

Theoretical study on consolidation of concrete pipe pile composite foundation considering permeability and time-varying load effects

Yuguo Zhang, Zilai Zhang*, Chunhao Zhao, Zhaobin Zhang

Institute of Civil Engineering, Zhongyuan University of Technology, Zhengzhou, Henan, 451191, China
**Corresponding author:3014256211@qq.com*

Abstract: *In response to the consolidation problem of concrete pipe pile composite foundation, the theory of equal strain assumption is used to consider the radial and vertical seepage of soil without sand filling in the pipe, as well as the semi permeability of the pile-soil interface and the influence of load changes over time. The analytical solution for the consolidation of concrete pipe pile composite foundation is derived. The correctness and rationality of the solution have been verified through degradation research. Finally, the influence of various related factors on the consolidation characteristics of concrete pipe pile composite foundation was studied through parameter analysis. Research has shown that the faster the loading, the faster the consolidation of the composite foundation, with the fastest consolidation occurring during instantaneous loading; The faster the loading, the stronger the influence of permeability rate, permeable hole radius, smearing effect, and wellbore diameter ratio on the consolidation of composite foundation.*

Keywords: *permeable; Composite foundation; Time-varying load; consolidation*

1. Introduction

The method of adding vertical piles to reinforce soil has been widely used in soft soil foundation treatment in China.^[1-2] Pile bodies can be divided into drainage piles and non drainage piles based on their permeability.^[3] The permeability of drainage piles is very good, and this type of pile has a good drainage effect on pore water in soft soil foundation; The stiffness of undrained piles is relatively high, and this type of pile has a good effect on improving the bearing capacity of weak foundations.^[4-5] Permeable concrete pipe piles have semi permeability and can combine the advantages of two types of piles.

At present, many scholars have conducted research on permeable concrete pipe piles. In terms of experimental research, scholars mainly conduct research on permeable concrete pipe piles in terms of ultra static pore pressure, bearing performance, permeability, and permeability forms.^[6-12] The experimental results show that permeable concrete pipe piles have excellent characteristics in improving the bearing capacity and consolidation rate of the foundation. In terms of theoretical research, Tang Xiaowu et al. assumed that the flow inside the pile hole satisfies the Poiseuille pipe flow condition and derived an analytical solution for the consolidation of permeable concrete pipe pile composite foundation.^[13-14] Chen Junbo et al. considered the boundary drainage characteristics of the pile-soil interface and derived an analytical solution for the consolidation of permeable concrete pipe pile composite foundation.^[15] Chen Jie et al. assumed that the pile-soil interface was completely permeable and derived an analytical solution for the consolidation of permeable concrete pipe pile composite foundation.^[16] Han Bolin et al. considered the radial seepage of piles and derived the corresponding analytical solution for the consolidation of permeable concrete pipe pile composite foundation.^[17]

Time varying loads are an important factor affecting the consolidation characteristics of soft soil foundations. Hu Min et al. considered non Darcy seepage and derived a one-dimensional nonlinear consolidation equation for saturated soft soil foundation under variable load conditions.^[18] Shi Lantian et al. considered the spatiotemporal evolution of wellbore resistance and derived an analytical solution for the consolidation of vertical shaft foundations under variable load conditions.^[19] Ge Shangqi et al. considered the variation of external loads over time and derived a one-dimensional large deformation consolidation solution for soft soil foundations.^[20] Wu Sisi et al. considered the effect of variable loads and derived the one-dimensional consolidation equation of soft soil foundation under continuous

drainage boundary conditions, and solved it.^[21] Chen Yu et al. considered the initial hydraulic slope and established a one-dimensional consolidation equation for soft soil foundation under variable load conditions, and solved it.^[22]

In summary, the current research on the consolidation theory of concrete pipe pile composite foundation is not yet perfect. In response to this issue, based on the theory of equal strain assumption, considering the radial and vertical seepage of the soil without sand filling in the pipe, as well as the semi permeability of the pile-soil interface and the influence of load changes over time, an analytical solution for the consolidation of concrete pipe pile composite foundation was derived. The correctness and rationality of the solution have been verified through degradation research. Finally, the influence of various related factors on the consolidation characteristics of concrete pipe pile composite foundation was studied through parameter analysis.

