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Analysis of Relationships between Common Distributions Based on Computing Science

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Abstract: *Random variables are the most fundamental concept in probability theory and mathematical statistics. Each random variable can have a distribution, different random variables can have different distributions, and different random variables can have the same distribution. There are many random variables, and there are many distributions that can be formed, but there are not many commonly used distributions. In general probability theory and mathematical statistics textbooks, each distribution is explained separately, and the relationship between distributions is occasionally discussed, and there is no detailed description. Establishing the correspondence between distributions plays an important role in mastering the theory and relationship of common distributions. This paper studies the relationship between distributions, and establishes some common distribution diagrams on the basis of distribution relationships, so as to illustrate the connections between some distributions with distribution diagrams.*

Keywords: *random variables, probability distributions, discrete distributions, continuous distributions, correspondence*

1. Introduction

Common distributions play a huge role in solving problems in probability theory and mathematical statistics. The distribution does not exist in isolation, there are some connections between distributions and distributions, and many scholars have studied some relationships between distributions.

Zhang Dan [1] In the article on the relationship between several common probability and statistics distributions, some correspondences between binomial distributions, Poisson distributions, normal distributions, hypergeometric distributions, and exponential distributions are mainly discussed. High color [2] In the study of gamma probability distribution properties, the relationship between gamma distribution, exponential distribution, order r Airron distribution, etc. was analyzed. Zhu Fang [3] In the article to study the relationship between exponential distribution and other distributions based on MGF, the relationship between exponential distribution, Poisson distribution, and Laplace distribution was discussed by using the kinetic difference generation function. Zhao Tianyu [4] used the parent function as a research tool in the study of the probability distribution of commonly used discrete random variables based on the parent function, and discussed the relationship between the probability distributions of commonly used discrete random variables. Wang Lifang [5] studied the close relationship between binomial distribution, Poisson distribution and exponential distribution, and studied the internal relationship between these distributions in the form of limit distribution. Jing Yingchuan [6] discusses the characteristics of some commonly used continuous distributions and the relationship between distributions in the summary of the properties of commonly used continuous distributions and their relationships. Hou Wen [7] and Tao Huiqiang [8] analyzed the relationship between commonly used probability distributions and drew a comprehensive relationship diagram. This paper further refines the relationship between distributions on the basis of previous research. The relationship between common distributions is analyzed from four aspects: limit equality, functional relationship, special and general relationship, and relationship of the same nature. In the research, some theories and concepts are used to refer to Mao Shisong's [9] book "Probability Theory and Mathematical Statistics Course".

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2. Preliminaries

2.1 Distribution of common discrete random variables

(1) Single point distribution (degradation distribution) $P(X=c)=1$

(2) Two-point distribution ($0-1$ distribution) $X \sim b(1,p)$

(3) Binomial distribution $X \sim b(n,p)$

(4) Poisson distribution $X \sim p(\lambda)$

(5) Hypergeometric distribution $X \sim h(n,N,M)$

(6) Geometric distribution $X \sim Ge(p)$

(7) Negative binomial distribution (Pascal distribution) $X \sim Nb(r,p)$

2.2 Distribution of common continuous random variables

(8) Standard normal distribution $X \sim N(0,1)$

(9) Normal distribution $X \sim N(\mu,\sigma^2)$

(10) Uniform distribution $X \sim U(a,b)$

(11) Exponential distribution $X \sim \exp(\lambda)$

(12) Gamma distribution $X \sim Ga(\alpha,\lambda)$

(13) Beta distribution $X \sim Be(a,b)$

(14) χ^2 Distribution $X \sim \chi^2(n)$

(15) F Distribution $X \sim F(m,n)$

(16) t Distribution $X \sim t(n)$

(17) Cauchy distribution $X \sim C_{au}(\mu,\lambda)$

(18) Standard Cauchy distribution $X \sim C_{au}(0,1)$

3. Main conclusion the relationship between distributions

3.1 The limit relationship between distributions

(1) Poisson approximation of binomial distribution (Poisson's theorem)

$X \sim b(n,p) \to p(\lambda)$, approximate for n Very Large, p Very Small.

