Innovation and Practice of the Linear Algebra Teaching Based on “the Unity of Knowledge and Action”

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Abstract: As the fruit of human thought and wisdom, the education law of “the unity of knowledge and action” is an intrinsic requirement for the harmonious and comprehensive development of the human body and mind. It is also the trend of modern education development. For the pain point problems in the traditional teaching of Linear Algebra (LA) in colleges and universities, the course team implemented the concept of "the unity of knowledge and action" based on professional needs. The team built the four innovation measures of "integrating teaching content, transforming teaching forms, optimizing teaching modes, and strengthening teaching practice" to guide students to learn in depth. The team also extended two methods of "project practice and mathematical modeling" to conduct teaching innovation exploration and practice. In this paper, we took the "inverse matrix" as a teaching example to demonstrate the innovative ideas and measures of promoting knowledge with action, promoting action with knowledge, and keeping common progress in knowledge and action. The teaching feedback showed that innovating reforms can effectively solve the pain points and achieve comprehensive improvement in knowledge-acquiring, capacity-cultivating, and value-leading.

Keywords: the unity of knowledge and action; pain point problems; teaching innovations

1. Introduction

Since ancient times, knowledge and action have been classic problems involving human growth in the philosophy and pedagogy fields. In the fourth year of Emperor Wuzong of the Ming Dynasty (1509), Shouren Wang gave a lecture at the Civilization Academy in Guiyang and first proposed "the unity of knowledge and action" theory [3]. The theory mainly includes two aspects: one is that there is action in knowledge, and vice versa, and the two are inseparable from each other; the second is that learning knowledge is acting, knowledge determines the action, knowledge is the beginning of the action, and action is the completion of knowledge. Chinese educator Xingzhi Tao's educational theories [9], such as "life is education", "society is school", and "teaching, learning, and doing are one" contain rich scholarly connotations of “the unity of knowledge and action." The relationship between knowledge and practice has also always been a fundamental issue in Western philosophy and education. In 1996, UNESCO proposed the four pillars of education in the Delors Report [7], including learning to know, learning to do, learning to live together, and learning to survive, and argued that education should conform to these four essential learning. With the rapid advancement of science and technology and the changes of the world pattern, strengthening innovation and practical teaching, reforming learning methods, and developing students' higher-order thinking have become the consensus and trend of international education reform.

Based on “the unity of knowledge and action,” and by combining theory and practice, the teaching innovation can make the initially boring mathematical knowledge more dynamic and make the knowledge easier for students to internalize by integrating “knowledge” into “action” to achieve teaching goals. The teachers should create a comprehensive performance environment for students in the teaching process to inspire students’ innovative thinking in activities easily. Besides cultivating students' cognitive ability of knowledge in the teaching process, it is also necessary for teachers to closely connect the activities in and out of the classroom to guide students’ actions with knowledge, to test their knowledge with action, and to achieve “the unity of knowledge and action.” Hui Liu introduced the teaching methods of the construction of the meaning of “the unity of knowledge and action”, “checking one’s thoughts, words and deeds to meet the moral requirements”, and "tempering and adjusting oneself to be in a stable
state gradually” [5]. The scholar pointed out that the teaching methods mentioned above have great value for realizing the characteristics of self-identity, self-control, and behavior habits in personal moral formation. Several Chinese scholars discussed the application and practice of “the unity of knowledge and action” in teaching [4,10-12]. The mixed teaching reform of college mathematics was investigated [2], and the teaching effects by employing artificial intelligence algorithms was evaluated [1]. The online teaching models in Chinese and American universities were compared [10]. Western education also attached great importance to practical education, such as “the curriculumization of the extracurricular activities,” “problem-oriented learning,” and “applied to learn.” Triantafyllou E. and Possani E. explored the teaching of college mathematics and linear algebra from the perspective of modes and games respectively [6,8].

