ISSN 2616-5775 Vol. 4, Issue 8: 67-72, DOI: 10.25236/AJCIS.2021.040813

Optimal Design of FAST Paraboloid Deformation Strategy Based on Single Objective Optimization

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Abstract: *Firstly, this paper establishes a single objective optimization model with the goal of minimizing the area of parabola and arc in the lighting area. The parameters are traversed and solved within a reasonable range to obtain the equation of paraboloid. Then, using the method of coordinate system transformation, a single objective optimization model is established to minimize the distance from the adjusted main cable node to the ideal paraboloid. The shortest radial distance from the adjusted main cable node to the ideal paraboloid is taken as the objective function, and the expansion amount of the actuator and the position of the adjusted main cable node are taken as the decision variables to traverse each main cable node. Finally, the obtained ideal paraboloid, its vertex coordinates and the final position of the main cable node are inversely transformed by the above coordinate transformation to obtain the equation of the ideal paraboloid when the azimuth of the celestial body is 36.795 °and the elevation is 78.169 °, and the vertex coordinates are (- 49.3841, - 36.9373, - 294.4002).*

Keywords: *FAST, single objective optimization, Spatial coordinate system transformation, Dimension reduction method*

1. Introduction

China SkyEye 500 m aperture spherical reflector radio telescope (fast) is the largest single aperture radio telescope with the highest sensitivity in the world with independent intellectual property rights. With a unique station site, active reflector technology and light cable drag mechanism, it has broken through the 100 Meter Engineering limit of the full movable telescope. The Chinese celestial eye is composed of active reflector system, measurement and control system, feed support system, receiver and terminal and observation base. The main reflector system is an adjustable spherical surface composed of reflector panel, lower cable, actuator, main cable network and support structure. Active reflection bread includes two forms: reference state and working state. In the reference state, the active reflecting surface is a sphere with a radius of about 300m and a diameter of 500m, which is called the reference sphere, and C is the center of the reference sphere. In the working state, the reflector is adjusted to an approximate rotating paraboloid with a diameter of 300m, which is called the working paraboloid. When fast observes a celestial target s in an air position, the center of the receiving plane of the feed cabin is moved to the focus P of the straight line SC and the focal plane of the working paraboloid. Adjusting some reflection panels on the reference sphere can form an approximate rotating paraboloid with P as the focus and SC as the axis of symmetry, so that the reflection of parallel electromagnetic waves from the celestial target converges to the effective area of the feed cabin.

2. Journals reviewed

As the largest and most sensitive radio telescope in the world, Chinese celestial eye has been widely studied by scholars at home and abroad. Literature [1] proposed that each node in the whole fast network is coupled with each other, and its independent displacement control can not operate completely according to the theoretical model, and will be affected and disturbed by many factors. The displacement control model in the movement process is multivariable and nonlinear. The established node displacement control model should be the model after screening the strong correlation parameters and decorrelating, and finally get the simplified model. Reference [2] proposed that when the reference sphere of the reflector changes into an instantaneous paraboloid according to the proposed displacement strategy, the displacement of the main cable node changes in three directions: spherical radius direction, longitude direction and latitude direction. The

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displacement of the main cable node in three directions is related to the deformation strategy of the instantaneous paraboloid. Different deformation strategies correspond to the focal ratio F of different paraboloid equations and the relative position h between the paraboloid vertex and the reference sphere. Reference [3] proposed a new scheme of reflector structure system: global surface integral tension structure scheme.

3. Ideal paraboloid model

3.1. Evaluation of the model

According to the requirements, considering the adjustment factors of the reflective panel, the ideal parabola is obtained and the ideal paraboloid is determined. Therefore, the parameters h and P of the parabola are selected as the decision variables, where h is the offset between the apex of the parabola and the point on the corresponding sphere, R is the radius of the reference sphere, and P is the quasi focal length [4].

(1) Objective function

It is required to find the ideal paraboloid under the constraint of reflection panel adjustment, that is, the paraboloid with the minimum adjustment degree of reflection panel. For this purpose, the objective function is set as the minimum area composed of parabola and arc in the lighting area. The formula is as follows:

$$\min S = \int_{-150}^{150} \left| Z_1(x) - Z_2(x) \right| dx \tag{1}$$

Where, $Z_1(x)$ is a parabolic function equation, $Z_2(x)$ is the arc function equation.

(2) Constraints

① Parabolic equation constraints:

The general parabolic equation is: $x^2 = 2 \times p \times (z + c)$. Where p is the quasi focal length and c is the down shift distance. In order to make the model more practical, c is decomposed into R and h, and the parabolic equation here is:

$$x_1^2 = 2 \times p \times (z_1 + R + h) \tag{2}$$

Where R is the radius of the reference sphere and h is the offset of the point corresponding to the arc at the vertex of the ideal parabola.

