Optimal Design Model of Thermal Protective Clothing Based on Heat Conduction Difference Equation

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ABSTRACT. In this paper, we study the temperature distribution of protective clothing when the design is known, and find the optimal design of protective clothing under different temperature protection requirements.

KEYWORDS: Thermal protective clothing design, One-dimensional heat conduction model, Difference equation, Heat balance analysis, Dichotomy search

1. Question Analysis

This study investigates the temperature distribution in the case of an experience temperature of 75°C, a thickness of II layer of 6 mm, a thickness of IV layer of 5 mm, and a working time of 90 minutes. Firstly, the problem is equivalent to one-dimensional heat conduction, and the appropriate layer thickness and time step are determined. According to the thermodynamic formula and Fourier's law, the heat conduction difference equation is derived, that is, the recursive equations of temperature at each grid change with time. Due to the small change in the surface temperature of the left and right sides of the layer structure in the time micro-element, it is necessary to correct the equations. Considering the boundary transition layer of the difference equation, a skin layer with a unit thickness is placed at the skin of the dummy, and the equivalent thermal conductivity of the layer is obtained according to the steady state condition of the heat conduction process. The initial temperature value of each surface is determined at the time, and the temperature distribution is derived from the recursive equation of the temperature distribution along time.

The area heat flow rate \( \dot{q}(x,t) \) is used to indicate the heat per unit area passing through the unit area per time, which is used to characterize the heat conduction; the function \( T(x,t) \) is used to indicate the temperature at the position \( x \) at time \( t \). According to the law of Fourier's thermodynamics, the temperature at which the i-th
layer rises, that is, the temperature difference $\Delta T_i$ between the left and right sides of the i-th layer is:

$$
\Delta T_i = \frac{k_i(T_{r,i} - T_i) - k_{r,i}(T_i - T_{l,i})}{\rho c_i(\Delta x)^2} \Delta t
$$

(1)

As can be seen from the title, the temperature changes with time. In the discrete process, in the selected micro-element $\Delta t$, the temperature $T_i$ of the i-th surface is treated as a certain value, and the temperature of each surface needs to be corrected to obtain the temperature $T_i'$ of the i-th layer. A recursive equation for the temperature change over time at each grid after correction:

$$
\Delta T_i' = \frac{k_i[T_{r,i} - T_i + \frac{1}{2}(\Delta T_{r,i} - \Delta T_i)] - k_{r,i}[T_i - T_{l,i} + \frac{1}{2}(\Delta T_i - \Delta T_{l,i})]}{\rho c_i(\Delta x)^2} \Delta t
$$

(2)

The function image of the outer skin temperature as a function of time is as follows:

![Temperature Curve of Lateral Skin](image)

**Figure. I Temperature Curve of Lateral Skin**

2. Establishment and Solution of the Optimal Thickness of II layer Material

When the experience temperature is 65°C, the thickness of the IV layer is 5.5 mm, and the working time is 60 minutes, the outer layer of the skin of the dummy does not exceed 47°C, and the time exceeds 44°C for less than 5 minutes. The optimal thickness of the material is an optimization design problem.
Only by changing the value of \( d_2 \), a series of curves of the skin outer temperature \( T_n \) as a function of time \( t \) are shown in Fig. 1.

Taking the thickness \( d_2 \) of the II layer as the search variable, using the dichotomy method with the smaller precision \( \Delta d \) as the standard, the thickness value satisfying the requirement is approximated, and the thickness \( d_2 \) of the II layer which satisfies the constraint condition is obtained as the minimum thickness. The specific steps are as follows:

A. manually determine the approximate range \([a, b]\) of the thickness of the II layer, \( d_2 = \frac{a + b}{2} \) as the initial value;

B. Substituting \( d_2 \) into the recursive equation of the grid temperature with time, and solving the temperature distribution under the newly known condition;

C. Judging whether the \( T_n \) is less than or equal to 47°C at the 60th minute, if it is less than, then proceed to the next judgment; otherwise, the value of \( d_2 \) is not large enough, update the value of \( d_2 \), repeat a, b;

D. Determine whether \( T_n \) is less than or equal to 44°C at the 55th minute. If it is less than, proceed to the next step to determine whether the search interval precision \( \Delta d = |a - b| \) at this time meets the requirements; otherwise, the value of \( d_2 \) is not
large enough, update the value of $d_2$, repeat a, b, c;

E. If the search interval precision $\Delta d = |a - b|$ meets the requirements, proceed to the next step; if not, then the value of $d_2$ is the upper bound, the $d_2$ value is decreased, and repeat the above steps;

F. Recording the layer II thickness value $d_2 = 16.2$ mm which just satisfies the above conditions at this time is the optimal solution.

3. Establishment and Solution of the Optimal Thickness of II, IV layer Material

When the experience temperature is 80°C, and the working time is 30 minutes, the outer layer of the skin of the dummy does not exceed 47°C, and the time exceeds 44°C for less than 5 minutes. The optimal thickness of the material is an optimization design problem.

\[
\begin{aligned}
\min & \quad d_1 + d_4 \\
\text{s.t.} & \begin{cases}
T_{n, 25} \leq 44, \\
T_{n, 30} \leq 47.
\end{cases}
\end{aligned}
\tag{4}
\]

In combination with the above, the optimal thickness of layer II is $d_2 = 17.8$ mm, and the optimal thickness of layer IV is $d_4 = 6.2$ mm, corresponding to $T_{n, 30} = 44.7561°C$ outside the skin at 30 minutes and $T_{n, 25} = 43.9122°C$ at 25 minutes, meeting the design requirements.

4. Model promotion

The model is designed for high-temperature work-specific protective clothing, which can be used for heat insulation in high-temperature environments and for heat preservation in low-temperature environments. According to the different external environment temperature, applying the built model, the protective clothing design scheme adapted to the environment can be obtained. At the same time, it is also possible to promote clothing design in a variety of special fields, such as the design of aerospace anti-pressure clothing and the design of fire-fighting clothing.

The model design scheme can also be extended to the storage of articles, and the special storage space for the items with high environmental temperature requirements is designed to meet the temperature requirement; even in the product processing industry, the design of a specific detachable insulation layer for a specific link greatly facilitates the production life.
References