

A Study of the Property Insurance Industry Based on the Property Insurance Underwriting Assessment (PIUA) Model

Minqi Lv[#], Jiahui Yin[#]

College of Control Science and Engineering, China University of Petroleum (East China), Qingdao, 266580, China

[#]These authors contributed equally.

Abstract: The frequent occurrence of extreme weather events has brought serious challenges to the world, and the insurance industry is facing a huge crisis. To determine whether the insurance industry should underwrite in areas with increasing extreme weather events, this paper establishes a profit prediction model based on the Cramer-Lundberg model and ARMA. Then, 7 extreme climate events, such as floods and wildfires, are selected from the collected data as secondary indicators, each divided into 5 indicators such as frequency and total loss. The AHP-EWM-CVM combination method is used to calculate the weights of each indicator, and a formula for the Extreme Weather Risk Index (EWRI) is constructed. All regions are classified into three classes severe, moderate, and slight risks, and a property insurance underwriting evaluation model is constructed. Then, the model is applied in two countries, China and Germany, to obtain the profit situation of the insurance industry in the next ten years, with EWRI of 82.8238 and 88.3605, respectively.

Keywords: ARMA, AHP-EWM-CVM, Property Insurance, Extreme Weather Risk Index

1. Introduction

Extreme weather events caused by climate change are having a very negative impact on the world today, and the situation will only get worse if people do not yet take the necessary measures.

Property owners and the insurance industry are facing serious challenges in the face of escalating extreme weather risks. Data from the National Oceanic and Atmospheric Administration (NOAA) shows that extreme weather events and natural disasters are causing significant losses for insurers. In the first half of 2023 alone, insurers lost more than \$50 billion in natural disaster coverage [1]. Record-breaking natural disaster losses have led to significant rate increases and capacity constraints in the personal and commercial property insurance markets. The American Property Casualty Insurance Association (APCIA) describes today's property insurance market as "the toughest market cycle in a generation" [2].

For property owners, on the one hand, the financial security of homeowners is at risk as more and more insurance companies cease operations in areas hit by natural disasters; on the other hand, when the cost of insurance is prohibitive, homeowners may decide not to take out insurance and, unless governmental or charitable assistance is put in place, they will bear the full brunt of the loss in the event of a disaster.

At present, the insurance industry is to some extent facing a dilemma: how to achieve a balance between the profitability of insurance companies and the affordability of property owners. If the two cannot be balanced, it may face a crisis, which will affect the sustainable development of the industry (data sources: <https://www.emdat.be/>).

This paper mainly establishes a profit prediction model based on the Cramer-Lundberg model and ARMA. At the same time, considering the different subjective and objective weighting methods, the AHP-EWM-CVM combination method is innovatively used to calculate the weights of each indicator, and a formula for calculating the Extreme Weather Risk Index (EWRI) is constructed. This paper analyzes the future profits and EWRI levels in different regions to determine whether the property insurance industry should underwrite in that region.

2. Property insurance underwriting assessment (PIUA) model

Extreme weather events can cause huge property losses and deaths, which will be transmitted to the insurance market through the "shock-transmission mechanism", and the insurance market is the first shockwave bearer of extreme weather events, and will face the severe challenges brought by extreme weather [3].

To enable the insurance industry to develop sustainably under extreme weather conditions, this paper establishes a profit forecasting model based on the Cramer-Lundberg model and Auto Regressive Moving Average (ARMA) Model and then builds an underwriting assessment model for the property-casualty insurance industry based on this model.

2.1 Profit forecasting model based on the Cramer-Lundberg model and ARMA

Based on the results of Cramer and Lundberg's study [4], the classical risk model has been referred to as the Cramer-Lundberg model, which is the most basic risk model in the process of bankruptcy theory research.