② Arc equation constraint:

The arc takes the origin as the center and R as the radius. The arc equation can be written as:

$$x_2^2 + z_2^2 = R^2, \ -150 < x_2 < 150 \tag{3}$$

The focus of the parabola must be in the central area of the feed cabin. Establish a coordinate system with the spherical center as the origin. It can be determined that the abscissa of the focus is x = 0 and the ordinate is $z = -R \times (1 - f)$, where f is the focal diameter ratio and the value is 0.466. So the focus coordinates are (0, - 160.2). Therefore:

$$\frac{p}{2} - R - h = -160.2 \tag{4}$$

(3) To sum up, the problem model is as follows:

The objective function is:

$$\min S = \int_{-150}^{150} \left| Z_1(x) + Z_2(x) \right| dx \tag{5}$$

The constraint conditions and relations are:

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$$\begin{cases} x_1^2 = 2 \times p \times (z_1 + R + h) \\ x_2^2 + z_2^2 = R^2 \\ \frac{p}{2} - R - h = -160.2 \end{cases} \tag{6}$$

3.2. Solution of the model

The above relationship is solved by multivariate equation, and the above model is solved by setting the idea of minimum step size ergodic solution.

Since h is the offset of the point corresponding to the spherical surface at the vertex, it can be seen that the adjustment range of the actuator has almost the same constraints as the offset |h|. Therefore |h| must be less than 0.6, and the step size is 0.001.

Step 1: Take h=-0.6.

Step 2: After h is fixed, the quasi focal length p value is obtained.

Step 3: Get the value of the objective function s and save it.

Step 4: If h < 0.6, increase h by one step, turn to step 2, otherwise turn to step 5.

Step 5: Find the parabolic equation corresponding to the minimum value of S, which is the solution.

The algorithm and model are used to solve, and considering the adjustment factors of the reflective panel, the ideal paraboloid equation is finally determined as:

$$x^2 + y^2 = 2 \times 280.75 \times z + 168676 \tag{7}$$

The schematic diagram of ideal paraboloid is given:

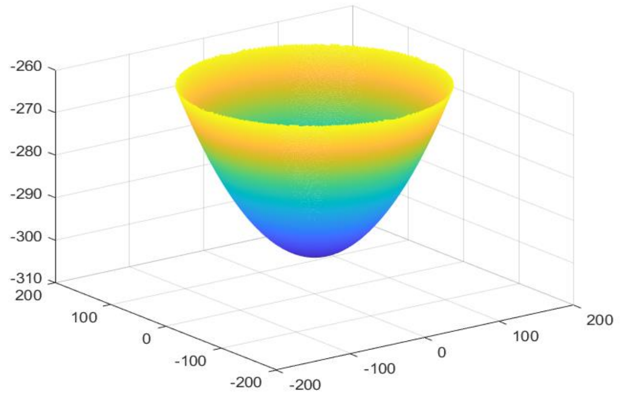

Figure 1: Schematic diagram of ideal paraboloid.

Through integral solution, the minimum difference s between the paraboloid area and the area obtained by circular arc rotation is 65.28. It can be seen from the figure that when the measured celestial body is above the reference sphere, the parabola rotates to obtain the paraboloid. The four vertices of the paraboloid are on the same horizontal plane, and its vertical projection is a circle. This paraboloid is an ideal paraboloid.

4. Ideal paraboloid model

4.1. Establishment of rotating axis coordinate system

After spatial coordinate transformation, the coordinate system * is recorded as a new coordinate system. In the case of $\alpha = 36.795°, \beta = 78.169°$, the following can be obtained through simple geometric analysis:

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$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} * \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, \quad c = \begin{pmatrix} 0.7838 & -0.5990 & 0.1642 \\ 0.5862 & 0.8008 & 0.1228 \\ -0.2050 & 0 & 0.9788 \end{pmatrix} \tag{8}$$

4.2. Evaluation of the model

(1) Objective function

The problem requires adjusting the expansion and contraction of the relevant actuator to make the reflector as close as possible to the ideal paraboloid, that is, the adjusted main cable node is closest to the ideal paraboloid. The objective function is set to:

$$\min = \left| M_i - Q_i \right| \tag{9}$$

Where M_i is the adjusted position coordinate of the ith main cable node, Q_i is the position coordinate of the ith main cable node corresponding to the ideal paraboloid, and I is the number set of main cable nodes in the lighting area.