As an introductory public course of mathematics in colleges and universities, Linear Algebra (LA) cultivates students’ ability to discover, propose, and solve discrete problems by adopting linear space theory and matrix modeling methods, and provides the necessary mathematical foundation for the learning of the subsequent professional courses and the pursuit of scientific research. There are some pain points in traditional mathematics teaching. The first is the insufficient integration of “basic teaching” and “professional education.” With the rapid development of science and technology, disciplines and specialties are constantly refined, the market demand for talents is diversified, the degree of mastering mathematical knowledge and focus requirements are different. Under the admission of profession priority, the difference between the mathematical foundation and learning ability of students in different majors is further expanded. As a public foundational course, LA should adjust its teaching objectives and teaching plan to meet different training requirements according to output orientation. According to professional characteristics, how to integrate disciplines by changing from "single" to "comprehensive" to improve students' professional quality? The second is the insufficient integration of “teaching students according to their aptitude” and “teaching environment.” Due to the limited educational resources, the universities usually arrange mathematics in a large class of more than 100 students. It is difficult for students to interact efficiently and maintain an active classroom atmosphere in such a large class. The large base is also not conducive to teachers' clear understanding of students' characteristics and personality differences, and the learning situation cannot be timely fed back. How to teach according to students’ aptitude by teaching renovation? How to change students’ passive learning to active learning? And how to develop students’ advantageous potential? The final is the insufficient integration of “teaching activities” and “innovative practices.” Because the problems that the teaching activities of the course only focused on doing exercises and the teaching evaluation only relied on exams, college mathematics education became an extension of high school cramming education. Thus, some students are only absorbed in simple subject acquisition and repeated exercises. How to cultivate students' creative ability according to reality? How do students change from "textbooks" to "practice" style?

Guided by “the unity of knowledge and action,” the research group (TRG) conducted the teaching reform targeting LA. The second section of the paper introduced the four innovative measures of “integrating teaching content, transforming teaching forms, optimizing teaching modes, and strengthening teaching practice” built by combining professional requirements and realizing the integration of fundamental teaching of LA with professional education, the conjunction of “teaching students according to their aptitude” and teaching environment, and the combination of teaching activities and innovative practice by extending two methods of "project practice and mathematical modeling." The third section dealt with the innovative exploration and practice of “the unity of knowledge and action” classroom teaching. TRG illustrated the teaching design scheme by taking the "inverse matrix" as an example and conducting deep learning by guiding students to think. The fourth section was the teaching feedback and reflection. The survey results showed that the teaching effect was outstanding, and students had highly recognized the innovation and reform. The last part was the summary. The result showed that the innovative reform scheme could effectively solve pain points and achieve comprehensive improvement in knowledge acquisition, capacity training, and value leadership. However, new problems will emerge, which need to be further explored.

2. Innovative Ideas and Measures for Teaching

First, the course teachers determined the teaching objectives of LA from three dimensions. The knowledge objective of LA was to enable students to master LA's basic knowledge, theories, and methods. The students could establish linear spatial awareness and master linear transformation ideas and matrix modeling methods by processing and analyzing discrete linear problems, such as linear equations. The ability objectives were to enable students to transform practical problems into linear ones, to solve problems with linear thinking and matrix methods, and to cultivate students' creative thinking ability,
abstract generalization ability, and logical reasoning ability to excel at induction, analogy, and association. The emotional goal was to develop students’ thinking habit of "describing the world by linear space, explaining phenomena by linear transformation, solving problems by matrix modeling, and understanding the essence through phenomena" to establish a rigorous and realistic style with awareness of daring to explore and innovate, and to cultivate a sense of responsibility and purpose to pursue truth and bravely climb the heights of science. The course team implemented teaching innovation reform to achieve the teaching goals and implement specific measures.