(2) Normal distribution approximation of binomial distribution (Demovre–Laplace central limit theorem)

$X \sim b(n,p) \to N(np,np(1-p))$, approximate for $n \to +\infty$.

(3) Normal approximation of the Poisson distribution

$X \sim p(\lambda) \to N(\lambda,\lambda)$, approximate for $\lambda \to +\infty$.

(4) Binomial approximation of hypergeometric distributions

$X \sim h(n,N,M) \to b\left(n,\frac{M}{N}\right)$, approximate for $n << N$.

(5) Standard normal approximation of the t-distribution

$X \sim t(n) \to N(0,1)$, approximate for $n << N$.

(6) Lindberg–Levy central limit theorem

$\{X_n\}$ is a sequence of independent and homogeneous random variables, regardless of whether $\{X_n\}$ is a discrete or continuous sequence of random variables,

$\sum_{n=1}^{+\infty} X_n \to N(n\mu,n\sigma^2)$, $E(X_i)=\mu,\ D(X_i)=\sigma^2>0$, approximate for $n \to +\infty$.

In other central limit theorems, there are also some distributions whose limit distributions are normal, and in statistics the limit distributions of medians and quantiles are also normal, in fact, many

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distributions and variables are ultimately normal.

3.2 Functional relationships between distributions

Functions of random variables can form distributions, so there are functional relationships between some distributions. This article mainly discusses some ordinary function relationships corresponding to the relationship between independent random variables and functions and distributions.

3.2.1 Independent random variables and functions

(1) The sum of r independent and homogeneous distributions $Ge(p)$ constitutes $Nb(r,p)$,

$X_i \sim Ge(p),\ X_i,\ iid,\ \sum_{i=1}^{r} X_i \sim Nb(r,p)$.

(2) the sum of n independent and identical $b(1,p)$ distributions $b(n,p)$,

$X_i \sim b(1,p),\ X_i,\ iid,\ \sum_{i=1}^{n} X_i \sim b(n,p)$.

(3) the sum of n independent and identical $\chi^2(1)$ distributions $\chi^2(n)$,

$X_i \sim x^2(1),\ X_i,\ iid,\ \sum_{i=1}^{n} X_i \sim x^2(n)$.

(4) the sum of n independent $\exp(\lambda)$ distributions $Ga(n,\lambda)$,

$X_i \sim \exp(\lambda),\ X_i,\ iid,\ \sum_{i=1}^{n} X_i \sim Ga(n,\lambda)$.

3.3.2 Other functional relationships

(1) $X \sim N(0,1),\ Y = X^2 \sim \chi^2(n)$.

(2) $X_i \sim N(0,1),\ t = \dfrac{X_i}{\sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n}}} \sim t(n)$.

(3) $X \sim t(n),\ t^2 \sim F(1,n)$.

(4) $X \sim F(m,n),\ Y = \dfrac{1}{X} \sim F(n,m)$.

(5) $X \sim N(\mu,\sigma^2),\ Y = \dfrac{X-\mu}{\sigma} \sim N(0,1)$.

(6) $X \sim U(0,1),\ Y = -2\ln X,\ Y \sim \exp\left(\dfrac{1}{2}\right)$.

(7) $X \sim Ga(\alpha_1,\lambda),\ Y \sim Ga(\alpha_2,\lambda),\ V = \dfrac{X}{X+Y} \sim Be(\alpha_1,\alpha_2)$.

(8) $X \sim p(\lambda_1),\ Y \sim p(\lambda_2),\ V = \dfrac{X}{X+Y} \sim b\left(n,\dfrac{\lambda_1}{\lambda_1+\lambda_2}\right)$.

(9) $X \sim N(0,1),\ Y \sim N(0,1),\ X,Y$ independent, $\dfrac{X}{|Y|} \sim C_{au}(1,0)$.

(10) $X \sim F(2,2),\ Y\dfrac{X}{1+X},\ Y \sim U(0,1)$.

(11) $X \sim \exp(\lambda),\ Y = [X+1],\ Y \sim Ge\left(1-e^{-\lambda}\right)$.