The classical risk model is defined as:

$$U(t) = u + C(t) - S(t), \forall t \geq 0 \quad (1)$$

Where, $U(t)$ is the profit of the insurance company up to the moment t ; u is known to be the initial fund of the insurance company; $C(t)$ is the total premium; $S(t)$ is the total claims and obeys a composite Poisson process with parameter λ ; $C(t) = ct$, while c denotes the premium income per unit of time, i.e., the premium rate. $S(t)$ is defined as:

$$S(t) = \sum_{i=1}^{N(t)} X_i \quad (2)$$

Where, X_i is the amount of claims; $N(t)$ is the counting process; $F(x)$ is the distribution function of claims with mean μ and variance $\sigma^2 < \infty$; and random processes $\{X_i, i = 1, 2, \dots\}$ and $\{N(t), t > 0\}$ are independent of each other.

This paper uses the data collected to derive the profitability of an insurance company for a particular year in the past and envisions using this model to predict whether an insurance company should write in a particular area in the coming year, i.e., whether it will be profitable. Therefore, this paper also adds an ARMA prediction model to the Cramer-Lundberg model. By analyzing the profit data calculated by the Cramer-Lundberg model for the years 2011-2021, the ARMA model is then used to predict the profits of the dataset ten years later.

First, since the time series data are found to be non-stationary, the difference method (the difference between two consecutive neighboring terms) is used for processing.

An autoregressive moving average model ARMA (p,q) is built [5-6]. If the time series x satisfies:

$$x_t - \phi_1 x_{t-1} - \dots - \phi_p x_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \quad (3)$$

Then the time series x is said to obey the autoregressive moving average model ARMA(p,q) of order (p,q).

Where, x_t is the value of the time series x at time t ; $\phi_1, \phi_2, \dots, \phi_p$ is the autoregressive coefficient; ε_t is a sequence of independently and identically distributed random variables, and satisfies: $\theta_1, \theta_2, \dots, \theta_q$ is the moving average coefficient.

2.2 Establishment of PIUA model

To determine whether insurance companies should provide insurance in areas with increased extreme weather events, this paper develops a three-level evaluation system. Considering that various evaluation methods have their limitations, a combination of hierarchical analysis method (AHP), entropy weight method (EWM) and coefficient of variation method (CVM) are used to determine the weights of each indicator [7].

2.2.1 Determination of Indicators and Their Weights

This paper first identifies the Extreme Weather Risk Index (EWRI) as a primary indicator and divides extreme weather into 5 categories in advance. To simplify the model, by analyzing the data, this paper finally chooses 7 extreme climate events, extreme temperature(ET), Flood(FD), Drought(DT), Storm(SM), Wildfire(WF), Earthquake(EQ), Mass movement(wet)(MM), as the second-level indicators.

For the risk of these events, by searching the literature and combining relevant data, this paper uses frequency, total number of events, total number of people affected, total number of people killed, and total number of people damaged (adjusted) as the third-level indicators of the model.

To simplify the model, this paper assumes that each extreme weather event has the same weights of the three-level indicators so that only the weight of the three-level indicator of one of the events needs to be found to represent all the events. The insurance evaluation indicator system is shown in Fig 1.