2. Numerical calculation model

As shown in Figure 1, the analytical model constructed in this chapter consists of three parts: permeable concrete pipe piles, coating area, and influence area. In Figure 1: H is the thickness of the soft soil foundation and the design depth of the pipe pile; r is the radial coordinate; z is the vertical coordinate; r_d is the outer diameter of the pipe pile; r_s is the radius of the disturbance zone; r_c is the radius of the influence zone of the vertical shaft; k_s , k_h are the horizontal permeability coefficients of the soil in the applied area and the affected area, respectively; k_v is the vertical permeability coefficient of the foundation soil; γ_w is the severity of water; u_s , u_n are the excess pore pressure at any point within the soil mass of the applied and affected areas, respectively.

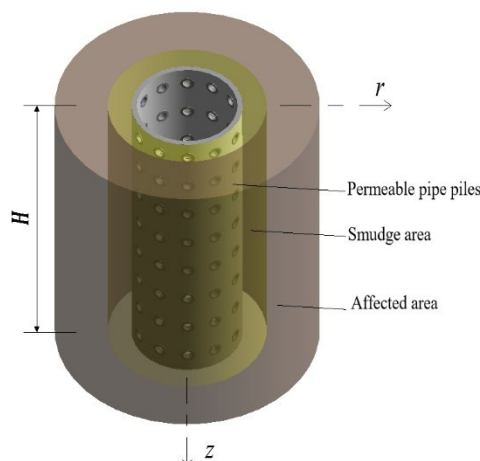


Figure 1: Analytical model for consolidation of permeable pipe pile composite foundation.

Considering the actual situation of the engineering site and the convenience of subsequent derivation, the basic assumptions are as follows:

- (1) The soil is saturated soil, and the soil particles and pore water are incompressible. The deformation of the soil is completely caused by the discharge of pore water;
- (2) Wait until the strain conditions are met;
- (3) The seepage of water in the soil completely conforms to Darcy's law;
- (4) When compressing the pile body, ignore the changes in the shape of the pile hole and blockage.

3. Deduction of Control Equations

According to Deng et al.'s research, the basic control equation for foundation consolidation is as follows^[23]:

$$\frac{k_s}{\gamma_w} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_s}{\partial r} \right) = D_\omega, \quad r_d \leq r \leq r_s \quad (1)$$

$$\frac{k_h}{\gamma_w} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_n}{\partial r} \right) = D_\omega, \quad r_s \leq r \leq r_e \quad (2)$$

Among them:

$$D_\omega = -\frac{k_v}{\gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} - \frac{\partial \varepsilon_v}{\partial t} \quad (3)$$

In the formula: ε_v is the volumetric strain of the soil.

Based on the equilibrium condition and the assumption of equal strain:

$$\pi(r_e^2 - r_d^2) \bar{\sigma}_s + \pi r_d^2 \bar{\sigma}_d = \pi r_e^2 q \quad (4)$$

$$\frac{\bar{\sigma}_s - \bar{u}}{E_s} = \frac{\bar{\sigma}_d}{E_e} = \varepsilon_v \quad (5)$$

$$\frac{\partial \varepsilon_v}{\partial t} = -\frac{1}{DE_s} \left(\frac{\partial \bar{u}}{\partial t} - \frac{dq}{dt} \right) \quad (6)$$

Among them:

$$q = g(t) q_u$$

E_e is the equivalent modulus of the pile considering the deformation of the pile body; E_s is the elastic modulus of the soil; $\bar{\sigma}_s$, $\bar{\sigma}_d$ are the average stresses borne by the soil and pile respectively in the composite foundation; q is the uniformly distributed variable load on the foundation surface, which is related to the time t ; $g(t)$ is a function that describes the loading process of a load, assuming the initial load is 0, that is $g(0) = 0$; q_u is the final value of load loading; \bar{u} is the average excess pore pressure of the soil; $Y = E_e / E_s$; $n = r_e / r_d$; $D = (n^2 - 1 + Y) / (n^2 - 1)$.