(12) $X_i \sim N(0,1),\ Y = \sum_{i=1}^{n} X_i^2 \sim \chi^2(n)$.

(13) $X \sim \chi^2(m)$, $Y \sim \chi^2(n)$, $F = \dfrac{\frac{X}{m}}{\frac{Y}{n}} \sim F(m,n)$.

(14) $X_1 \sim N(0,1)$, $X_2 \sim \chi^2(n)$, $t = \dfrac{X_1}{\sqrt{\frac{X_2}{n}}} \sim t(n)$.

(15) $X \sim N(\mu,\sigma^2)$, $Y = e^X$, $Y \sim LN(\mu,\sigma^2)$ Lognormal distribution.

(16) $X \sim F(n,m)$, $\dfrac{1}{1+X} \sim Be\left(\dfrac{m}{2},\dfrac{n}{2}\right)$.

(17) $X_i \sim N(0,\sigma^2)$, X_1, X_2 Independent with distribution, $Y = \sqrt{X_1^2 + X_2^2}$ is a Rayleigh distribution.

(18) $\theta \sim U\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$, $X = \tan\theta \sim Gau(0,1)$ is a standard Cauchy distribution.

(19) $X \sim W(\alpha, r, \mu)$, $Y = \dfrac{X-\mu}{\alpha}$, $Y \sim \exp(1)$.

(20) $X \sim F_X(x)$, $Y = F_X(x)$, $F_X(x)$ is a strictly monotonic increasing function, if its inverse $F_X^{-1}(y)$ exists, there is $Y \sim U(0,1)$.

(21) $X \sim C_{au}(\mu,\lambda)$, $\text{Y}=\dfrac{1}{X} \sim C_{au}(\mu,\lambda)$.

3.3 Special vs general relationship

When the variables in some distributions change, the distribution can be reduced to some commonly used simple distributions. There is a special and general relationship between these distributions, and there are some specific relationships in such relationships.

(1) $X \sim b(n,p)$, when $n=1$, that is $b(1,p)$, that is the two-point distribution;

In a two-point distribution, when $p=1$, is a single-point distribution $p(X=c)=1$.

(2) $X \sim Nb(r,p)$, when $r=1$, $Nb(1,p)$ is the geometric distribution $Ge(p)$.

(3) $X \sim Ga(\alpha, X)$, when $\alpha = \dfrac{n}{2}$, is $\chi^2(n)$; When $\alpha = 1$, that is $\exp(\lambda)$.

(4) $X \sim Be(a,b)$, when $a=1, b=1$, $Be(1,1)$ is $U(0,1)$.

(5) $X \sim t(n)$, when $n=1$, $t(1)$ is the Cauchy distribution $Cau(0,1)$.

(6) $X \sim \chi^2(n)$, when $n=2$, $\chi^2(2)$ is $\exp\left(\dfrac{1}{2}\right)$.

3.4 Identical relationship

3.4.1 No memory

No memory means that in the first m experiments, under the condition that event A does not appear, in the next n experiments, the probability of A not appearing is only related to n, and has no relationship with the first m experiments, and the first m experiments have no experience for subsequent experiments. Just like the previous m experiments didn't happen, this is no memory.

The distributions with or without memory in the probability distribution are geometric and exponential.

(1) $X \sim Ge(p)$, $\exists m,n \in N^+$, $P\left(\dfrac{X > m+n}{X > m}\right) = P(X > n)$.

(2) $X \sim \exp(\lambda)$, $\exists m,n > 0$, $P\left(\dfrac{X > m+n}{X > m}\right) = P(X > n)$.

The main difference between the memory of the geometric distribution and the exponential distribution is similar is that the geometric distribution is a discrete random variable and studies the probability problem of scatters. Exponential distributions are continuous random variables. The study is on probability problems on intervals, using slightly different fields.

3.4.2 Additivity of distributions

In probability theory and mathematical statistics, the sum of independent random variables of the same class of distribution is still called additive in the property of such distributions.

Additivity is not unique to a distribution. In probability theory and mathematical statistics, many distributions are additive, and of course there are many distributions that are not additive.

