ISSN 2616-5775 Vol. 2, Issue 1: 63-69, DOI: 10.25236/AJCIS.010018

Multi-objective Dynamic Programming Model for Three-dimensional Packing Problems

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ABSTRACT. *Natural disasters occur from time to time. In order to increase the resilience to sudden natural disasters, this paper takes Puerto Rico as an example to establish a three-dimensional packing problem model. First, we discussed the problem of medical bag and drones loading into containers, and constructed a gradually in-depth three-dimensional packing problem model. We systematically analyzed the whole process of packing, introduced 0-1 variables and other methods to describe the constraints mathematically, and constructed a multi-objective dynamic programming model for three-dimensional packing problems of medical packages, drones and containers.*

KEYWORDS: *0-1 variables, Multi-objective, Dynamic programming model*

1. Introduction

In 2017, the worst hurricane hit the United States territory of Puerto Rico, which caused severe damage to this area, resulting in dozens of areas isolated and without communication. With the rising demand for medical supplies, HELP.Inc, one of Non-governmental organizations (NGOs), designed a transportable disaster response system called “DroneGo” to deliver pre-packed medical supplies and provide high-resolution aerial video reconnaissance simultaneously or separately.

2. Initial model

First, we can establish model one through the multi-objective dynamic programming equation. For convenience, we refer to all drones and medical packages as cargo, and standard containers as containers. We simplify the model and discuss only a single homogeneous container type which can be used to deliver cargo. And the cargo are all the same kind, the same size, and the same shape. So this can be regarded as a simple three-dimensional Bin packing problem with a

ISSN 2616-5775 Vol. 2, Issue 1: 63-69, DOI: 10.25236/AJCIS.010018

single container and single cargo. Next, we construct decision variables, constraints, and objective functions [1].

Suppose the container is rectangular with length, width and height of L, W and H respectively, and the amount of container is I. So the container number is a set $\boldsymbol{I} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \dots, |\boldsymbol{I}|\})$. Suppose the cargo is rectangular with length, width and height of l, w, h respectively, and the amount of cargo is J. So the cargo number is a set $\boldsymbol{J} = \{\mathbf{1}, \mathbf{2}, \dots |\boldsymbol{J}|\}$.

$$x_i: \textbf{The quantity of cargos contained in container } \boldsymbol{i};$$

$$Y_{ij} = \begin{cases} 1, \textit{cargo } j \textit{ loaded into container } i; \\ 0, \textit{otherwise} \end{cases}$$

$$Y_i = \begin{cases} 1, \textit{container } i \textit{ is not empty} \left(\sum_{j=1}^{|j|} Y_{ij} > 0\right) \\ 0, \textit{otherwise} \end{cases}, \quad \frac{\sum_{j=1}^{|j|} Y_{ij}}{\sum_{j=1}^{|j|} Y_{ij} + 1} \le Y_i \le \sum_{j=1}^{|j|} Y_{ij}$$

$$l_{xj} = \begin{cases} 1, \textit{the length of the cargo } j \textit{ is parallel to the } x \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$l_{yj} = \begin{cases} 1, \textit{the length of the cargo } j \textit{ is parallel to the } y \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$l_{zj} = \begin{cases} 1, \textit{the length of the cargo } j \textit{ is parallel to the } z \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$w_{xj} = \begin{cases} 1, \textit{the width of the cargo } j \textit{ is parallel to the } x \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$w_{yj} = \begin{cases} 1, \textit{the width of the cargo } j \textit{ is parallel to the } y \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$w_{zj} = \begin{cases} 1, \textit{the width of the cargo } j \textit{ is parallel to the } z \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$h_{xj} = \begin{cases} 1, \textit{the height of the cargo } j \textit{ is parallel to the } x \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$h_{yj} = \begin{cases} 1, \textit{the height of the cargo } j \textit{ is parallel to the } y \textit{ axis} \\ 0, \textit{otherwise} \end{cases},$$

$$h_{zj} = \begin{cases} 1, \textit{the height of the cargo } j \textit{ is parallel to the } z \textit{ axis} \\ 0, \textit{otherwise} \end{cases}$$

