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Study of light pollution risk level based on TOPSIS and integer programming summary

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Abstract: *High-intensity lighting has become an inevitable phenomenon in cities, so it is very important to establish an index system to measure the degree of light pollution risk. We selected the entropy weight method to determine the weight of each indicator that affects the risk of light pollution, and then used the TOPSIS model to quantitatively analyze the degree of light pollution risk and calculate the score. Finally, it was concluded that among the seven cities of Shanghai, Guangzhou, Kunming, Lhasa, Ningbo, Hohhot and Yinchuan, Shanghai had the highest light pollution risk score of 0.77, while Yinchuan had the lowest light pollution risk score of 0.23. This paper constructs a light pollution risk measurement system, which is of great significance to the measurement and prevention of light pollution.*

Keywords: *Entropy method, TOPSIS, Light pollution*

1. Introduction

With the rapid development of urbanization and industrialization, artificial light sources are increasing and the degree of light pollution is intensifying. The degree of light pollution has a very obvious impact on people's health [1], biodiversity [2] and astronomical observations.

So, it's essential to construct an index system for evaluating the risk level of light pollution in the region, to quantify and analyze the risk of light pollution according to each index of the region, and finally to give the risk level of light pollution in the location.

To address this problem, we first reviewed various relevant characteristics of the city from China Statistical Yearbook, Light Pollution Map, etc., and refined a total of six indicators of population, GDP, per capita consumption expenditure, DN value, sky brightness, and radiation, respectively, to make a comprehensive assessment of light pollution risk level from three aspects: population density, economic level, and optical indicators, and established a light pollution risk The evaluation index system of light pollution risk level was established. We then adopted the entropy weighting method to assign weights to each index and used the TOPSIS method to quantify the light pollution risk of each city.

2. Methods

2.1 Construction of light pollution risk level evaluation index system

Table 1: Evaluation index system of light pollution risk level

Tier 1 indicators	Tier 2 indicators
Population indicators	Population H_1
Economic indicators	GDP H_2
	Per capita consumption expenditure H_3
Optical indicators	sky brightness H_4
	DN H_5
	Radiation intensity H_6

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According to existing research conclusions, population, economy, and optical indicators can all reflect the risk level of light pollution to a certain extent [3] [4] The goal of our model is to evaluate the risk level of light pollution in the region. The selected influencing factors are considered from population indicators, economic indicators and optical indicators.The light pollution risk level evaluation index system is shown in Table 1.

2.2 Entropy weight method

EWM assesses value by measuring the degree of differentiation. The higher the degree of dispersion of the measured value is, the higher the degree of differentiation of the index is, and more information can be obtained. [5]

The entropy weight method calculates the weight steps as follows [6]:

1) According to the statistical data selected by the evaluation index system, an initial matrix is constructed, with m an evaluation object, n an evaluation index, and an initial matrix R:

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{1}$$

($r_{ij}, i=1,2,\cdots,n; j=1,2,\cdots m$ It is the value of the th evaluation index of the th m evaluation object n).

2) To standardize the data of various indicators that affect the risk level of light pollution, a normalization method can be used:

$$a_{ij}=\frac{r_{\max}-r_{ij}}{r_{\max}-r_{\min}} \tag{2}$$

($r_{\max}$ The maximum value under the same indicator, $r_{\min}$ the minimum value under the same indicator, a_{ij} and the normalized value of each indicator).

3) Calculate the proportion of the i evaluation object in the 1st j indicator p_{ij}:

$$p_{ij}=\frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \tag{3}$$

4) Calculate j the information entropy of the th indicator e_j:

$$e_j=-k\sum_{i=1}^{m} p_{ij}\cdot \ln p_{ij} \tag{4}$$

Among them $k=\frac{1}{\ln m}$.

5) Compute j the entropy weight for the th index w_j:

$$w_j=\frac{(1-e_j)}{\sum_{j=1}^{n}(1-e_j)} \tag{5}$$

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2.3 TOPSIS method

For the problem conditions, we give six TOPSIS indicators, and the model construction steps are as follows [7]:

1) The original matrix $R=(r_{ij})_{m\times n}$ is positively transformed to establish m an evaluation n system consisting of evaluation objects and evaluation indicators:

$$X=(x_{ij})_{m\times n}(i=1,2,\cdots m; j=1,2,\cdots n) \tag{6}$$

Normalize it to form a decision matrix $H=(h_{ij})_{m\times n}$, where:

$$h_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{n}x_{ij}^2}}(i=1,\cdots,m; j=1,\cdots n) \tag{7}$$