The distributions with additive properties mainly include, Poisson distribution, binomial distribution, normal distribution, gamma distribution, chi-square distribution, exponential distribution, negative binomial distribution, Cauchy distribution, the specific theory is as follows.

(1) $X \sim p(\lambda_1)$, $Y \sim p(\lambda_2)$, X and Y independent of , then $Z = X + Y \sim p(\lambda_1 + \lambda_2)$.

(2) $X \sim b(n,p)$, $Y \sim b(m,n)$, X and Y independent of , there is $Z = X + Y \sim b(m+n, p)$.

(3) $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, X and Y independent of , there is $Z = X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

(4) $X \sim Ga(\alpha_1, \lambda)$, $Y \sim Ga(\alpha_2, \lambda)$, X and Y independent of , there is $Z = X + Y \sim Ga(\alpha_1 + \alpha_2, \lambda)$.

(5) $X \sim \chi^2(m)$, $Y \sim \chi^2(n)$, X and Y independent of , there is $Z = X + Y \sim \chi^2(m+n)$.

(6) $X \sim Nb(r,p)$, $Y \sim Nb(s,p)$, X and Y independent of , there is $Z = X + Y \sim Nb(r+s, p)$.

(7) Additivity of Cauchy distribution

$X \sim C_{au}(\mu_1, \lambda_1)$, $Y \sim C_{au}(\mu_2, \lambda_2)$, X+Y= $\sim C_{au}(\mu_1 + \mu_2, \lambda_1 + \lambda_2)$.

Of course, it can be seen from the above conclusions that the requirements for additivity of different distributions are different. The binomial distribution, the negative binomial distribution requires the same p , the gamma distribution requires the same λ , and there are no special requirements for other distributions.

The above conclusions are written as additiveness of the two distributions. In fact, the above conclusions can be generalized to the case of multiple distributions, and the relevant conclusions can be easily demonstrated by mathematical induction, and some of the above additiveness can be generalized to subtraction situations.

For example, if the normal distribution additive can have $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, X and Y independent, then there is $Z = X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$, and subtraction is also true , but the variances must be added.

In fact, the conclusion of the normal distribution can also be generalized to the case of linear combination, that is $X \sim N(\mu_i, \sigma_i^2)$, a_i are arbitrary real numbers, then there is $\sum_{i=1}^{n} a_i X_i \sim N(\mu_0, \sigma_0^2)$, where $\mu_0 = \sum_{i=1}^{n} a_i \mu_i$, $\sigma_0^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2$. The additivity of other distributions cannot be generalized to subtraction cases, Poisson distributions, binomial distributions, gamma releases, etc. are not additive for subtraction.

Distributions that do not have additiveness, mainly $0-1$ distribution, geometric distribution, uniform distribution, exponential distribution, beta distribution. The independent $0-1$ distribution is added to the binomial distribution and is no longer the $0-1$ distribution. The independent geometric distributions are added together to form a negative binomial distribution, which is not a geometric distribution, so it is not considered additive.

4. Diagram of common distributions

Distribution Figure 1 Relationship between distributions centered on multiple Bernoulli

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experiments

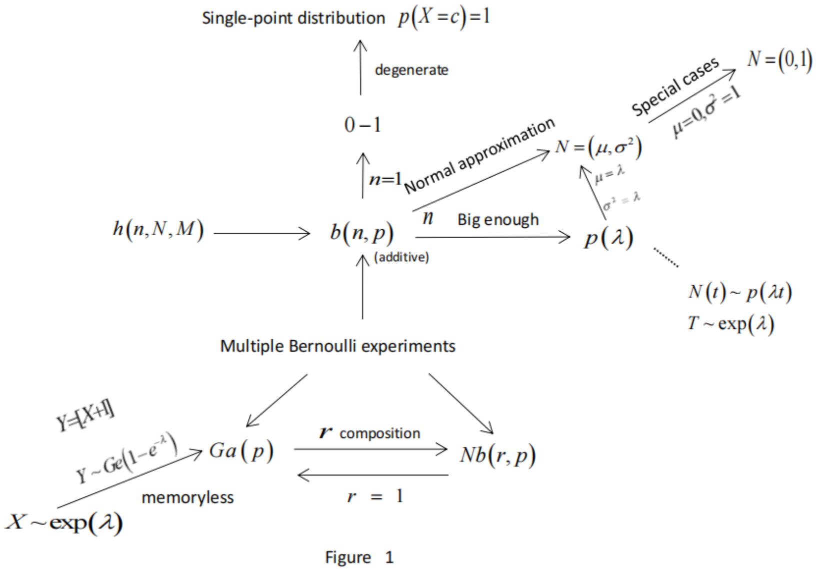

Figure 1: Relationship between distributions centered on multiple Bernoulli experiments

