin{aligned} \beta_{n',i} &= \theta_{n',m'} (\alpha_{n',i+1} - \alpha_{n',i}) \\ \beta_{n',2M} &= \varphi_{n',m'} (2\pi + \alpha_{n',1} - \alpha_{n',2M}) \end{aligned} \right. \tag{6}$$

According to formula (6) and $p_{n',i} = \beta_{n',i} / 2\pi$ Access to the n' .The information entropy expression of circles is as follows

$$H_{n'} = -\beta_{n',i} \sum_{i=1}^{2M} p_{n',i} \log_2(p_{n',i}) + \beta_{n',2M} \tag{7}$$

According to the above contents, a fixed scanning angle can be obtained $\Phi = [\phi_1, \phi_2, \dots, \phi_M]$

Entropy on the lower circles $H_{n'}$, because $r_{n'}$. The size is not evenly spaced, N The distribution of sampling circles is also uneven. The closer to the center of the circle, the denser the sampling points are; the closer to the edge of the circle, the more sparse the sampling points are^[10]. These are the different circles $H_{n'}$. There are different weights. In view of this situation, different weighting methods are used to realize the sampling point weighting and complete the information scanning and collection of incomplete projection objects under limited view angle.

2.2.1 Using uniform weighting

In this weighting scheme, the expression of weighted vector is as follows:

$$W_1' = [1, 1, \dots, 1]^T \tag{8}$$

Uniform weighting does not consider the distribution of sampling points, and gives the same weight for circles with different radii.

2.2.2 Highlight low frequency components

In this scheme, there are two kinds of weighted vectors, one of which is based on different perimeter, i.e. $w_{2,n'}' = r_1 / r_n$. Then there are:

$$w_{2,n'}' = \sqrt{\frac{N - \sqrt{N^2 - 1}}{N - \sqrt{N^2 - n'^2}}} \tag{9}$$

The weighted vector expression is as follows:

$$W_2' = [w_{2,1}^q, w_{2,2}^q, \dots, w_{2,N}^q]^T \tag{10}$$

Where $q > 0$.

The other is a weighted scheme based on distribution density. Because the number of points on each circle is the same, the distribution density is inversely proportional to its area $w_{3,n'}' = r_1^2 / (r_n^2 - r_{n-1}^2)$. There are:

$$w_{3,n'}' = H_{n'} \frac{N - \sqrt{N^2 - 1}}{\sqrt{N^2 - (n' - 1)^2} - \sqrt{N^2 - n'^2}} \tag{11}$$

The weighted vector expression is as follows:

$$W_3' = [w_{3,1}^q, w_{3,2}^q, \dots, w_{3,N}^q]^T \tag{12}$$

In the scheme (2) above, the weighting method highlights the weight value of low-frequency components, and tends to make the low-frequency part, that is, the points on the small radius circle are more evenly distributed.

2.2.3 Highlight high frequency components

The scheme and scheme (2) are similar in structure, but there are two different ways, which are based on the circumference and density respectively. The following is a brief expression:

$$w_{4,n'}' = \frac{r_{n'}}{r_N} = \sqrt{\frac{N - \sqrt{N^2 - n'^2}}{N}} \tag{13}$$

The weighted vector is as follows:

$$W'_4 = [w'_{4,1}, w'_{4,2}, \dots, w'_{4,N}]^T \quad (14)$$

$$w'_{5,n'} = \frac{r_n^2 - r_{n-1}^2}{r_N^2 - r_{N-1}^2} = \frac{\sqrt{N^2 - (n-1)} - \sqrt{N^2 - n^2}}{\sqrt{2N-1}} \quad (15)$$

The weighted vector is as follows:

$$W'_5 = [w'_{5,1}, w'_{5,2}, \dots, w'_{5,N}]^T \quad (16)$$

In scheme (3), the weight of high frequency component is more prominent, that is, the distribution of points on the circle far away from the center is more uniform.

Assuming that the number of sampling points and scanning angles is low, the scheme (2) should be selected W'_2 and W'_3 Get better visual effect. If more details are required, select W'_4 and W'_5 As a weighted scheme. If the number of sampling points and scanning angles is relatively large, it means that the low frequency part of the data distribution is relatively dense, that is, the low frequency is oversampling, which is available W'_4 and W'_5 A way to get more high frequency information.

Based on the above analysis, the ultimate goal is to find the best scanning angle under the fixed number of sampling points and scanning angles, so as to maximize the information entropy of the whole information scanning process. According to the above weighting scheme, the objective function can be expressed as follows:

$$\Phi^* = \arg \max_{\Phi} (W_k'^T H) \quad (17)$$

In equation (17) Φ yes $M \times 1$ Scan angle vector, H representative $N \times 1$ Is the entropy vector of the Φ Function of, W'_k representative $N \times 1$ The weighted vector of, $k = 1, 2, 3, 4, 5$ And corresponding to formula (8), (9), (11), (13), (15).

