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Differential Equation-Based Study on Sex Ratio and Ecosystem Interaction of Seven Gill Eel

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Abstract: *Focusing on the key trait of sex ratio in the lampreys, this study constructed an eco-dynamic model to examine the effects of multiple organisms on the lampreys. The biomass changes of lampreys, parasitic fish and competing organisms were simulated under different resource conditions, revealing the effects of sex ratio on the ecosystem and its own population biomass. Through the blocked growth model and competition model, we explored how the resource changes affect the sex ratio of lampreys and how the interactions between different organisms affect the stability of the ecosystem. The experimental results showed that the models were reasonable and provided an important reference for understanding the population dynamics of the lampreys and the evolution of the ecosystem.*

Keywords: *Differential Equations, Ecological Dynamics Model, Competition Models*

1. Introduction

As an important player in the ecosystem, changes in the sex ratio of the lampreys have a profound impact on the ecological balance [1]. In this study, an ecological dynamics model was constructed based on the key trait of sex ratio of the lampreys, and the interactions between the lampreys and other organisms were investigated under different resource conditions. The effects of resource changes on the sex ratio of the lampreys and the competitive relationship between different organisms on the stability of the ecosystem were explored through the blocked growth and competition models. The experimental results revealed the importance of sex ratio changes on the ecosystem and population biomass and provided new perspectives for understanding the population dynamics and ecosystem evolution of the lampreys. This study will contribute to better management and conservation of the lampreys and its ecosystem and provide a useful reference for the fields of ecology and resource management.

2. Model I: block growth model

2.1 Establishment of differential equation modeling

2.1.1 Changes in lamprey biomass at stage $0-T_0$

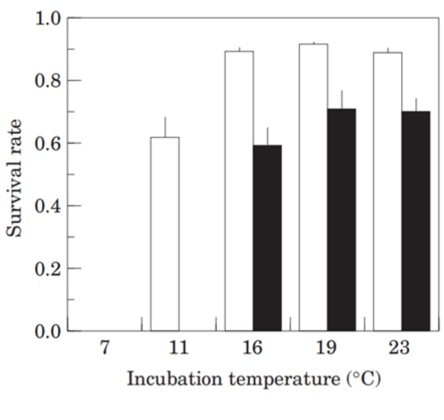

Figure 1: Lamprey larval survival rate during the stage $0-T_0$

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We have obtained some data on the survival rates of lampreys at different temperature stages [2]. The hatching rate of the lamprey and the survival rate of the larvae successfully burrowing into the nursery area vary at different temperatures. Lampreys cannot hatch at temperatures below 10°C. They can hatch at 11°C, but since the yolk is consumed during the hatching process, they are unable to successfully burrow into the nursery area. The survival rate of lampreys is relatively stable between 15°C and 23°C, as shown in Figure 1.

To better illustrate the decline in lamprey biomass during the $0-T_0$ stage, we chose the average survival rate of 15°C to 23°C when establishing the model, which is 0.65. This means that the biomass of lampreys that successfully burrow into the nursery area is reduced to 65% of its original value. The change in lamprey biomass is represented by $N(t)$:

$$\begin{cases} N(t) = N(0) + \dfrac{0.65 \cdot N(0) - N(0)}{T_0} \\ N|_{t=0} = N(0) \end{cases} \tag{1}$$

2.1.2 The change in lamprey biomass between stage $T_0 - T_1$ and $T_1 - T$

The moment when the lamprey larvae successfully burrow into the nursery area is recorded as T_0, and we use λ to represent the growth rate of the lamprey larvae during this period, which is the $T_0 \le t < T_1$ time required for the lamprey larvae to grow to maturity. According to research, we have found that the growth period for lampreys mainly ranges from 2 to 6 years. To simplify the model, we fix the time required for lamprey larvae to grow to maturity at 4 years, which means $T_1 = 4$. Due to factors such as food supply affecting the growth rate of lamprey larvae during their development, a logistic growth model can be introduced [3]. Our Logistic Model is represented as:

$$\begin{cases} \dfrac{dN}{dt} = \lambda \cdot N(1 - \dfrac{N}{N_1}) \\ N|_{t=T_0} = N(T_0) \end{cases} \tag{2}$$