There are the following relative position variables for any two pieces of caogos: $\mathbf{j_1}, \mathbf{j_2}$, the goal of the three-dimensional bin packing problem is to minimize the number of containers:

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$$\boldsymbol{min}\sum_{i=1}^{|I|}\boldsymbol{Y_i}$$

In conclusion, the model is:

$$\begin{cases}
\sum_{i=1}^{|2|}X_i=|J|,\sum_{i=1}^{|I|}Y_{ij}=1\\
l_{xj}+l_{yj}+l_{zj}=1,w_{xj}+w_{yj}+w_{zj}=1,h_{xj}+h_{yj}+h_{zj}=1\\
l_{xj}+w_{xj}+h_{xj}=1,l_{yj}+w_{yj}+h_{yj}=1,l_{zj}+w_{zj}+h_{zj}=1\\
Y_{ij}\cdot\left[x_{ij}+\left(l\cdot l_{xj}+w\cdot w_{xj}+h\cdot h_{xj}\right)\right]+f_{ijj_1}\cdot\left(l\cdot l_{xj_1}+w\cdot w_{xj_1}+h\cdot h_{xj_1}\right)\le L\\
Y_{ij}\cdot\left[y_{ij}+\left(l\cdot l_{yj}+w\cdot w_{yj}+h\cdot h_{yj}\right)\right]+r_{ijj_2}\cdot\left(l\cdot l_{yj_2}+w\cdot w_{yj_2}+h\cdot h_{yj_2}\right)\le W\\
Y_{ij}\cdot\left[z_{ij}+\left(l\cdot l_{zj}+w\cdot w_{zj}+h\cdot h_{zj}\right)\right]+u_{ijj_3}\cdot\left(l\cdot l_{zj_3}+w\cdot w_{zj_3}+h\cdot h_{zj_3}\right)\le H\\
u_{ij_tj_1}\cdot\left[x_{ij_{t_2}}-x_{ij_1}+l_{t_2}-x_{ij_{t0}j_1}\cdot\left(x_{ij_{t0}}-x_{ij_1}\right)\right]\ge s_{ij_{t2}j_1}\cdot\frac{1}{2}\cdot l_1\\
\frac{\sum_{J=1}^{|J|}Y_{ij}}{\sum_{J=1}^{|J|}Y_{ij}+1}\le Y_i\le\sum_{J=1}^{|J|}Y_{ij}\\
\mu_{ij_tj_1\cdot\left[y_{ij_{t3}}-y_{ij_1}+W_{t_3}-y_{ij_{t1}j_1\cdot\left(y_{ij_{t1}}-y_{ij_1}\right)}\right]\ge t_{ij_{t3}}}\\
i\in I;j,j_1,j_2,j_3\in J,j_t,j_{t0},j_{t1},j_{t2},j_{t3}\in K
\end{cases}$$

Therefore, we obtain the mathematical model for the simplest three-dimensional bin packing problem with a single container and single cargos. However, obviously such a model is not required for our model building, so next we will establish a single container, a variety of cargos packing model [2].

3. Improved Model

By analyzing the mathematical model of simple three-dimensional bin packing problem, we can further study and obtain a mathematical model that can solve the more complex three-dimensional packing problem with single cargo and multiple cargos.

Suppose the container is rectangular with length, width and height of L, W and H respectively, and the amount of container is I. So the container number is a set $I=\{1,2,\ldots\}$. Suppose the cargo $r(\sum r=R)$is rectangular with length, width and height of l_r,w_r,h_r respectively, and the amount of cargo is J_r. So the cargo number is a set $J_r=\{1,2,\ldots|J_r|\}$, $|J|=\sum_R|J_r|$, A collection of cargos numbered in descending order of volume$J=\{1,2,\ldots|J|\}$.