Based on the orientation of the university and professional settings, TRG optimized the course teaching content. The course gave full play to the educational function of the curriculum geared toward professional needs. It deeply explored the emotional elements contained in LA, from philosophical thought, scientific spirit, scientific thinking, scientific consciousness, and the beauty of mathematics. TRG designed featured cases related to the profession to achieve the goals of the profession and features educating people. The course was designed categorically and applied the teaching of typical cases to stimulate the "demand traction" motivation by integrating the theoretical knowledge of LA with students’ professions. The course met the needs of students’ subsequent professional courses and high-level matrix computing in engineering problems, illustrated MATLAB to achieve “technology promotion.” Based on the multi-dimensional teaching content meeting the needs of output-oriented professional talent training, the four-in-one “basic theory, emotional elements, application cases, and mathematical software” integrated knowledge, ability, and value cultivation in an all-round way. This effectively solved the pain points of the single structure in teaching content and decoupling from professional education. Table 1 showed the teaching content of LA, where the application cases can be chosen independently based on the different majors.

Table 1: The four-in-one Teaching Content of LA

<table>
<thead>
<tr>
<th>Chapter Contents</th>
<th>Emotional Elements</th>
<th>Application Cases</th>
<th>Mathematical Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determinant/ Matrix/ Vector and Vector Space/ Linear Equations/ Similar Matrix/ Quadratic Form</td>
<td>From unique to general / Change and invariability, unity of opposites / Qualitative by quantity/consciousness of rules/ Deformation and invariance /Local to the whole/Finite and infinite/Universal connection/Principle of primary contradiction / Reductive thinking/Process and result / General and special/Rigorous/Patient persistence/Fighting spirit/Innovation consciousness/Team consciousness / Concise beauty / Symmetrical beauty / Structural beauty</td>
<td>Calculation of graphic area and object volume / Linear combination of atomic orbitals into suborbital / Pixel grayscale matrix / Image transformation / Ceramic formula design /Cryptography problem/ Three primary colors / Meteorological observation station problem / Allocation problem of proprietary Chinese medicine / Chemical composition structure / Chemical reaction equation balance/ Nutritional recipe problem / Economic price balance / Web ranking algorithm / Infectious disease model / Application of matrix diagonalization in differential equations / Temperature distribution during cooling of chemical machinery / Conditional extrema of quadratic function / Application of positive definite quadratic form in linear regression model/ Application of quadratic form in degrees of freedom/Enterprise device update</td>
<td>Use MATLAB to calculate determinants/ perform matrix operations/ solve vector problems/solve linear equations/ solve matrix problems/ perform quadratic operations.</td>
</tr>
</tbody>
</table>

The course pursued academic support for curriculum construction by relying on experts and platform resources. The course team invited famous teachers and lecturers of related majors to build the "virtual teaching and research office" together. Besides, they also regularly conducted teaching and research activities aiming at problem-oriented, cross-integration, forward-looking research, and dynamic construction. They also promoted the exchanges and cooperation of cross-college and cross-university and the sharing of high-quality resources to integrate LA teaching and professional education. The primary teacher of the course was a member of the lecturer group of an excellent LA online open course in Chinese University MOOC and implemented blended teaching through high-quality course resources.

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The course team transformed the teaching form, implemented online and offline blended teaching, replaced the "large class teaching" with "online self-learning," and adopted flipped teaching in offline teaching. The course team also organized "group discussions," conducted deep learning by guiding students to think, built a high-level smart classroom, and effectively tracked students' learning through the information-based teaching network platform. Table 2 showed the teaching activities and evaluation of different stages (pre-class, in-class, and post-class) in blended teaching design.