The average excess pore pressure of the soil in the analysis unit at any depth and any time is:

$$\bar{u} = \frac{1}{\pi(r_e^2 - r_d^2)} \left(\int_{r_d}^{r_s} 2\pi r u_s dr + \int_{r_s}^{r_e} 2\pi r u_n dr \right) \quad (7)$$

The analysis unit is not drained around, except for the radial boundary condition of the pile-soil interface, which is:

$$r = r_e, \quad \frac{\partial u_n}{\partial r} = 0 \quad (8)$$

$$r = r_s, \quad u_s = u_n \quad (9)$$

$$r = r_s, \quad k_s \frac{\partial u_s}{\partial r} = k_h \frac{\partial u_n}{\partial r} \quad (10)$$

According to Chen et al.'s research,^[24] the permeability performance of the pile-soil interface can be represented by the semi permeable boundary interface parameter TY. After considering the semi permeability of the pile-soil interface and disturbance effects, the boundary conditions of the pile-soil interface in this paper are as follows:

$$r = r_d, \frac{\partial u_s}{\partial r} - \mu \frac{u_s}{r_d} = 0 \tag{11}$$

Among them:

$$\mu = \frac{0.43 + 2.03e^{\frac{-\theta_s}{0.123}} + 0.55e^{\frac{-\theta_s}{0.76}} + 0.17e^{\frac{-\theta_s}{7.01}}}{\omega^{1.28} - 1.31 \times 10^{-3}} \rho_s$$

In the formula: r_h is the radius of the permeable hole in the pile body; ρ_s is the permeability rate of the pile body; m is the number of permeable holes in the pile body; $\theta_s = k_h / k_s$; $\rho_s = mr_h^2 / 2r_dH$; $\omega = r_h / r_d$.

Combining equations (1) and (2), and solving conditional equations (8) and (10), we obtain:

$$\frac{\partial u_s}{\partial r} = \frac{\gamma_w}{2k_s} \left(\frac{r_e^2}{r} - r \right) (-D_\omega), r_d \leq r \leq r_s \tag{12}$$

$$\frac{\partial u_n}{\partial r} = \frac{\gamma_w}{2k_h} \left(\frac{r_e^2}{r} - r \right) (-D_\omega), r_s \leq r \leq r_e \tag{13}$$

Combining equations (12), (13), and boundary condition equation (9) yields:

$$u_s = \frac{\gamma_w}{2k_s} G_w (-D_\omega) + u_s |_{r=r_d}, r_d \leq r \leq r_s \tag{14}$$

$$u_n = \frac{\gamma_w}{2k_h} G_s + \frac{\gamma_w}{2k_s} G_{ws} (-D_\omega) + u_s |_{r=r_d}, r_w \leq r \leq r_s \tag{15}$$

Among them:

$$G_w = r_e^2 \ln \frac{r}{r_w} - \frac{r^2 - r_w^2}{2}$$

$$G_s = r_e^2 \ln \frac{r}{r_s} - \frac{r^2 - r_s^2}{2}$$

$$G_{ws} = r_e^2 \ln s - \frac{r_s^2 - r_w^2}{2}$$

In the formula: $s = r_s / r_d$.

