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Construction and analysis of the stochastic model of the influence of industrial emissions on human

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ABSTRACT. *Based on the differential equation model of the influence of environmental toxins on biological population, the objective random interference factors of Brownian motion simulation are introduced, and the objective delay factors are simulated by using the delay term, so as to construct the stochastic delay differential equation model of the influence of toxins in industrial emissions on human beings, and prove the existence of the global positive solution of the model, thus proving the established model Type A has certain theoretical feasibility.*

KEYWORDS: *Brownian motion; delay; Industrial emissions.*

1. Introduction

In recent years, the situation of haze in China has become more and more serious, affecting 25 provinces and more than 100 large and medium-sized cities. The air quality is shocking. The harmful substances such as sulfur dioxide, nitrogen oxide, organic chloride, arsenic, lead, mercury and inhalable particulate matter have led to the proliferation of major malignant diseases in the respiratory system, reproductive system and other aspects, and even lead to fetal malformation during pregnancy An increase in the rate.

Today’s era is a multi information age. In the field of science, the development of each discipline is no longer independent, but intertwined and promoted.

In recent years, with the aggravation of global environmental pollution and the improvement of environmental awareness, it has become an important content of Biomathematics to study the impact of air pollution on biological population by establishing mathematical model.

Some progress has been made in the research on the models of environmental toxins affecting biological populations in the existing literature [1]. Among them, some achievements have been made in the research and analysis of two closely related models of environmental toxins affecting biological populations [2], the

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model of species living conditions in the context of industrial pollution [3], the model of n-dimensional polluted ecosystem [4], etc., and the results of existing literature retrieval have been obtained The results show that the models for studying the effects of environmental toxins on biological populations are all deterministic differential equations.

However, the development of mathematical model is a process that is gradually and objectively close to the truth of things. With the development of environmental pollution control technology and related discipline theory, there is a lot of space for the exploration of mathematical model of environmental toxins on biological population.

When we use mathematical methods to study biological and ecological problems and establish appropriate mathematical models to study the effects of environmental toxins on biological populations, we usually need to consider the consistency between the models and the actual situation and the feasibility of dealing with the problems.

In fact, in the process of ecosystem evolution and development, various forms of random interference are ubiquitous. For example, the changes of air relative humidity, atmospheric pressure, wind speed, air inversion layer, sunlight, exhaust emissions of life and industry, vegetation adsorption capacity and other random factors will affect the uncertainty changes of haze concentration, state, harmfulness and other aspects as well as biological population The unpredictable results of immunity, birth rate and mortality rate [5], these factors can not be controlled under the existing technical conditions, can not be ignored and have random variability. The interference factors of this kind of random variation can be regarded as the environment white noise (the synthesis of all kinds of small noise interference in the environment, from the law of large numbers, these common all kinds of small noise interference The synthesis will obey the normal distribution, which is called white noise.

Because the ideal white noise does not exist in mathematics, the best approximation of white noise is to simulate it with the form derivative of Brownian motion or Wiener process. Research [6] shows that the environmental white noise will affect the growth rate, environmental capacity, competition coefficient and other system parameters to varying degrees, and the number of biological population individuals often does not reach the approximation For the requirements of deterministic systems, if these random factors are ignored in the research process, there may be a large deviation. The random fluctuations in the ecosystem are obvious and can not be ignored. Therefore, it is necessary to introduce the brown motion model of environmental white noise on the basis of the deterministic differential equation model.

On the other hand, generally speaking, the toxicity of heavy metal toxins such as arsenic, lead and mercury in the environment can only be reflected after they enter the biological population through the accumulation and combination process, which has a certain amount of delay in time [7], so that the physiological indicators and symptoms of the biological population not only depend on the current state, but also

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on the past state at a certain time A state or period of time. In addition, in real life, the serious pollution haze outbreak is discontinuous and only occurs at some time of the year. We can consider using pulse [8] to simulate the serious haze outbreak.

Under the background of air pollution, the original biological model is expanded and deepened from the perspective of randomness and delay, and the stochastic delay differential equation model of environmental toxin's influence on biological population is established. The stochastic dynamic properties of the model and its practical application are studied, hoping to provide a more reasonable and practical theoretical basis for the treatment of environmental pollution.