Distribution Figure 2 Relationship between distributions established with $\chi^2(n)$ as the center

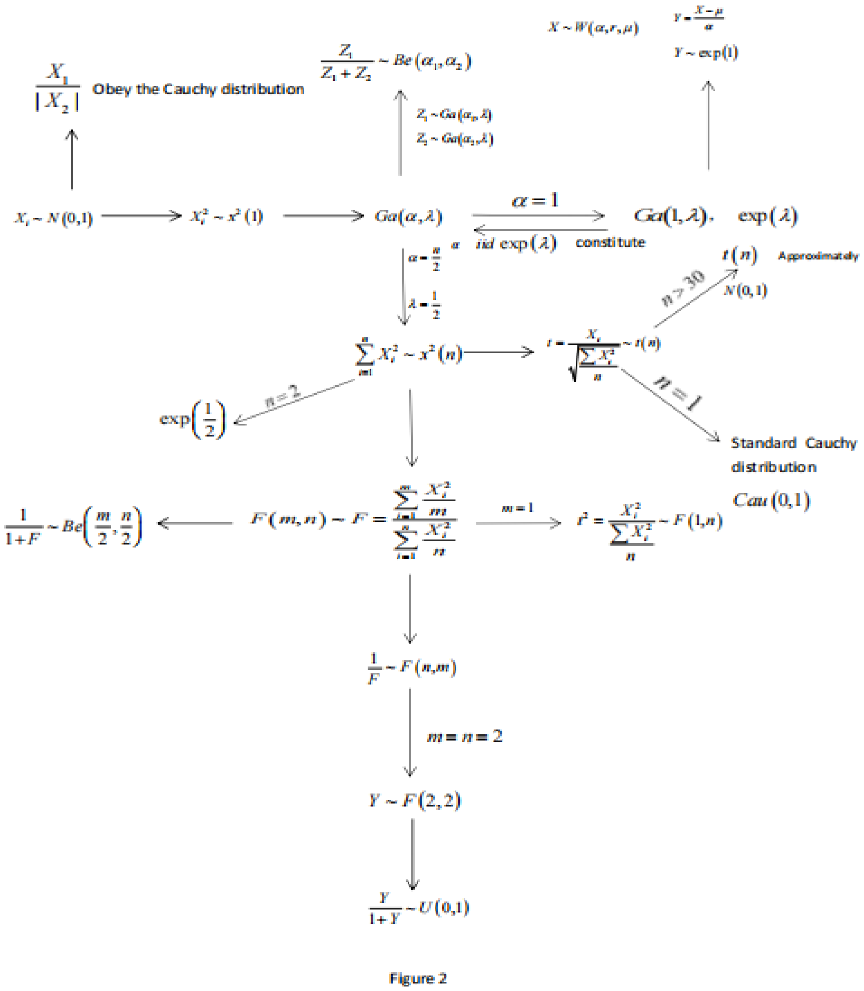

Figure 2: Relationship between distributions established with $\chi^2(n)$ as the center

Distribution Figure 3 Relationship between distributions centered on the normal distribution