Substituting equations (14) and (15) into equation (7) yields:

$$\bar{u} = -\frac{1}{DE_s} \frac{\gamma_w}{k_h} \frac{r_e^2 F}{2} (-D_\omega) + u_s |_{r=r_d} \tag{16}$$

Among them:

$$F = \left(\ln \frac{n}{s} + \frac{k_h}{k_s} \ln s - \frac{3}{4} \right) \frac{n^2}{n^2 - 1} + \frac{s^2}{n^2 - 1} \cdot \left(1 - \frac{k_h}{k_s} \right) \left(1 - \frac{s^2}{4n^2} \right) + \frac{k_h}{k_s} \frac{1}{n^2 - 1} \left(1 - \frac{1}{4n^2} \right) \tag{17}$$

Substituting the values of equation (12) at time and equation (16) into equation (11) yields the governing equation for this paper:

$$\frac{\partial \bar{u}}{\partial t} - D \frac{E_s k_v}{\gamma_w} \left[\frac{\partial^2 \bar{u}}{\partial z^2} - \frac{\theta_v F_\mu}{H^2} \bar{u} \right] = \frac{dq}{dt} \tag{18}$$

Among them:

$$F_{\mu} = 2\mu\zeta^2 / [\theta_s(1 - 1/n^2) + \mu F]$$

In the formula: $\zeta = H / r_e$; $\theta_v = k_h / k_v$.

4. Solution and degradation verification of control equations

4.1. Solution to the control equation

The bottom of the foundation is not drained, and the upper part is drained on one side. The initial and vertical boundary conditions are:

$$z = 0, \bar{u} = 0 \tag{19}$$

$$z = H, \frac{\partial \bar{u}}{\partial z} = 0 \tag{20}$$

$$t = 0, \bar{u}(z, 0) = q_0 \tag{21}$$

In the formula: $q_0 = g(0)q_u = 0$, q_0 is the initial value of the load applied to the surface of the foundation.

According to the method of separating variables and impulse theorem, combined with boundary condition equations (19)~(21), the expression can be obtained:

$$\bar{u} = \int_0^t \frac{dq}{d\tau} \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{Mz}{H}\right) e^{-\alpha_m(t-\tau)} d\tau \tag{22}$$

Among them:

$$\alpha_m = \frac{Dk_v E_s}{H^2 \gamma_w} (M^2 + \theta_v F_{\mu}) \tag{23}$$

$$M = \frac{2m+1}{2} \pi, \quad m = 0, 1, 2, \dots \tag{24}$$

According to the definition of consolidation degree, the average consolidation degree of composite foundation can be expressed as:

$$\bar{U} = 1 - \frac{1}{q_u} \int_0^t \frac{dq}{d\tau} \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-\alpha_m(t-\tau)} d\tau \tag{25}$$

4.2. Discussion and degradation verification

(1) When the ground load is uniformly loaded at a single level, the load curve is shown in Figure 2.

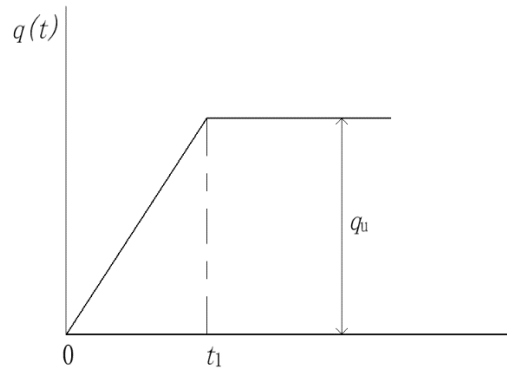


Figure 2: Schematic diagram of single-stage constant speed loading

The mathematical expression for single-stage constant velocity loading is:

$$g(t) = \begin{cases} \frac{q_u}{t_1} (0 \leq t \leq t_1) \\ 1 (t > t_1) \end{cases} \quad (26)$$

When $0 \leq t \leq t_1$,

$$\bar{u} = \frac{q_u}{t_1} \sum_{m=0}^{\infty} \frac{1}{\alpha_m} \frac{2}{M} \sin \frac{Mz}{H} (1 - e^{-\alpha_m t}) \quad (27)$$

$$\bar{U} = \frac{t}{t_1} - \frac{1}{t_1} \sum_{m=0}^{\infty} \frac{1}{\alpha_m} \frac{2}{M^2} (1 - e^{-\alpha_m t}) \quad (28)$$

