

# A Blind Calibration Algorithm of DOA Estimation for Uniform Antenna Array

Han Cui<sup>1</sup>, Sib0 Huang<sup>2,\*</sup>

<sup>1</sup> School of Electronic Information and Electrical Engineering, Huizhou University, Huizhou 516007, China

<sup>2</sup> Network and Information Center, Huizhou University, Huizhou 516007, China

\*Corresponding author e-mail: [\\_hsb@hzu.edu.cn\\_](mailto:_hsb@hzu.edu.cn_)

**ABSTRACT.** *The errors of antenna array element position and orientation affect phase and gain of received array signal, respectively. Direction of arrival (DOA) algorithm based on eigenvalue decomposition is very sensitive to gain/phase errors of array signal. Thus, it is necessary to correct the gain/phase errors of array signal. Considering uniform linear antenna array model, this paper proposes a “blind” calibration algorithm. It estimates array’s gains and phases errors using the subspace method, no needing known reference signals. DOA of the incident signal and error parameters are estimated separately using iterative approach. The simulation results show that its convergent rate is fast and its convergent precision is high.*

**KEYWORDS:** *array antenna, gain/phase errors, subspace principle, calibration*

## 1. Introduction

By estimating direction of arrival (DoA) and dynamically generating the beam patterns, a smart antenna can greatly reduce the interference and increase the system capacity. However, it is very sensitive to sensors’ errors, such as position, orientation, gain and phase errors, and the array manifold is often known imprecisely in practice. Therefore it is necessary to calibrate sensor’s errors. At present, various array calibration methods with or without known sources’ directions have been proposed. [1, 2] Traditional calibration of gain/phase response was done by sending reference signals from known directions, which is not practical indeed. Blind calibration algorithms without reference signals are available in the literature. It estimate DoAs and antenna errors simultaneously, but it causes huge computational complexity and can’t promise global convergence. A number of blind calibration algorithms have been proposed to calibrate the sensor gain and phase errors using some presumed structures, and examples of such methods include. [1, 3, 4] An algorithm was proposed for estimating the unknown sensor gain and phase perturbation by exploiting the structure of the array covariance matrix. It is

developed for linear equispaced array. [3] The method applies to array with arbitrary sensor geometries and an iterative procedure is proposed to compensate for gain and phase perturbations and to simultaneously estimate the unknown DOAs. [1] In [4] it proposed a simple, robust linear LS-based algorithm for self-calibration of LES arrays. To solving defects of the current methods this paper proposes an improved “blind” calibration algorithm, no needing reference signals. It estimates the DoAs of the incident signals and error parameters separately and searches solutions iteratively. The algorithm can estimate the phase perturbation accurately by removing the estimated errors of DOA from the phase response. The simulation results show that this method works well.

## 2. Array Signal Model

### 2.1 Ideal model

Suppose that antenna array system has six evenly placed elements with a distance of half wavelength between adjacent elements. The array manifold with regard to this  $k$  th source signal equals: [5]

$$\mathbf{a}_k = [1 \quad e^{j\pi \cos \varphi_k} \quad e^{j2\pi \cos \varphi_k} \quad e^{j3\pi \cos \varphi_k} \quad e^{j4\pi \cos \varphi_k} \quad e^{j5\pi \cos \varphi_k}]^T \quad (1)$$

The receiving signal can be written as:  $\mathbf{r}(n) = \sum_{k=1}^K \mathbf{a}_k s_k(n) + \mathbf{w}(n)$ , where  $K$  represents the number of users,  $\varphi_k, \mathbf{a}_k, s_k(n)$  represent DoA, array manifold and user signal respectively,  $\mathbf{w}(n)$  represents the noise.