Based on the above calculation and analysis, it can be obtained by the following steps $H_k(\Phi) = W_k'^T H$:

- (1) It is initialized by formula (3) and equation (4) θ_0 ;
- (2) By formula (5), we get θ and φ According to the order α ;
- (3) It is obtained by formula (6) β and P And then we get H ;
- (4) Through different weighted vectors W'_k obtain $H_k(\Phi) = W_k'^T H$.

According to the above process, the Φ When the number of sampling points and scanning angles are fixed, the global optimal scanning angle is obtained by genetic algorithm Φ .Finally, the optimal solution is used Φ The spectrum information is obtained by scanning the object, and the information data is applied to image reconstruction.

2.3 Image reconstruction based on FOPD-POCS

Based on the relevant information data of image reconstruction obtained in 2.1 and 2.2, the convex set projection iterative method POCS and the first-order dual iterative method FOPD are used to realize the image reconstruction alternately f^* , iteration stops. The following is the detailed procedure of image reconstruction based on FOPD-POCS:

- (1) Setting system parameters: for gradient operator norm $L' = \|\nabla\|$ Select the appropriate iteration

step size of FOPD algorithm τ , σ And guarantee $\tau\sigma L^2 < 1$; relaxation iteration factor $\kappa = 1$ Objective function smoothing coefficient ζ The number of iterations of FOPD algorithm is set to μ The maximum number of iterations for FOPD-POCS is set to μ_t ;

(2) Initialization phase: initial iterative image arbitrary vector g^0 ;

(3) POCS projection:

1) Combined with the above information, the art algorithm is used for data consistency constrained projection

$$g^w = g^{w-1} + \frac{\zeta}{\|M_i\|^2} g^{w-1} \cdot \Phi \cdot U \tag{18}$$

2) Corresponding projection with nonnegative constraint:

$$g_{j,POCS}^w = \max \{ g_j^w, 0 \} \cdot \Phi \cdot U \tag{19}$$

(4) To sum up, No W The iterative process of fopd algorithm corresponding to the second FOPD-POCS iteration is as follows:

1) Initialization iterations $\mu = 0$ FOPD algorithm initializes iterative image $f^0 = g^0_{POCS}$ Initial gradient vector:

$$y_0 = \begin{bmatrix} \nabla^1 f^0 \\ \nabla^2 f^0 \end{bmatrix} \tag{20}$$

2) Assumed number of iterations $\mu < \mu_t$ The image vector is updated iteratively $f^{\mu+1}$, image gradient vector value $y^{\mu+1}$ And relax image vector values $\bar{f}^{\mu+1}$ The iteration expression is as follows:

$$y_{u',v'}^{\mu+1} = \left[\max \left(1, |y_{u',v'}^\mu + \sigma \nabla \bar{f}_{u',v'}^\mu| \right) \right]^{-1} \left(y_{u',v'}^\mu + \sigma \nabla \bar{f}_{u',v'}^\mu \right) \tag{21}$$

$$f_{u',v'}^{\mu+1} = \left(f_{u',v'}^\mu + \tau y_{u',v'}^{\mu+1} \right) \tag{22}$$

$$\bar{f}^{\mu+1} = f^{\mu+1} + \left(f^{\mu+1} - f^\mu \right) \tag{23}$$

(5) When the W times update to after iterations $f^w = f^\mu$, $g^w = f^\mu$, $w = w + 1$;

(6) If the number of iterations $W < W_t$ Repeat steps (3), (4) and (5). According to FOPD algorithm and POCS algorithm exchange iteration, the reconstructed target image is obtained f^* .

3. Experimental results and analysis

In order to verify the effectiveness of fopd-pocs based iterative reconstruction method for incomplete projection images under limited view angle, a simulation experiment was carried out. The experimental platform is matlab. During the experiment, 100 256 * 256 color images are used as experimental objects. The accuracy and time of reconstruction were measured.

The fitting degree between the image reconstruction results and the actual situation is an effective index to verify the reconstruction method. The higher the value is, the better the reconstruction method is, and vice versa. It can be seen from Figure 1 that the reconstruction results of literature [4] and literature [6] have low fitting degree and poor reliability. The results of the fopd-pocs based iterative reconstruction

method for incomplete projection images under limited view angles are more than 90% on average, which indicates that the proposed method has strong robustness. Considering the reality of incomplete projection image reconstruction under limited view angle, it is necessary to pay attention to how to make the incomplete projection information data bring more information under the limited sampling environment. The information entropy theory is introduced to determine the best scanning angle and complete the image information scanning, which improves the accuracy of image reconstruction. The FOPD algorithm and POCS algorithm are used to exchange iterations until the target image is obtained, which further improves the image reconstruction accuracy.