When $t > t_1$,

$$\bar{u} = \frac{q_u}{t_1} \sum_{m=0}^{\infty} \frac{1}{\alpha_m} \frac{2}{M} \sin \frac{Mz}{H} (e^{\alpha_m(t_1-t)} - e^{-\alpha_m t}) \quad (29)$$

$$\bar{U} = 1 - \frac{1}{t_1} \sum_{m=0}^{\infty} \frac{1}{\alpha_m} \frac{2}{M^2} (e^{\alpha_m(t_1-t)} - e^{-\alpha_m t}) \quad (30)$$

(2) When the surface load of the foundation is not considered to vary over time, it remains constant at 1. At this point, equations (22) and (25) revert to:

$$\bar{u} = \sum_{m=0}^{\infty} \frac{2u_0}{M} \sin \left(\frac{Mz}{H} \right) e^{-\alpha_m t} \quad (31)$$

$$\bar{U} = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-\alpha_m t} \quad (32)$$

For the convenience of text calculation and drawing, let:

$$T_v = \frac{C_v t}{H^2} \quad (33)$$

$$T_{v1} = \frac{C_v t_1}{H^2} \quad (34)$$

$$\alpha_m t = D(M^2 + \theta_v F_\mu) T_v \quad (35)$$

$$\alpha_m t_1 = D(M^2 + \theta_v F_\mu) T_{v1} \tag{36}$$

In the formula: $C_v = E_s k_v / \gamma_w$; T_v , T_{v1} are a dimensionless time factor.

5. Consolidation behavior analysis

This article uses Fortran software to program the calculated analytical solution into a computer program, and uses the program to analyze the rationality and correctness of the obtained analytical solution. Select different parameters for quantitative analysis and explore the influence of various related factors on the consolidation characteristics of concrete pipe pile composite foundation under single-stage loading. The basic parameters selected in this article are shown in Table 1.

Table 1: Basic parameters.

$k_h/(m/s)$	$k_v/(m/s)$	Y	r_d/m	H/m	s
6×10^{-7}	3×10^{-7}	30	0.25	20	3

(1) The impact of T_{v1} on the consolidation process reflected in Figure 3 was compared with the consolidation curve under instantaneous loading. From the figure, it can be seen that the length of loading time has a significant impact on the consolidation process; The smaller the T_{v1} value, the faster the loading rate, and the faster the consolidation; During instantaneous loading, consolidation occurs the fastest.

(2) Figure 4 reflects the influence of T_{v1} on the consolidation process when n is different. From the graph, it can be seen that the smaller the n and T_{v1} values, the faster the consolidation rate; As the n value decreases, the influence of T_{v1} on the consolidation rate becomes more pronounced.

(3) Figure 5 reflects the influence of T_{v1} on the consolidation process when k_s / k_h is different. From the graph, it can be seen that the smaller the smearing effect, the smaller the T_{v1} value, and the faster the consolidation rate; As the smearing effect weakens, the influence of T_{v1} on the consolidation rate becomes more pronounced.

(4) Figure 6 reflects the influence of T_{v1} on the consolidation process when ρ_s is different. From the graph, it can be seen that the higher the permeability, the smaller the T_{v1} value, and the faster the consolidation rate; As the permeability increases, the influence of T_{v1} on the consolidation rate becomes more significant.

(5) Figure 7 reflects the influence of different ω and T_{v1} on the consolidation process. From the graph, it can be seen that the smaller the radius of the permeable hole, the smaller the T_{v1} value, and the faster the consolidation rate; As the radius of the permeable hole decreases, the influence of T_{v1} on the consolidation rate becomes more pronounced.