The optimization model for three-dimensional packing bin problems of containers, cargo specifications is established as follows:

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$$\begin{cases} \min \sum_{i=1}^{|I|} Y_i \\ \sum_{i=1}^{I} Y_{ij} \geq \sum_{n=1}^{|5|} R_{nj} \end{cases}$$

$$\begin{cases} \sum_{i=1}^{|I|} X_i = \sum_{r=1}^{R} |J_r|, \sum_{i=1}^{|I|} Y_{ij} = 1 \\ l_{xj} + l_{yj} + l_{zj} = 1, w_{xj} + w_{yj} + w_{zj} = 1, h_{xj} + h_{yj} + h_{zj} = 1 \\ l_{xj} + w_{xj} + h_{xj} = 1, l_{yj} + w_{yj} + h_{yj} = 1, l_{zj} + w_{zj} + h_{zj} = 1 \\ Y_{ij} \cdot \left[x_{ij} + \left(l \cdot l_{xj} + w \cdot w_{xj} + h \cdot h_{xj}\right)\right] + f_{ijj_1} \cdot \left(l \cdot l_{xj_1} + w \cdot w_{xj_1} + h \cdot h_{xj_1}\right) \leq L \\ Y_{ij} \cdot \left[y_{ij} + \left(l \cdot l_{yj} + w \cdot w_{yj} + h \cdot h_{yj}\right)\right] + r_{ijj_2} \cdot \left(l \cdot l_{yj_2} + w \cdot w_{yj_2} + h \cdot h_{yj_2}\right) \leq W \\ Y_{ij} \cdot \left[z_{ij} + \left(l \cdot l_{zj} + w \cdot w_{zj} + h \cdot h_{zj}\right)\right] + u_{ijj_3} \cdot \left(l \cdot l_{zj_3} + w \cdot w_{zj_3} + h \cdot h_{zj_3}\right) \leq H \\ u_{ij_tj_1} \cdot \left[x_{ij_{t_2}} - x_{ij_1} + l_{t_2} - x_{ij_{t0}j_1} \cdot \left(x_{ij_{t0}} - x_{ij_1}\right)\right] \geq s_{ij_{t2}j_1} \cdot \frac{1}{2} \cdot l_1 \\ \frac{\sum_{J=1}^{|J|} Y_{ij}}{\sum_{J=1}^{|J|} Y_{ij} + 1} \leq Y_i \leq \sum_{J=1}^{|J|} Y_{ij} \\ u_{ij_tj_1} \cdot \left[y_{ij_{t_3}} - y_{ij_1} + w_{t_3} - y_{ij_{t_1}j_1} \cdot \left(y_{ij_{t_1}} - y_{ij_1}\right)\right] \geq t_{ij_{t3}j_1} \cdot \frac{1}{2} \cdot w_1 \\ i \in I; j, j_1, j_2, j_3 \in J, j_t, j_{t0}, j_{t1}, j_{t2}, j_{t3} \in K \end{cases}$$

And so we have a model that we can use to figure out how we put the right amount of medical packages into the right amount of cargo bay [3].

The goal of the three-dimensional bin packing problem is to minimize the number of containers:

$$\min \sum_{i=1}^{|I|} Y_i$$

Meanwhile, for each container, we must ensure that there is at least one drone can take videos. From the table, we know only type seven drone can not take videos:

$$J_7 + \sum_{i=1}^{5} J_i \neq 0$$

We need to ensure that the number of cargos (medical package) is greater than the hospital needs:

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$$\sum_{i=1}^{\mathrm{k}} Y_{ij} \geq \sum_{n=1}^{|5|} R_{nj}$$

In this inequality, $\sum_{n=1}^{|5|} R_{nj}$ refers to the number of medical packages required by five hospitals, k refers to the types of medical packages.