In reality, antenna array elements usually have gain/phase deviation, position deviation and orientation deviation. Position deviation is the parallel shift of an antenna array element on the basis of the original position. Parallel shift generates phase error, so position deviation will affect the phase of the array manifold. Orientation deviation means that antenna array element’s orientation is not strictly in the vertical direction. Even though it won’t give rise to changes in DoAs, it will affect gain of array manifold by affecting array element’s equivalent length in the vertical direction. All deviations affect received signals by affecting array manifold  $\mathbf{a}$ . When gain/phase deviation, position deviation and orientation deviation exist at the same time, the array manifold would become:  $\tilde{\mathbf{a}}_k = \Gamma \mathbf{a}_k$ . Where

$$\Gamma = \text{diag}[g_1 \cos \theta_1 e^{j(\omega_1 + \frac{2\pi f_c \delta_1 \cos \varphi_k}{c})}, g_2 \cos \theta_2 e^{j(\omega_2 + \frac{2\pi f_c \delta_2 \cos \varphi_k}{c})}, \dots, g_6 \cos \theta_6 e^{j(\omega_6 + \frac{2\pi f_c \delta_6 \cos \varphi_k}{c})}]$$

$g_l, \omega_l, \delta_l, \theta_l$  represent gain deviation, phase deviation, position deviation and orientation deviation. The signal received is given by

$$\mathbf{r}(n) = \sum_{k=1}^K \tilde{\mathbf{a}}_k s_k(n) + \mathbf{w}(n) = \sum_{k=1}^K \Gamma \mathbf{a}_k s_k(n) + \mathbf{w}(n) \quad (2)$$

Let  $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_K]$ ,  $\mathbf{s}(n) = [s_1(n)s_2(n)\dots s_K(n)]^T$ , then:  $\mathbf{r}(n) = \Gamma \mathbf{A} \mathbf{s}(n) + \mathbf{w}(n)$

### 3. Algorithm

#### 3.1 Identification

Consider the number of unknown parameters versus the number of equations at the antenna array: The entire array consists of  $L$  sensors, Let  $K$  be the number of incident sources. Each sensor involves one unknown non-ideal gain parameter, plus one unknown non-ideal phase parameter. Each incident source incurs an unknown DOA parameter. The equations we want to solve are  $\mathbf{E}_s = \tilde{\Gamma} \mathbf{A} \mathbf{T}$ , where  $\mathbf{T}$  is a unique  $K \times K$  nonsingular complex matrix which involves another  $2K^2$  unknowns. [6] Hence, the total number of unknowns equal  $(2L^2 + K + 2K^2)$ ,  $(2L^2 + 2K)$  of which are need to be estimated. On the other hand, as our above analysis,  $\mathbf{E}_s$  is a  $L \times K$  complex matrix, thus the total number of equations equal  $(2LK)$ . As a result of all the above, a proper value for  $K$  could provide the necessary number of equations to evaluate all unknowns, i.e.,  $2LK > 2L + K + 2K^2$ .

#### 3.2 Removing the effects of DOA error from the phase response

In the existing methods, the DoAs and array gain/phase response are estimated jointly, [6] therefore, the estimated error of DoAs will affect the estimation of array gain/phase response. We can see from (1), DoAs will affect array phase response because it only contains the phase factor. Hence, we must remove the effects of DOA error from the phase response.

Let DoA of the  $k$  th source be  $\varphi_k$ , and the estimated result be  $\tilde{\varphi}_k$ . The estimated error is  $\Delta\varphi_k = \varphi_k - \tilde{\varphi}_k$ . In this case, the error of array manifold is

$$\mathbf{a}_k = [1 \quad e^{j\pi \cos(\Delta\varphi_k)} \quad e^{j2\pi \cos(\Delta\varphi_k)} \quad e^{j3\pi \cos(\Delta\varphi_k)} \quad e^{j4\pi \cos(\Delta\varphi_k)} \quad e^{j5\pi \cos(\Delta\varphi_k)}]^T \quad (3)$$

The effect it caused to phase response is  $[0 \quad \Delta\varphi_k \quad 2\Delta\varphi_k \quad 3\Delta\varphi_k \quad 4\Delta\varphi_k \quad 5\Delta\varphi_k]$ . So, the effect of all sources' DoAs is  $[0 \quad \sum_{k=1}^K \Delta\varphi_k \quad \sum_{k=1}^K 2\Delta\varphi_k \quad \sum_{k=1}^K 3\Delta\varphi_k \quad \sum_{k=1}^K 4\Delta\varphi_k \quad \sum_{k=1}^K 5\Delta\varphi_k]$ . At the same time, there

will be a constant bias  $\beta_1$  in the estimation of phase response. Hence the total estimated errors of phase response are

$$[\beta_1 + 0 \quad \beta_1 + \sum_{k=1}^K \Delta\varphi_k \quad \beta_1 + \sum_{k=1}^K 2\Delta\varphi_k \quad \beta_1 + \sum_{k=1}^K 3\Delta\varphi_k \quad \beta_1 + \sum_{k=1}^K 4\Delta\varphi_k \quad \beta_1 + \sum_{k=1}^K 5\Delta\varphi_k] \quad (4)$$