When lampreys reach maturity, they acquire the ability to parasitize, feeding on the flesh of other fish. At this stage, we simplified all the fish species that lampreys can parasitically to M for modeling, and r_1 and r_2 were expressed as the mortality rate of lampreys at maturity and the growth rate of parasitically parasitized fish, respectively. When lampreys parasitically inhabit fish, their biomass will increase, and the biomass of the host fish will decrease. σ_{12} represents the coefficient of change in the number of lampreys caused by the host fish, and σ_{21} represents the coefficient of change in the biomass of the host fish caused by the lampreys. When lampreys reach maturity, they will be fished by humans, so their biomass will be reduced to a certain extent, as will the parasitic fish. The differential equation between lamprey biomass $N(t)$ and parasitic fish biomass $M(t)$ is as follows:

$$\begin{cases} \dfrac{dN}{dt} = r_1 \cdot N(-1 - \dfrac{N}{N_1} + \sigma_{12}\dfrac{M}{M_1}) - \rho_1 \cdot N \\ \dfrac{dM}{dt} = r_2 \cdot M(1 - \dfrac{M}{M_1} - \sigma_{21}\dfrac{N}{N_1}) - \rho_2 \cdot M \\ N|_{t=T_1} = N(T_1), M|_{t=T_1} = M(T_1) \end{cases} \tag{3}$$

2.1.3 Relationship between resources and male proportion of lampreys

Biomass can reflect the trend of population change and thus reflect the quantity change of population. Since the sex ratio of lampreys has some adaptive changes with the change of food resources, and the size of female lampreys is larger than that of male lampreys [4], it can be inferred that the demand for resources of female lampreys when the larvae mature into female lampreys is greater than that of male lampreys. Due to different resources, the highest proportion of lamprey's larvae growing and maturing into males can be 78%, and the lowest is 56%. Therefore, the male proportion R of lampreys is shown as follows:

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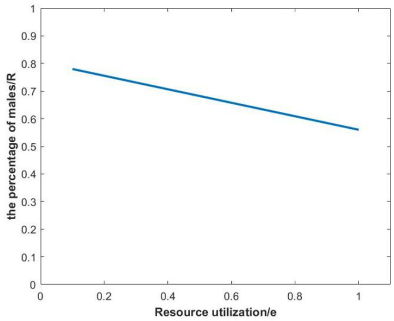
$$R = 0.78 - e \cdot \frac{(0.78 - 0.56)}{1 - 0.1} \tag{4}$$

In order to adapt to the ecological environment when resources are scarce, lamprey populations tend to produce more males. When resources are sufficient, the reproduction and survival of lamprey's populations are no longer limited, and the sex ratio can be maintained at a relatively stable level, approximately 1:1. Figure 2 shows the change in the proportion of males in lamprey larvae that grow to maturity when resources change.



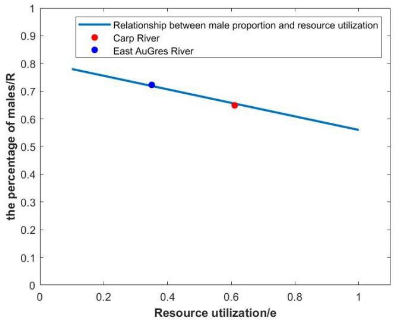


Figure 2: Change in the proportion of males   *Figure 3: Male proportion*

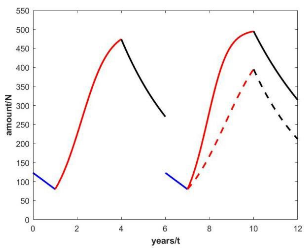
We selected the data from Carp River and East AuGres River respectively, calculated the male proportion of lampreys by MATLAB, and drew the points corresponding to the male proportion of lampreys in the resources in these two rivers, as shown in Figure 3, and found a curve very close to Figure 2, which can demonstrate the effectiveness of our model.