Finally, to save costs the use of drones being transported should be reduced:

$$min \sum_{i=1}^{|t|} Y_{ij}$$

In conclusion, we can obtain the object function:

$$\begin{cases} min \sum_{i=1}^{|I|} Y_i \\ J_7 + \sum_{i=1}^{5} J_i \neq 0 \\ \sum_{i=1}^{\mathrm{k}} Y_{ij} \geq \sum_{n=1}^{|5|} R_{nj} \\ min \sum_{i=1}^{|t|} Y_{ij} \end{cases}$$

4. Model results

First we need to calculate the size of each cargos. As shown in the table below:

Table 1 The size of each cargos

Content(J_r)	Length(l_r)	Width(w_r)	High(h_r)	Volume
A	45	45	25	50625
B	30	30	22	19800
c	60	50	30	90000
d	25	20	25	12500
e	25	20	27	13500
f	40	40	25	40000
g	32	32	17	17408
MEDI	14	7	5	490
MEDII	5	8	5	200
MEDIII	12	7	4	336

We use the genetic algorithm to look for the optimal solution. Finding non - inferior solution set is the basic method of multi - objective programming? In order to solve the problem of multi-objective programming, the multi-objective problem is quantified into a single-objective problem, and then the non-inferior solution set is derived directly based on the genetic algorithm according to the preference information condition [4].We used MATLAB software to calculate the above models. We can obtain this figure about the optimization process:

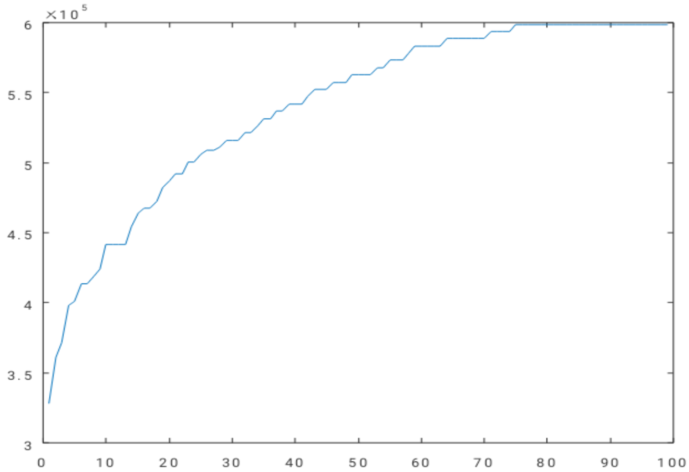

Figure. 1 The figure of the optimization process

We can see that the optimal value is basically reached around the 80th iteration. Meanwhile, we combine the data from the model for putting the medical packages into the cargo bay with this iterative model to get the following figure:

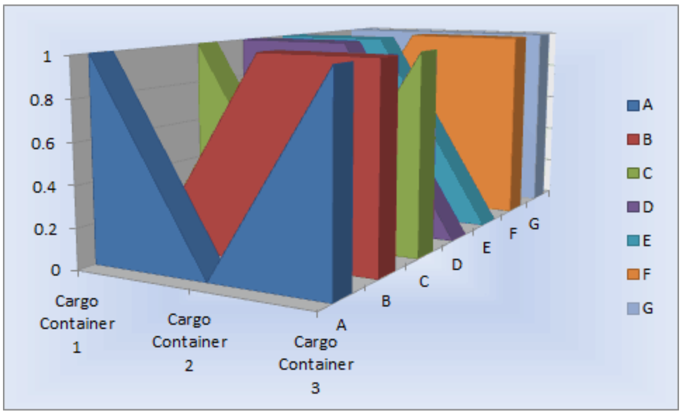

Figure. 2 Iterative model results

ISSN 2616-5775 Vol. 2, Issue 1: 63-69, DOI: 10.25236/AJCIS.010018

Finally, we obtain appropriate quantity and type of drone fleets and medical packages which can meet the requirements of the Puerto Rico hurricane scenario.

5. Conclusion

This paper takes Puerto Rico as an example to establish a three-dimensional packing problem model. We first consider the single type of goods, single type of box three-dimensional packing problem model. Then, a three - dimensional packing problem model for multi - type cargo and single - type container was constructed. And finally we establish the multi-objective dynamic programming model and the model is calculated based on the genetic algorithm by MATLAB.

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