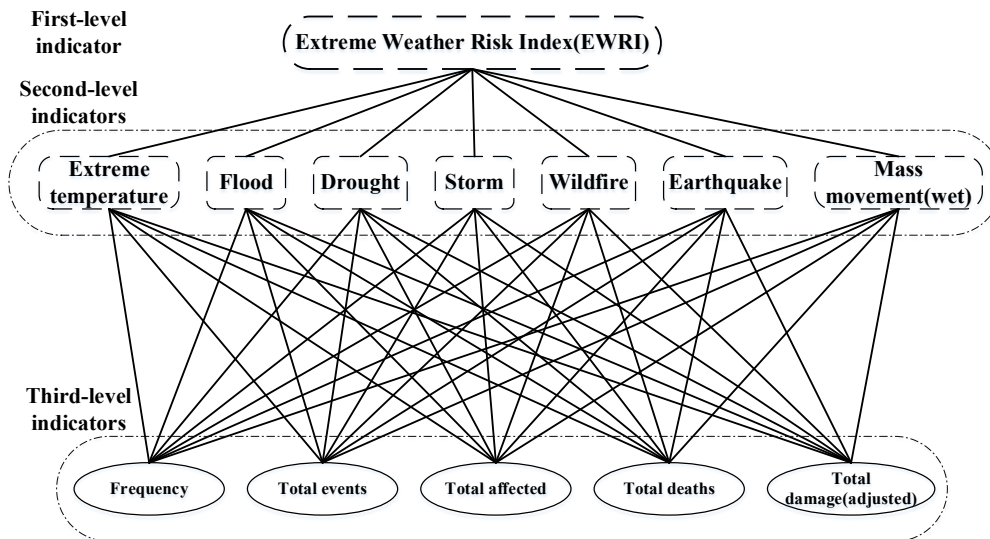


Figure 1: Insurance evaluation indicator system

Before calculating the weights, this paper first performs Data Normalization. Suppose n represents the number of extreme climate events for the second-level indicator, and m is the number of third-level indicators. Then this paper gets an $n \times m$ matrix, let it be X , where denotes the data in row i and column j .

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix} \quad (4)$$

Since all three levels of indicators in this paper are negative indicators, i.e., the smaller the data type, the better, they are treated as negative indicators:

$$x'_{ij} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)} \quad (5)$$

Where, x'_{ij} denotes the data in row i and column j after normalization; $\max(x_j)$ denotes the maximum data in column j ; $\min(x_j)$ denotes the minimum data in column j . After processing, all

indicators x_{ij} are converted to positive indicators between [0,1]. Next this paper calculates the weights of the indicators.

This paper first uses AHP to calculate the weights of the third-level indicators [8]. By finding the literature and analyzing the related data, each element is compared two by two and a judgment matrix is constructed:

$$A = \begin{bmatrix} 1 & 1/2 & 5 & 7 & 3 \\ 2 & 1 & 7 & 9 & 4 \\ 1/5 & 1/7 & 1 & 2 & 1/2 \\ 1/7 & 1/9 & 1/2 & 1 & 1/4 \\ 1/3 & 1/4 & 2 & 4 & 1 \end{bmatrix} \quad (6)$$

This paper uses MATLAB to analyze and find the weight vector between frequency, total events, total affected, total deaths, and total damage as $\omega_{AHP} = [0.30 \quad 0.47 \quad 0.07 \quad 0.04 \quad 0.12]$.

Then, this paper uses the entropy weight method (EWM) to calculate the weights of each indicator. Before calculating the weights, the indicators are normalized and a probability matrix is calculated as follows:

$$p_{ij} = \frac{x'_{ij}}{\sum_{i=1}^n x'_{ij}} \quad (7)$$

Where n denotes the number of extreme climate events for the second-level indicator and m is the number of third-level evaluation indicators.

Then, for the jth indicator, its information entropy is calculated as:

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^n p_{ij} \ln(p_{ij}), (j = 1, 2, \dots, m) \quad (8)$$

Based on the information entropy, this paper further calculates the weights of each indicator:

$$\omega_{EWM} = \frac{1 - e_j}{\sum_{i=1}^m (1 - e_j)} \quad (9)$$

After analyzing with MATLAB, the weight vector is obtained as $\omega_{EWM} = [0.18 \quad 0.18 \quad 0.24 \quad 0.22 \quad 0.18]$.

Besides, this paper uses CVM to find the weights again [9]. Because the standard deviation can describe the degree of dispersion of the values, this paper uses the standard deviation to define the weight of the indicators and calculate the coefficient of variation as follows:

$$V_j = \frac{S_j}{A_j} \quad (10)$$

Where, $A_j = \frac{1}{n} \sum_{i=1}^n x'_{ij}$ indicates the mean value for each indicator; $S_j = \sqrt{\frac{1}{n} \sum_{i=1}^n (x'_{ij} - A_j)^2}$ denotes the standard deviation of each indicator.

This paper further calculates the weights:

$$\omega_{CVM} = \frac{V_j}{\sum_{j=1}^n V_j} \quad (11)$$

The same is analyzed using MATLAB to get $\omega_{CVM} = [0.19 \ 0.19 \ 0.23 \ 0.21 \ 0.18]$.