Multiply the weight calculated by the entropy weight method $w=(w_1,w_2,\cdots w_n)^T$ with the normalized decision matrix to obtain the weighted decision evaluation matrix:

$$Z=(z_{ij})_{m\times n},\ z_{ij}=w_j\cdot h_{ij} \tag{8}$$

2) Determine the best Z_j^+ and worst vectors Z_j^-:

$$Z_j^+=\max_{1\le i\le m}|z_{ij}|,\ Z_j^-=\min_{1\le j\le n}|z_{ij}| \tag{9}$$

Compute the distance of the selected $i(i=1,2,\cdots,m)$ city's index from the optimal vector D_i^+ and the worst vector D_i^-:

$$D_i^+=\sqrt{\sum_{j=1}^{n}(Z_j^+-z_{ij})^2},\ D_i^-=\sqrt{\sum_{j=1}^{n}(Z_j^--z_{ij})^2} \tag{10}$$

Computes $i(i=1,2,\cdots,m)$ the unnormalized score for the th city S_i:

$$S_i=\frac{D_i^-}{D_i^++D_i^-},\ S_i\in[0,1] \tag{11}$$

S_i is the light pollution degree of the first city, generally called the fit degree, and its range is $[0,1]$. S_i The smaller the value, the lower the degree of light pollution in the city, and S_i the larger the value, the higher the degree of light pollution in the city.

According to the existing research results, the degree of fit is graded to represent the risk level of urban light pollution, as shown in the Table 2:

Table 2: Criteria for Judging Urban Light Pollution Risk Level

Close degrees	Level of risk
(0,0.3]	Low
(0.3,0.65]	Medium
(0.65,0.8]	High
(0.8,1]	Higher

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3. Results

According to the formulas (1)~(5), we calculated the weight and information entropy of the six indicators, as shown in the Table 3:

Table 3: Index weight table

Indicators	H_1	H_2	H_3	H_4	H_5	H_6
Weights	0.1582	0.2094	0.2655	0.1151	0.1342	0.1176
Information entropy	0.768	0.694	0.611	0.803	0.828	0.832

We choose Shanghai C_1, Guangzhou C_2, Kunming C_3, Lhasa C_4, Ningbo C_5, Hohhot C_6 and Yinchuan C_7 as the evaluation objects to calculate the light pollution risk level. According to the formulas (6)~(10), the sum of the distances from the evaluation object to the optimal vector and the worst vector from 2018 to 2021 is calculated as shown in the Table 4:

Table 4: The distance between the evaluation vector and the optimal and worst vector

Indicators	C_1	C_2	C_3	C_4	C_5	C_6	C_7
D_i^+	0.27751	0.52395	0.74756	0.93661	0.68149	0.85806	0.86864
D_i^-	0.94270	0.60765	0.36332	0.33927	0.36700	0.28470	0.27310

According to the formula (11) to calculate the fitting degree, the results are as shown in the Table 5:

Table 5: Normalized score of urban light pollution risk level

Cities	Close degrees
C_1	0.7725
C_2	0.5369
C_3	0.3271
C_4	0.2660
C_5	0.3500
C_6	0.2492
C_7	0.2393

4. Discussions

TOPSIS model contains several constant parameters. When determining the parameters, we use the entropy weight method to determine the weight of each index. In this section, we test the sensitivity of the TOPSIS model based on the entropy weight method by changing the parameter values and comparing the difference between the original results and the changed results to show its reliability. We fluctuate the weight of GDP by 5%, and keep other indicator parameters unchanged. Each curve in the Figure 1 represents the score of each city when the weights are different values. It is found that the curves all show the same trend and the fluctuation of the score value is small, so the TOPSIS model is not sensitive to the weight value of GDP and has good robustness.

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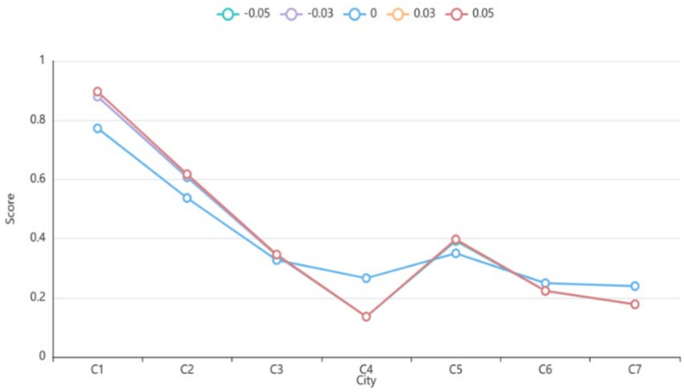

Figure 1: Sensitivity analysis

5. Conclusion

There are many factors that affect the degree of light pollution risk. In this paper, the entropy weight method is used to process the data of each influencing factor, so that the weight of each indicator is objective. After that, the TOPSIS model is used to construct an evaluation system for the degree of light pollution risk. The light pollution risk score and determination of its level are of great significance for the measurement and evaluation of light pollution risk.

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