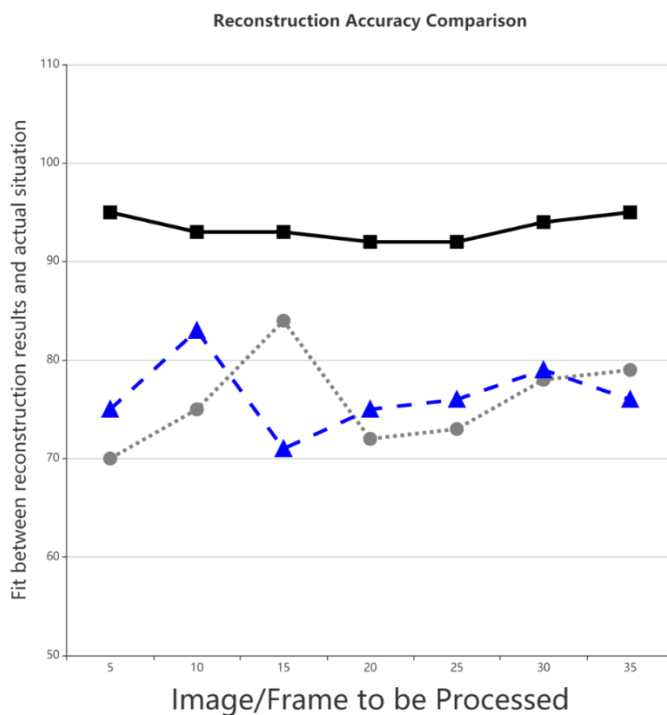


Figure 1: Comparison of image reconstruction accuracy of different research results

Reference [5] and reconstruction time-consuming comparison chart base

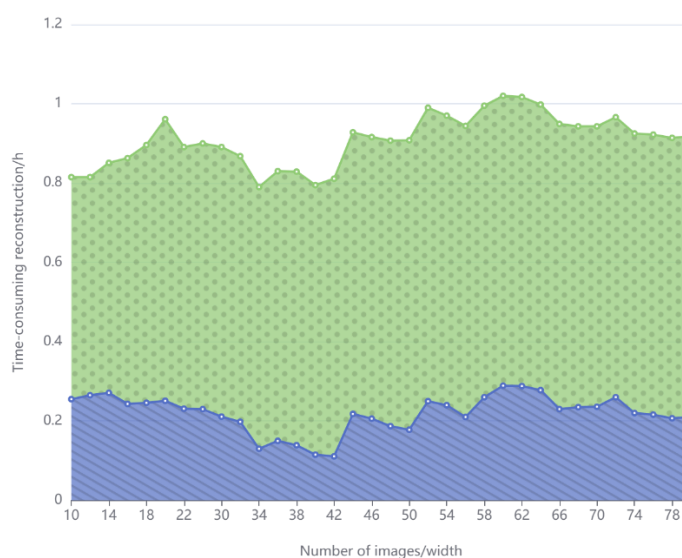


Figure 2: Comparison of image reconstruction time of different research results

It can be seen from Figure 2 that the reconstruction time of incomplete projection image iterative reconstruction method based on FOPD-POCS can be controlled under 0.3 h, which is more stable and real-time than the research results in literature^[5]. Before the acquisition of image information and

reconstruction, the proposed method uses adaptive change threshold algorithm to process the image, which effectively reduces the image reconstruction time and provides reliable support for high-precision image reconstruction.

4. Conclusion

For the problem of incomplete projection image reconstruction with limited view angle, the main consideration is to get more available information under limited sampling conditions. Based on this, an iterative reconstruction method based on FOPD-POCS is proposed. In the process, adaptive median filter is used to suppress image noise and image information scanning sampling based on information entropy is used to lay the foundation for image reconstruction. The target image is obtained by exchanging iteration between FOPD algorithm and POCS algorithm. The experimental results show that the proposed method has advantages in both reconstruction accuracy and efficiency. For the next step of research, the following suggestions are put forward: redundant dictionary and sparse representation are the research hotspots with strong applicability at present, and they can be used in image reconstruction in future work. In order to improve the reconstruction accuracy, the special structure of the reconstructed image should be fully considered. FOPD iteration is mainly based on the image pixel by pixel. In the next step, parallel computing concept can be introduced to improve the reconstruction speed.

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