We assume the real phase response are  $[\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \quad \alpha_5 \quad \alpha_6]$ , then the estimation of phase response will be:

$$[\alpha_1 + \beta_1 \quad \alpha_2 + \beta_1 + \sum_{k=1}^K \Delta\varphi_k \quad \alpha_3 + \beta_1 + \sum_{k=1}^K 2\Delta\varphi_k \quad \alpha_4 + \beta_1 + \sum_{k=1}^K 3\Delta\varphi_k \quad \alpha_5 + \beta_1 + \sum_{k=1}^K 4\Delta\varphi_k \quad \alpha_6 + \beta_1 + \sum_{k=1}^K 5\Delta\varphi_k] \quad (5)$$

Because the estimated error of every array element phase response are not same, it is impossible to estimate accurate relative error if we only set one reference element. To solve this problem, the paper proposes a rotating method that is we rotate the antenna array  $180^\circ$  and estimate the antenna array response in the same method. Then, the estimation of phase response will be

$$[\alpha_1 + \beta_2 + \sum_{k=1}^K 5\Delta\varphi_k \quad \alpha_2 + \beta_2 + \sum_{k=1}^K 4\Delta\varphi_k \quad \alpha_3 + \beta_2 + \sum_{k=1}^K 3\Delta\varphi_k \quad \alpha_4 + \beta_2 + \sum_{k=1}^K 2\Delta\varphi_k \quad \alpha_5 + \beta_2 + \sum_{k=1}^K \Delta\varphi_k \quad \alpha_6 + \beta_2] \quad (6)$$

Where  $\beta_2$  is the constant bias in the second estimation of phase response. We put (5) and (6) together, then it equals:  
 $[2\alpha_1 + \beta_1 + \beta_2 + \sum_{k=1}^K 5\Delta\varphi_k \quad 2\alpha_2 + \beta_1 + \beta_2 + \sum_{k=1}^K 5\Delta\varphi_k \dots 2\alpha_6 + \beta_1 + \beta_2 + \sum_{k=1}^K 5\Delta\varphi_k]$ . We can see from it, the estimation of every array element's phase response have the same bias, then we can estimate the relative error of phase response accurately.

#### 4. Simulation Results

Here we mainly simulate the joint estimation of DoAs and array gain/phase response. We consider two signal sources, both with unity power and equal frequency  $f = 0.2kHz$ . The DOAs are 0.5(rad) and 0.75(rad) respectively. Note the estimates of the proposed algorithm have unknown scalars. Here we present the estimates of the relative parameters. The relative gain response of the  $l$ th array element is defined as its estimated gain response divided by the gain response estimates of the reference array element. The relative phase response of the  $l$ th array element is defined as its estimated phase response subtracted by the phase response estimates of the reference array element. Figure 1 shows the convergent properties of the estimates of the arriving angles. It can be observed that the estimation of the arriving angle has robust and fast convergence. The DOAs estimations MSEs before and after calibration are compared in Figure.2. Figure.3 shows MSEs of phase response estimates, where they are all displayed versus SNR. We also compare MSEs of DOA estimates of the proposed algorithm with maximum likelihood approach. It is observed that the deviation of the DOA estimates is close to the estimates of maximum likelihood approach but reduces the computational complexity considerably. The results are over 200 independent Monte

Carlo trials. They demonstrate that the algorithm proposed in the paper have substantially performance.