After calculating the weights of these three different assignment methods, they are simply weighted and averaged to finally obtain the average weight of the three-level indicator as:

$$\begin{aligned} \bar{\omega} &= \frac{\omega_{AHP} + \omega_{EWM} + \omega_{CVM}}{3} \\ &= [0.22 \ 0.28 \ 0.18 \ 0.16 \ 0.16] \end{aligned} \quad (12)$$

On this basis, the scores for the seven natural hazards in the secondary indicators are averaged to obtain the weights as $\omega = [0.18 \ 0.06 \ 0.16 \ 0.07 \ 0.20 \ 0.14 \ 0.19]$. The weights of the secondary and tertiary indicators are shown in Fig 2.

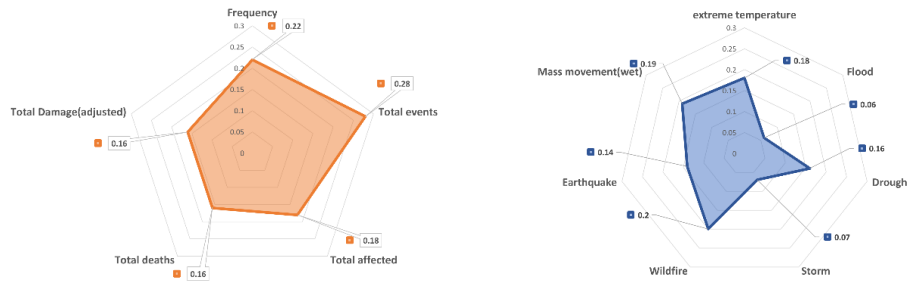


Figure 2: Weights of indicators

2.2.2 Calculation of EWRI

Based on the weights derived from the combination weighting method described above, the formula for EWRI is constructed:

$$\begin{aligned} EWRI &= 100 \cdot (\omega_{ET} \cdot ET + \omega_{FD} \cdot FD + \omega_{DT} \cdot DT + \\ &\quad \omega_{SM} \cdot SM + \omega_{WF} \cdot WF + \omega_{EQ} \cdot EQ + \omega_{MM} \cdot MM) \end{aligned} \quad (13)$$

Where, $\omega_{ET}, \omega_{FD}, \omega_{DT}, \omega_{SM}, \omega_{WF}, \omega_{EQ}, \omega_{MM}$ denote the weights of each of the second-level indicators obtained above; ET, FD, DT, SM, WF, EQ, and MM have the following formula:

$$\begin{cases} ET = \bar{\omega}_1 \cdot ET_1 + \bar{\omega}_2 \cdot ET_2 + \bar{\omega}_3 \cdot ET_3 + \bar{\omega}_4 \cdot ET_4 + \bar{\omega}_5 \cdot ET_5 \\ FD = \bar{\omega}_1 \cdot FD_1 + \bar{\omega}_2 \cdot FD_2 + \bar{\omega}_3 \cdot FD_3 + \bar{\omega}_4 \cdot FD_4 + \bar{\omega}_5 \cdot FD_5 \\ DT = \bar{\omega}_1 \cdot DT_1 + \bar{\omega}_2 \cdot DT_2 + \bar{\omega}_3 \cdot DT_3 + \bar{\omega}_4 \cdot DT_4 + \bar{\omega}_5 \cdot DT_5 \\ \vdots \\ MM = \bar{\omega}_1 \cdot MM_1 + \bar{\omega}_2 \cdot MM_2 + \bar{\omega}_3 \cdot MM_3 + \bar{\omega}_4 \cdot MM_4 + \bar{\omega}_5 \cdot MM_5 \end{cases} \quad (14)$$

Where, $\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3, \bar{\omega}_4, \bar{\omega}_5$ denote the weights of each of the third-level indicators obtained above, and the weight is the same for any weather event; Taking ET as an example, $ET_1, ET_2, ET_3, ET_4, ET_5$ denote frequency, total events, total affected, total deaths, and total damage(adjusted), respectively. Each of the other rows corresponds to ET and represents the same metric.

This paper substitutes the data organized in the previous section into the established model to find out the EWRI of each country. Then K-means algorithm is used to categorize all the countries into three classes, Class I: Severe; Class II: Moderate; and Class III: Slight. The specific classification criteria are

shown in Table 1.