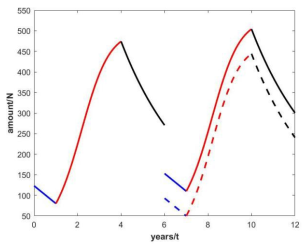
2.2 Relationship between sex ratio and lampreys in the next cycle

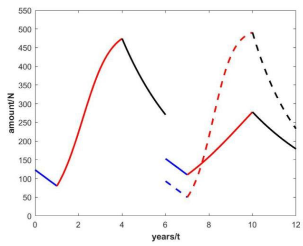
Now we can get the biomass $N(T)$ of lampreys at the end of a cycle where the proportion of females is $1-R$. The number of eggs a female lamprey can lay is c. The initial biomass of lampreys in the next cycle is related to the biomass of lampreys at the end of the previous cycle, the proportion of females, and the number of female eggs per unit. Therefore, the initial biomass $N(T^{0,next})$ of the next cycle is shown as follows:

$$N(T^{0,next}) = N(T)(1-R) \cdot c \tag{5}$$

When considering the next cycle, we simulated the biomass change of lamprey in the next cycle, assuming that the previous cycle was abundant and scarce. The simulated results are as follows.







Note: Figures 4 and 5: The left side shows the first cycle, and the right (solid line) shows the change in lampreys' biomass when resources are sufficient in the first cycle. The right (dashed line) shows the change in lampreys' biomass during the first cycle when resources were scarce. Figure 6: The left side shows the changes in lampreys' biomass when resources are abundant in the first cycle and when resources are scarce in the next cycle (solid line). The left side shows the change in lampreys' biomass when the first cycle is resource scarce and when the next cycle becomes resource scarce (dashed line).

Figure 4: When the sex ratio is unchanged (1:1) (Left)

Figure 5: Theoretical plot when the sex ratio changes (Middle)

Figure 6: Actual figure when the sex ratio changes (Right)

Figure 4 shows that when the sex ratio of lampreys does not change, that is, the sex ratio is approximately 1:1, the initial biomass of lampreys in each cycle remains unchanged. When resources are abundant (solid line), lampreys' biomass is similar to that of the first cycle. When resources are scarce (dashed line), the increase in the male ratio of lampreys results in a decrease in the biomass of lampreys.

Figure 5 shows that when the sex ratio of lamprey's changes, the biomass of lampreys in the next cycle will be similar to that in the first cycle if there is no lag in the spawning capacity of lampreys. When resources are sufficient (solid line), the initial biomass of lampreys in each cycle is theoretically greater than the initial biomass of the previous cycle, so the biomass of lampreys at this time is similar to that of the first cycle. When resources are scarce (dashed line), the initial biomass of lampreys in each cycle is theoretically smaller than the initial biomass of the previous cycle, so the biomass of lampreys at this time is similar to that of the first cycle.

Figure 6 shows that when the sex ratio of lamprey's changes, the biomass of lampreys has a lag. The first cycle has sufficient resources → Resource scarcity in the next cycle (solid line) : The initial biomass of lampreys increases in the next cycle, but due to resource scarcity in the next cycle, the male ratio of lampreys will increase, resulting in a greater decrease in the biomass change of lampreys compared to the previous cycle. The first cycle is resource-poor → Next cycle Shortage of resources (dashed line): The initial biomass of lampreys will decrease in the next cycle, but as the resources of lampreys are abundant in the next cycle, the proportion of males in lampreys will decrease, resulting in a larger increase in the biomass of lampreys compared to the previous cycle.