2. Model building

The model of stochastic delay differential equation driven by Brown motion is given

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = \alpha_1 x_1(t)x_1(t-\tau) + \beta_1 x_1(t)x_3(t-\tau) + \sigma_1 x_1(t)\mathrm{dB}(t) \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = k_1 x_2(t) + \alpha_2 x_2(t)x_1(t-\tau) + \beta_2 x_2(t)x_3(t-\tau) + \sigma_2 x_2(t)\mathrm{dB}(t) \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = k_2 x_3(t) + \alpha_3 x_3(t)x_1(t-\tau) + \beta_3 x_3(t-\tau)x_2(t-\tau) + \sigma_3 x_3(t)\mathrm{dB}(t) \end{cases} \quad (1)$$

Among them, $B(t)$ is Brown motion. $x_1(t),\ x_2(t),\ x_3(t)$ respectively represent the toxic amount of industrial waste gas, population and body toxin amount. $\tau > 0$ is delay. $\alpha_i,\ \beta_i,\ \sigma_i,\ i = 1,2,3$ and $k_i,\ i = 1,2$ are parameters.

The model (1) reflects the random effects of toxins in Industrial emissions on human beings.

3. Model analysis

Theorem The model (1) has a unique positive solution almost everywhere.

Proof. Let $\boldsymbol{X}(t) = (x_1(t),\ x_2(t),\ x_3(t))^T$..

For a given initial function $\{\boldsymbol{X}(t): -\tau \le t \le 0\} \in \mathrm{C}\left([-\tau,\ 0];\ \mathbf{R}_+^3\right)$, the model (1) satisfy the local Lipschitz condition (see [25]) and there is a maximum local solution $\boldsymbol{X}(t)$, $t \in [-\tau,\ \tau_e)$,where τ_e refers to the time of explosion.

Let stopping time is $\tau_k = \inf\left\{t \in [0,\ \tau_e) : S(t),\ T(t),\ U(t) \notin \left(\dfrac{1}{k},\ k\right)\right\}$.

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We define the functional

$$V(\boldsymbol{X})=\left(\sqrt{S}-0.5\log\ S-1\right)+\left(\sqrt{T}-0.5\log\ T-1\right)+\left(\sqrt{U}-\log\ U-1\right).$$

There we can get

$$\mathrm{d}\left(\int_{t-\tau}^{t}|\boldsymbol{X}(s)|^2\mathrm{d}s+V(\boldsymbol{X}(t))\right)\le K_1\,\mathrm{d}t+0.5\sigma_1\left(S^{0.5}(t)-1\right)\mathrm{d}W_1(t)+ 0.5\sigma_2\left(T^{0.5}(t)-1\right)\mathrm{d}W_2(t)+0.5\sigma_3\left(U^{0.5}(t)-1\right)\mathrm{d}W_3(t)$$

By integrating the two sides of the above formula and taking the expectation, we get

$$\mathrm{E}\left[\int_{(\tau_k\wedge T)-\tau}^{\tau_k\wedge T}|\boldsymbol{X}(s)|^2\mathrm{d}s+V\left(\boldsymbol{X}\left(\tau_k\wedge T\right)\right)\right]\le K_1\,\mathrm{E}\left[\boldsymbol{X}\left(\tau_k\wedge T\right)\right]+\int_{-\tau}^{0}|\boldsymbol{X}(s)|^2\mathrm{d}s+ V(\boldsymbol{X}(0))$$

And we can deduce that

$$K_1T+\int_{-\tau}^{0}|\boldsymbol{X}(s)|^2\mathrm{d}s+V(\boldsymbol{X}(0))\ge\mathrm{E}\left[\mathrm{I}_{\{\tau_k\le T\}}(\omega)V\left(\boldsymbol{X}\left(\tau_k,\ \omega\right)\right)\right]\ge \mathrm{P}\left(\tau_k\le T\right)\left\{\left[\sqrt{k}-1-0.5\log\ k\right]\wedge\left[0.5\log\ k-1+\sqrt{\frac{1}{k}}\right]\right\}$$

Therefore let $k\to\infty,$ We can have $\mathrm{P}\left(\tau_\infty=\infty\right)=1.$

From the above process, we get the model (1) has a unique positive solution almost everywhere.

4. Conclusion

Under the background of industrial pollution, the original biological model is extended and deepened from the perspective of randomness and delay, and the stochastic delay differential equation model of the impact of industrial emissions toxins on human beings is established. The stochastic dynamic properties of the model are studied, and the theoretical feasibility of the model is proved theoretically.

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