(2) Constraints

① Lighting area constraints:

The lighting area is a part of the datum sphere, and the main cable node in the lighting area needs to be determined. The position coordinates of the main cable node in the annex are the coordinates in the original coordinate system. The spatial coordinates are transformed into the position coordinates in the coordinate system *. The ith main cable node is in the lighting area, and the formula is as follows:

$$\sqrt{\left(x'^2_i + y'^2_i \right)} \le 150 \tag{10}$$

② Radial constraint:

For any main cable node in the lighting area, its radial direction vector is the direction vector of the transformed main cable node coordinates and the spherical center O (0,0,0):

$$\overrightarrow{a_i} = \left(0 - x'_i, 0 - y'_i, 0 - z'_i \right) \tag{11}$$

③ Final position constraint of main cable node:

The final position is the sum of the initial position and the adjustment change of the actuator, that is:

$$M_i = N_i + L \times a_i \tag{12}$$

④ Actuator expansion constraint:

It can be seen that under the reference state, the radial expansion amount of the top of the actuator is 0, and its radial expansion range is -0.6 ~ + 0.6m. Namely:

$$-0.6 \le L \le 0.6 \tag{13}$$

⑤ Ideal paraboloid position constraint:

Let the parameter equation of the ith radial straight line be:

$$\begin{cases} x_i = x'_i + k_x t \\ y_i = y'_i + k_y t \\ z_i = z'_i + k_z t \end{cases} \tag{14}$$

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The equation of parabola in coordinate system * is:

$$x^2 + y^2 = 2 \times 280.75 \times z + 168676 \tag{15}$$

(3) To sum up, the model is as follows:

The objective function is:

$$\min = \left| M_i - Q_i \right| \tag{16}$$

The constraint conditions and relations are:

$$\begin{cases} \sqrt{\left(x'^2_i + y'^2_i\right)} \le 150 \\ I = \left\{ i \middle| \sqrt{x'^2_i + y'^2_i} \le 150 \right\} \\ a_i = \left(0 - x'_i, 0 - y'_i, 0 - z'_i\right)\left(i \in I\right) \\ M_i = N_i + L \times a_i \left(i \in I\right) \\ -0.6 \le L \le 0.6 \\ x_i = x'_i + k_x t \\ y_i = y'_i + k_y t \\ z_i = z'_i + k_z t \\ x^2 + y^2 = 2 \times 280.75 \times z + 168676 \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \times \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \end{cases} \tag{17}$$

The final position of the main cable node and the vertex coordinates of the ideal paraboloid are transformed into the original coordinate system by using the rotation axis formula.

4.3. Solution of the model

Perform the following steps for each main cable node:

Step 1: judge whether the main cable node is in the lighting area. If so, turn to step 2. Otherwise, replace the next main cable node and repeat step 1.

Step 2: get the radial direction vector corresponding to the main cable node.

Step 3: calculate the point Q_i on the ideal paraboloid corresponding to the main cable node.

Step4: multiple values of corresponding M_i are obtained when l traverses [- 0.6, 0.6].

Step 5: find the M_i that minimizes the objective function among the multiple M_i obtained, and save all the corresponding data.

Step 6: change to the next main cable node and turn back to step 1. If there is no next main cable node, it ends.

The final position and vertex coordinates of the main cable node are transformed into the original coordinate system using the rotation axis formula. The specific transformation formula is as follows:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} * \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \tag{18}$$

Solve and determine that the object to be measured is located on the ideal paraboloid with

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azimuth $\alpha = 36.795°$ and elevation $\beta = 78.169°$, as shown in the following figure:

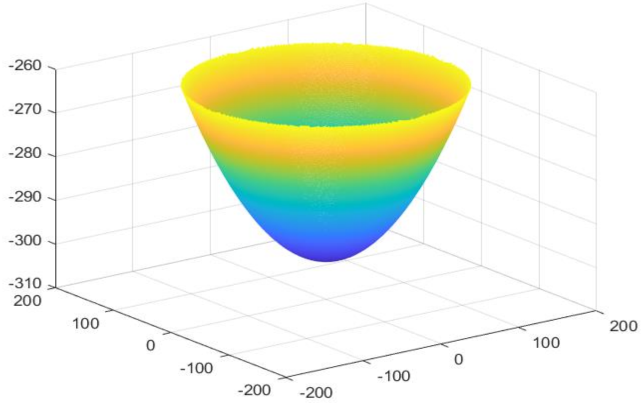

Figure 2: Ideal paraboloid after angle transformation.

Meanwhile, the equation of ideal paraboloid:

$$\begin{cases} \sqrt{x'^2 + z'^2} = 2 \times 280.75 \times z + 168676 \\ \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = c^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{cases} \tag{19}$$

The change of actuator expansion and contraction with the center distance of 300m caliber is almost consistent with strategy 2 in literature [5], indicating that the result is true and reliable.

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