When the sex ratio of lampreys is different under different resources, the initial biomass of the next cycle will change accordingly. According to the above analysis, the advantages and disadvantages of the change in the sex ratio of lamprey's population can be obtained as follows:

Advantages: Under different resource conditions, changing the sex ratio of lampreys allows them to adjust their population numbers in extreme environments (resource scarcity), thus reducing the number of lampreys at maturity, thereby improving resource utilization and preventing their own extinction. If the sex ratio of lampreys does not change according to the environment, and the initial value of lamprey's biomass does not change per cycle, then the average allocation of resources per unit of larvae may be very low when resources are extremely scarce, leading to the endangered or even extinct lampreys' species.

Disadvantages: Lampreys have a lag in egg production. Lampreys can only change their sex ratio based on current resources, thus adjusting their biomass for the next cycle. However, due to the unknown resources of the next cycle, it may lead to a large decrease in the number of eggs laid by lampreys when the current resources are scarce, but the actual resources in the next cycle are very abundant, resulting in a waste of resources. On the contrary, it causes resource shortage.

3. Model II: competition model

3.1 Competition modeling

In ecological processes, biodiversity is an important feature of ecosystem complexity, so there are multiple parasitic organisms in the ecosystem that compete with lampreys for the same species of fish [5]. In building the model, we reduced the parasitic species that compete with lampreys to species Q. The influence coefficient of species Q on lampreys is denoted as σ_{31}, and the influence coefficient of species Q on parasitized fish M is denoted as σ_{32}. In order to more accurately describe the relationship between lampreys in the ecological environment, we established a competition model based on Model I, as shown follows:

$$\begin{cases} \dfrac{dN}{dt} = n_1 N(-1 - \dfrac{N}{N_1} + \sigma_{12} \cdot \dfrac{M}{M_1} - \sigma_{13} \cdot \dfrac{Q}{Q_1}) - \rho_1 \cdot N \\ \dfrac{dM}{dt} = n_2 M(1 - \dfrac{M}{M_1} - \sigma_{21} \cdot \dfrac{N}{N_1} - \sigma_{23} \cdot \dfrac{Q}{Q_1}) - \rho_2 \cdot M \\ \dfrac{dQ}{dt} = n_3 Q(-1 - \dfrac{Q}{Q_1} - \sigma_{31} \cdot \dfrac{N}{N_1} + \sigma_{32} \cdot \dfrac{M}{M_1}) - \rho_3 \cdot Q \end{cases} \tag{6}$$

Where $Q(t)$ is the parasitic biomass that competes with lampreys, and n_3 is the mortality rate of

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the species.

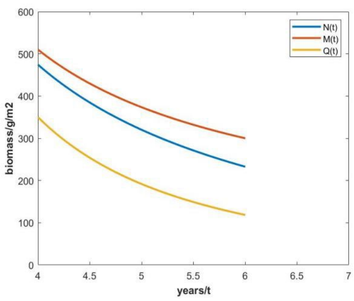
3.2 Solutions of Model II

We assume that when they have sufficient resources, when the competition between Lampreys and the parasite Q and the parasite M is suitable, the influence coefficient of lampreys on the other species is as follows: $\sigma_{12}=0.2$, $\sigma_{13}=2$, $\sigma_{21}=2.5$, $\sigma_{23}=1.5$, $\sigma_{31}=3$, $\sigma_{32}=0.8$. When the competition between lampreys and the parasite Q over the host M is excessive, the coefficients of their influence on the other species are as follows: $\sigma_{12}=0.15$, $\sigma_{13}=4$, $\sigma_{21}=15$, $\sigma_{23}=12$, $\sigma_{31}=3$, $\sigma_{32}=0.6$ .When the competition between lampreys and the parasitic species Q and the parasitized species M is suitable, the influence coefficient of lampreys on the other species is as follows: $\sigma_{12}=0.08$, $\sigma_{13}=6$, $\sigma_{21}=2$, $\sigma_{23}=3$, $\sigma_{31}=2.5$, $\sigma_{32}=1.2$. When the competition between lampreys and the parasite Q over the host M is excessive, the coefficients of their influence on the other species are as follows: $\sigma_{12}=0.12$, $\sigma_{13}=6.5$, $\sigma_{21}=12$, $\sigma_{23}=15$, $\sigma_{31}=2$, $\sigma_{32}=1.5$. The other parameter changes are shown in Table.1 below.