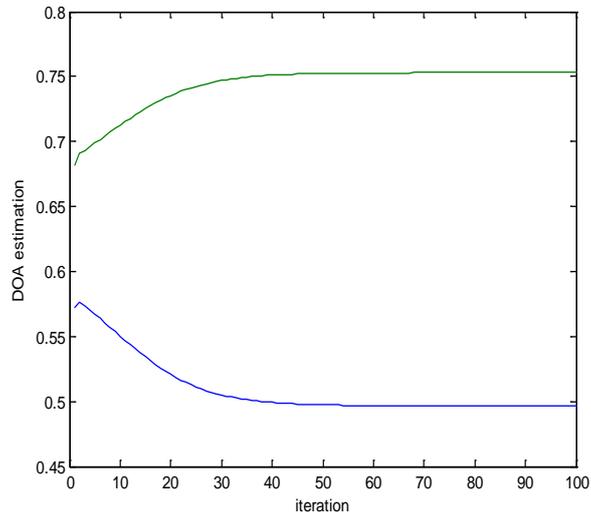


Figure. 1 Estimates of DOAs versus iteration true values are (0.5rad, 0.75rad).

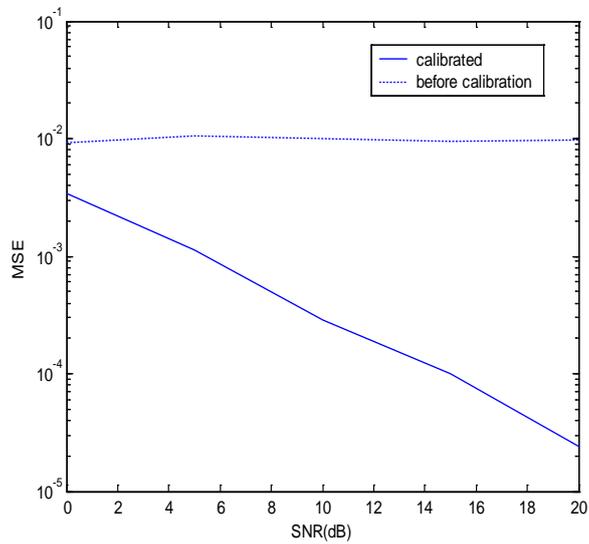


Figure. 2 MSE of DOA estimates versus SNR.

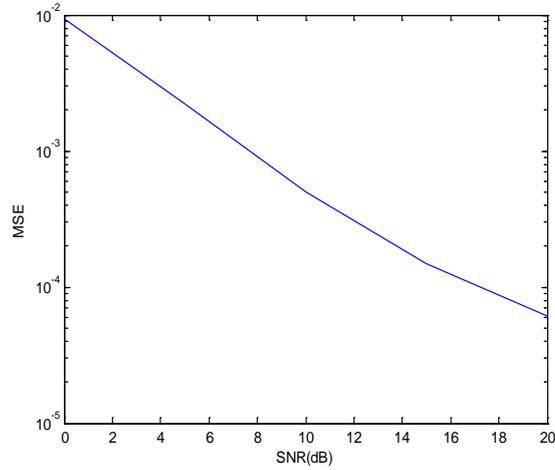


Figure. 3 MSE of phase response estimates versus SNR.

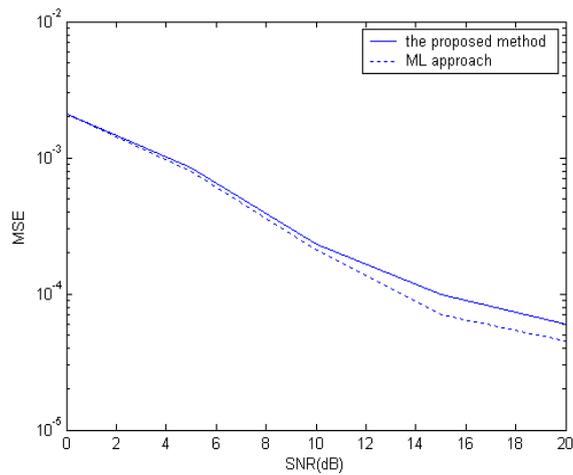


Figure. 4 MSE of DOA estimates of the proposed method and ML approach.

## 5. Conclusion

A blind calibration algorithm for unique model of antenna array under unknown antenna array gains and phases is proposed. The method can calibrate the gains and phases response accurately without increasing the complexity of system. We can see from simulation result that the algorithm works well and the estimation of gains and phases response has fast convergence. Using the estimation of the algorithm to

calibrate the receiving signals can significantly reduce the error rate after demodulation and improve the performance of base station.

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