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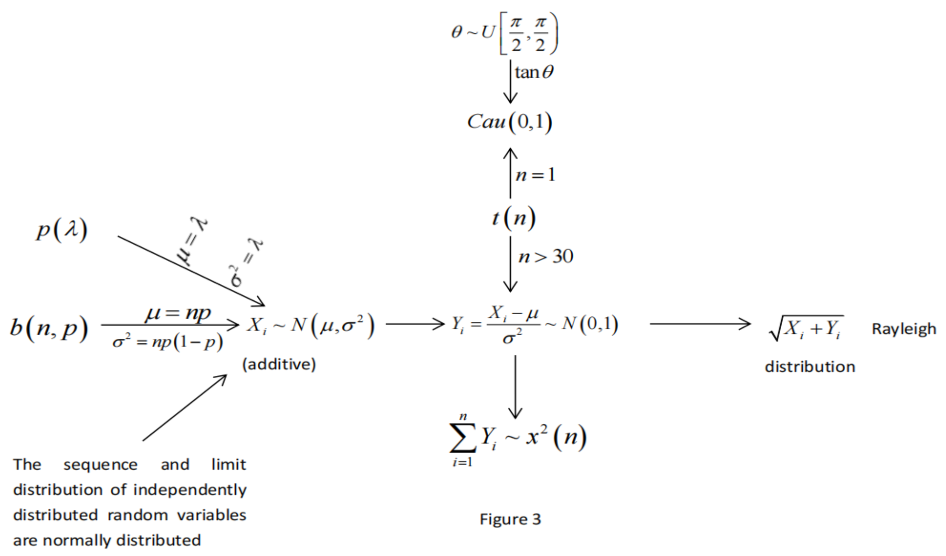

Figure 3: Relationship between distributions centered on the normal distribution

Distribution Figure 4 Establish a relationship distribution map centered on uniform distribution

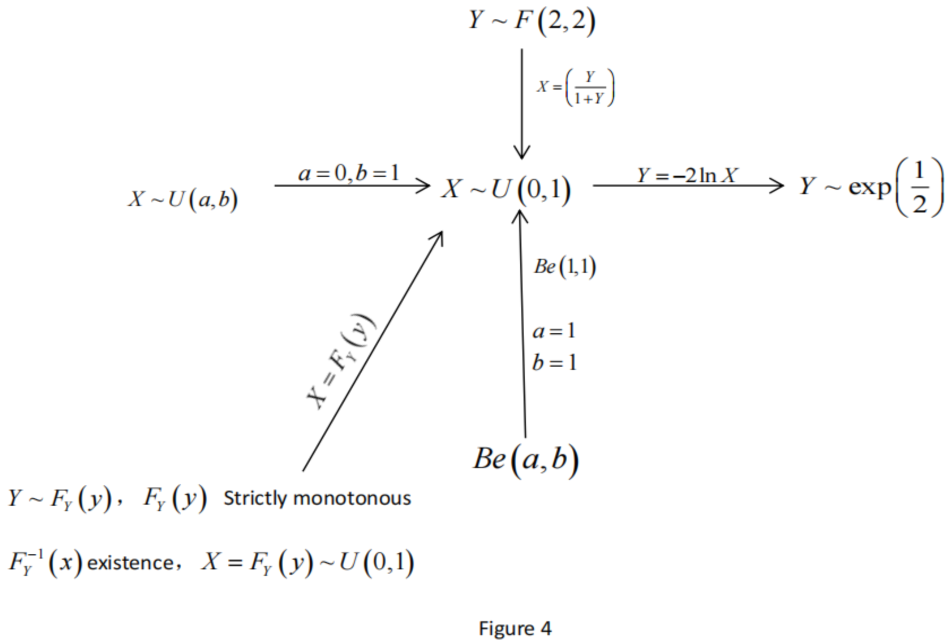

Figure 4: Establish a relationship distribution map centered on uniform distribution

5. Conclusion

In this paper, the relationship between elementary probability theory and common distributions in quantitative statistics is systematically analyzed. The relationship between distributions is mainly summarized into limit relations, functional relations, special and general relations, and relationships of the same nature, and the relationship between the distributions formed by these common discrete random variables and continuous random variables is given in the form of relations and relationship graphs.

Of course, the connections between distributions are inextricably linked, far beyond these relationships studied in this paper, and some relationships, such as the relationship between Poisson distribution and exponential distribution, are difficult to include in the four relationships listed in this article. Therefore, it has not been involved, but the research in this paper plays an important role in the discussion of the relationship between distributions, and has guiding significance for the subsequent deeper discussion of the relationship between distributions.

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