Table 1: Table of parameter changes when competitiveness changes

Symblo	Plenty of resources		Lack of resources	
	Suitable competition	Excessive competition	Suitable competition	Excessive competition
N_1	500	500	300	300
M_1	600	600	500	500
Q_1	400	400	250	250
n_1	0.1	0.1	0.1	0.1
n_2	0.08	0.08	0.08	0.08
n_3	0.11	0.11	0.11	0.11

According to the parameters, we simulated the results as follows:



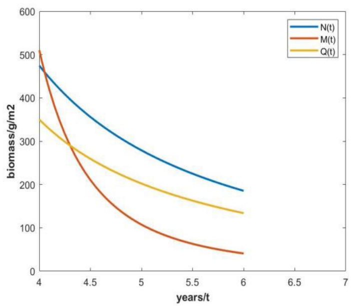
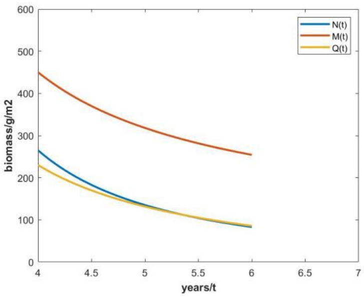


Figure 7: With adequate resources and suitable competition (Left)

Figure 8: With adequate resources and excessive competition (Right)



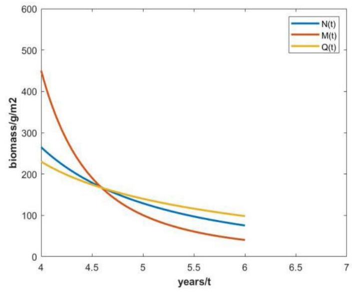


Figure 9: Lack of resources and competition suitable time (Left)

Figure 10: When lack of resources and excessive competition (Right)

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Under the condition of sufficient resources: When the competition between Lampreys and parasitic species Q and parasitic organism M is suitable and when the competition is excessive, the biomass changes of the three organisms are shown in Figure 7 and Figure 8 .When the competition is suitable, the changes of the three kinds of biomass will gradually flatten out, and when the competition is excessive, it may lead to the continuous decline of the parasite M , and even lead to its extinction.

In the condition of low resources: The initial values of the three kinds of biomass decreased, and the biomass changes of the three kinds of organisms when the competition between Lampreys and the parasitic species Q and the parasitic organism M was suitable and when the competition was excessive were shown in Figure 9 and Figure 10. When the competition is suitable, the changes of the three kinds of biomass will gradually flatten out, but when the competition is excessive, the extinction rate of the parasitized organisms will be accelerated.

Summarize the above conclusions, we find that when the ecosystem resources change, the sex ratio of lamprey is easily influenced by the outside world, which has advantages and disadvantages for the stability of the ecosystem: can increase the resilience of the ecosystem, lampreys' ability to change sex ratio is an adaptive variation to nature. The stability of the ecological balance may be destroyed due to excessive competition or excessive use of resources, resulting in the endangered or even extinct parasitic species or themselves. When the ecological balance fluctuates, the lamprey's population reacts violently, potentially resulting in a sex ratio imbalance, which can affect the biomass of the population, the biomass of the fish population with which it is parasitically related, changes in the parasitic biomass that compete with lampreys, and possibly the endangerment or even extinction of a species [6]. The greater the degree of excessive competition and sex imbalance, the less stable the ecosystem.