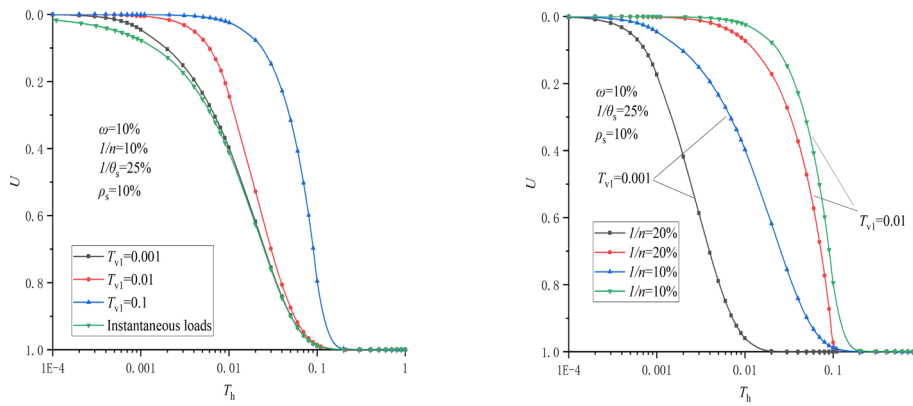


Figure 3: The influence of T_{v1} on the consolidation process (Left)

Figure 4: The influence of T_{v1} on the consolidation process at different times of n (Right)

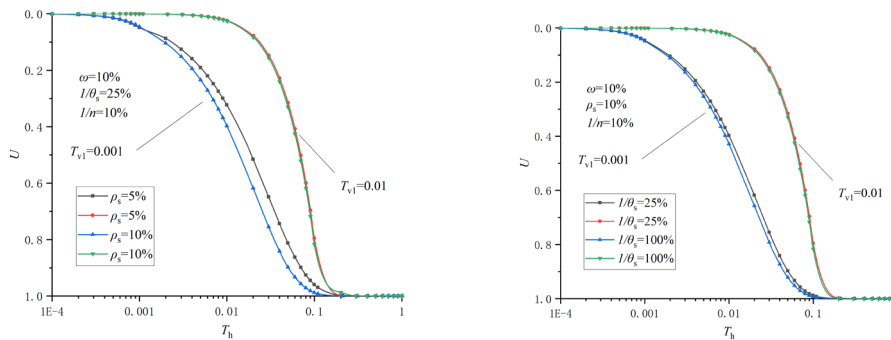


Figure 5: The influence of T_{v1} on the consolidation process at different times of k_s / k_h (Left)

Figure 6: The influence of T_{v1} on the consolidation process at different times of ρ_s (Right)

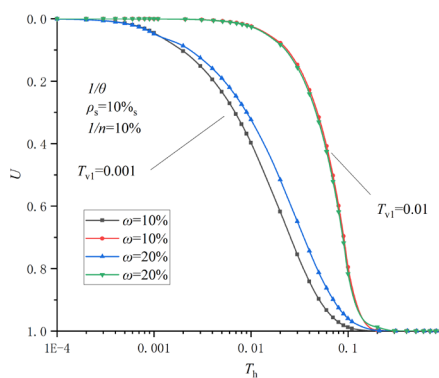


Figure 7: The influence of T_{v1} on the consolidation process at different times of ω

6. Conclusions

(1) In response to the consolidation problem of concrete pipe pile composite foundation, the theory of equal strain assumption is used to consider the radial and vertical seepage of soil without sand filling in the pipe, as well as the semi permeability of the pile-soil interface and the influence of load changes

over time. The analytical solution for the consolidation of concrete pipe pile composite foundation is derived. The correctness and rationality of the solution have been verified through degradation research. Finally, the influence of various related factors on the consolidation characteristics of concrete pipe pile composite foundation was studied through parameter analysis.

(2) The faster the loading (the smaller the T_{v1} value), the faster the consolidation of the concrete pipe pile composite foundation; During instantaneous loading, consolidation occurs the fastest.

(3) The higher the permeability rate, the smaller the permeable hole radius, the weaker the smearing effect, and the smaller the well diameter ratio, the faster the consolidation rate of the concrete pipe pile composite foundation; The faster the loading (the smaller the T_{v1} value), the stronger the influence of permeability rate (ρ_s), permeable hole radius (ω), coating effect (k_s / k_h), and well diameter ratio (n) on the consolidation of concrete pipe pile composite foundation.

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