Table 1: Classification criteria for EWRI

Class	I	II	III
EWRI	Severe	Moderate	Slight
	<15	15~85	>85

From this table, it is possible to find the EWRI for any country to determine its extreme weather risk level.

2.2.3 Criteria for insurance risk-taking under the model

After the model was built, this paper further analyzes the model to derive the following criteria for the insurance industry covered by the model.

The standards established in this paper are very simple: regardless of the EWRI in the region, insurers will underwrite if they make a profit, i.e., $U(t) > 0$. In the event of a loss, the insurer is at risk of insolvency.

In the case of "insuring the insurance industry", insurance companies can choose to take risks. It is necessary to build a perfect risk protection system consisting of a direct insurance company + catastrophe co-insurance body + reinsurance company + capital market + catastrophe special fund [10], i.e., to establish a multi-level risk diversification mechanism. Thus realizing the risk transfer of insurance companies under catastrophe and maintaining the sustainability solvency of the insurance industry.

The relevant members mentioned above need to make the following initiatives to share risks with insurance companies.

(1) The direct insurance company continuously innovates and improves its insurance product innovation system, and realizes the transition from post claim settlement to pre-event prevention and emergency response, gradually strengthening the service of reducing catastrophic risks.

(2) Major insurance companies have jointly funded the formation of catastrophe co-insurers to spread catastrophe risks.

(3) Reinsurance companies, on the one hand, utilize the additional underwriting capacity of reinsurance, and on the other hand, can diversify risks to the international market, thereby enhancing the overall underwriting capacity of the insurance industry.

(4) In the capital market, the issuance of catastrophe bonds (bonds with returns linked to designated catastrophic losses) involves trading some of the underwriting risks of insurance companies as tradable asset categories in the capital market. This serves both the purpose of fundraising in the capital market and the transfer of risks to the capital market.

To summarize, the risk of insurance companies has been greatly reduced, so they can choose to bear the risk.

2.3 Application of the PIUA model

In this section, this paper chooses two countries, China and Germany, which are located on different continents and do not have the same level of frequency of extreme weather events, to demonstrate the feasibility of the PIUA model.

This paper substitutes the premium data collated for China and Germany (which have been converted to US dollars) into the Cramer-Lundberg model to get the profitability of China and Germany over 11 years.

Then it is substituted into the ARMA model to get the profitability in the next ten years. Profit forecast for China and Germany in the next ten years are shown in Fig 3.

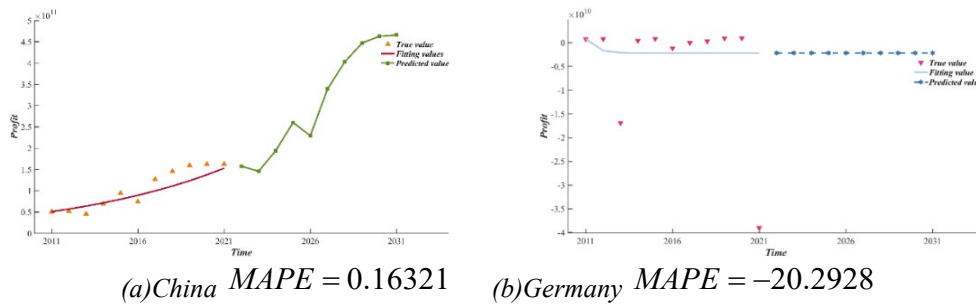


Figure 3: Profit forecast in the next ten years

3. Conclusions

With the increasing number of extreme weather events, the difficulty of underwriting in the property insurance industry is increasing, and the sustainability of property insurance deserves our attention. This paper constructs a Property Insurance Underwriting Assessment (PIUA) model using Profit Forecasting Model Based on the Cramer Lundberg Model, ARMA, and combination weighting method. When evaluating the risk of extreme weather, this paper adopts the integrated assessment method of Cramer-Lundberg and ARMA models, which makes the results more convincing. The experimental results show that the PIUA model has good predictive and robustness, and has certain practical application value.

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