3.3 Lamprey advantage a competitive parasitic species

In the above analysis, we respectively consider the biomass changes between lampreys and parasitic fish M under the condition of sufficient resources and scarcity of resources, and the biomass changes between lampreys and parasitic fish M under the condition of addition of a parasitic species Q that competes with lampreys, as shown in Figure 11.

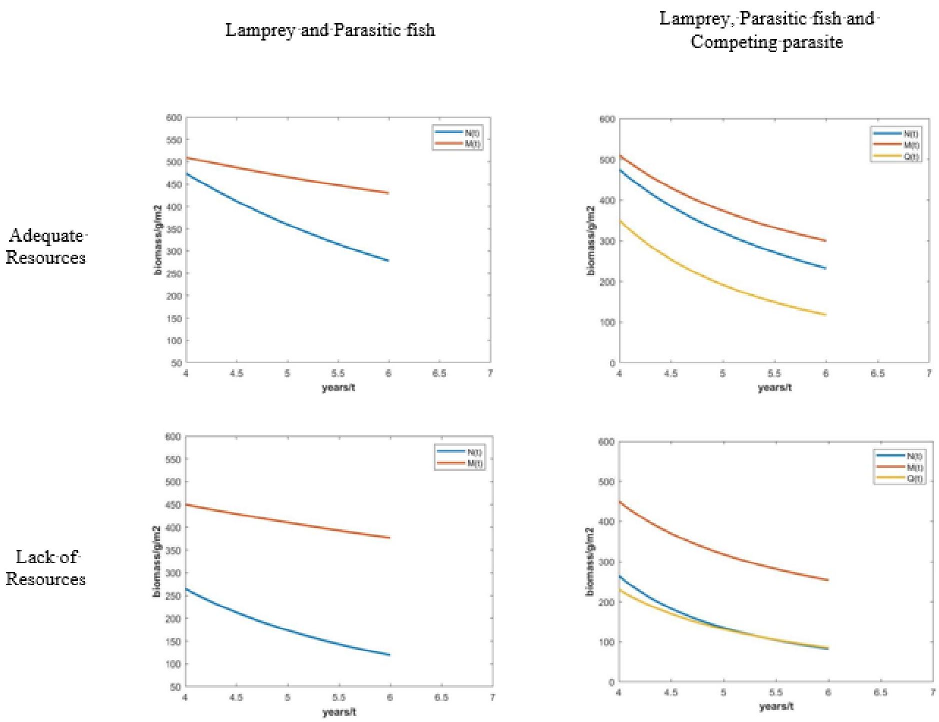

Figure 11: Comparison of biomass changes of the three species at competition suitability

When the competition is suitable, we find that: In the case of sufficient resources, the biomass of lampreys and the parasitic fish M will decrease more dramatically when there is competition than when there is no competition. However, due to the current sufficient resources, the species numbers of the three

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species (lampreys, parasitized fish M, and parasitized species Q) will change more and more but will gradually reach a stable value. In the case of resource scarcity, when there is competition, the competition between lampreys and parasitic species Q may cause the fluctuation of their biomass to be smaller than in the case of no competition, and gradually tend to a stable value.

4. Conclusions

Through this study, we delved into the effects of changes in the sex ratio of the lampreys on the ecosystem. Our modeling results suggest that the sex ratio of the lampreys is influenced by resource conditions, and that this change in sex ratio has important implications for the stability of the entire ecosystem. The lamprey's population showed a more balanced sex ratio when resources were sufficient, whereas the sex ratio may fluctuate more under resource scarcity. In addition, our study found that changes in the female-to-male ratio of lampreys had a significant impact on their competitive relationships with other species, further affecting the biomass and stability of the entire ecosystem. These findings provide important clues for an in-depth understanding of the population dynamics and ecosystem evolution of the lampreys, as well as substantial references for future ecological and resource management efforts.

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