**Table 2: Blended Teaching Design**

<table>
<thead>
<tr>
<th>Stages</th>
<th>Activities(S: student, T: teacher)</th>
<th>Evaluation Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-class: self-study phase of the blended</td>
<td>S: Watch teaching videos or other teaching materials</td>
<td>Summarize basic knowledge points and form a preview report</td>
</tr>
<tr>
<td>teaching design</td>
<td>S: Put forward questions in the online discussion area</td>
<td>Two points/question, each person submits at least one piece per chapter, calculates the average score</td>
</tr>
<tr>
<td></td>
<td>S: Complete concept dictionary online</td>
<td>Two points/time, each person completes each chapter at least once, calculates the average score</td>
</tr>
<tr>
<td></td>
<td>S: Answer 2-5 pre-test questions</td>
<td>Two points/valid answer, each person submits at least one answer per chapter and calculates the average score</td>
</tr>
<tr>
<td></td>
<td>T: Review students' preview report</td>
<td>Three points/question, automatically remarked by the system.</td>
</tr>
<tr>
<td>In-class: teaching phase of the blended</td>
<td>T: Summary of online learning</td>
<td>Three grades: excellent, good, and qualified</td>
</tr>
<tr>
<td>teaching design</td>
<td>T: Analysis of key points and difficulties</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T: Explanation of typical examples</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S&amp;T: Classroom activities (choice questions, judgment questions, etc.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T: Group discussion, problem exploration, and solution</td>
<td></td>
</tr>
<tr>
<td>Post-class: summary and improvement phase of</td>
<td>S: Summary</td>
<td>Encourage students with fewer points to report</td>
</tr>
<tr>
<td>the blended teaching design</td>
<td>T: Online operation</td>
<td>Automatically remarked by the system.</td>
</tr>
<tr>
<td></td>
<td>T: Offline operation</td>
<td>Four grades: A, B, C, and D</td>
</tr>
<tr>
<td></td>
<td>T: Online group work (mutual evaluation within the group)</td>
<td>Up to ten points</td>
</tr>
<tr>
<td></td>
<td>T: Submit chapter mind maps</td>
<td>Up to five points/article</td>
</tr>
<tr>
<td></td>
<td>T: Submit a monograph (optional)</td>
<td></td>
</tr>
</tbody>
</table>

The course group studied the history of science and technology and biographies and analyzed the worldview and methodology embodied in LA. In addition, the course team excavated LA's ideological and educational materials. TRG also explored the moral education elements embodied in the extension of course knowledge and concepts by combining course teaching with innovative practice and guiding project-based research. Moreover, TRG cultivated students’ scientific spirit to make students observe the real world with mathematical eyes, think about the natural world with mathematical thinking, and express the real world in mathematical language. TRG evaluated the simultaneous improvement of "knowledge transfer, ability improvement, and value leadership" in necessary teaching materials such as training plans and syllabuses. TRG conducted science popularization activities to help students deeply understand and promote mathematical theory and its application knowledge. Furthermore, TRG also carried out the learning style by relying on curriculum construction and teaching activities to change students' learning concepts and effectively reduce the homogenization of the coursework.

TRG implemented the integration of theory, practice, competition, and teaching, strengthening the teaching practice and enhancing students' innovation ability. TRG selected students to join the scientific research teams to make them systematically complete the teaching practice process through classroom teaching, competition training, and various innovation and entrepreneurship projects. TRG formed a "platform + module + project" internship training mode that integrated theory and practice, and expanded with modeling cases and big data projects, accurately serving the cultivation of students’ creative, practical abilities. And the course also took modeling activities as the platform, combined various majors to solve different practical cases and implement different projects, and finally built a modeling multi-module case library based on mathematical practice. The course strengthened the cooperation with the faculties and departments where the major was located, allowed students to participate in projects directly, realized the seamless connection of teaching, competition, and practice, and effectively improved students' creative practice ability.
3. The Teaching Innovation Practice Cases

In this part, TRG took the "inverse matrix (IM) " in LA as an example to carry out the exploration and practice of "the unity of knowledge and action" teaching innovation. After learning the addition, subtraction, and multiplication of matrices, the inverse matrix was a new operation, a basic but abstract concept. It’s difficult for students to learn IM, mainly because of the abstraction of the concept, the unknown source of the concept, and the ambiguity of the role of the concept in matrix operations. The concept was the basis of subsequent course learning. Only the students who understood the essential role of IM in matrix operations could lay a solid foundation for subsequent knowledge learning.

This section's teaching content was essential to algebra theory and specific assessment content in previous postgraduate examinations in China. Most universities arranged the course in the first year, and students were enthusiastic about learning, active in thinking, and fast in accepting information-based teaching methods; however, students had differences in their learning control ability and showed the characteristics of stratification. The teaching key points were the judgment and calculation method of particular IM and abstract IM. The teaching difficulties were the calculation of IM and the judgment and verification methods of IM.

The teaching process was student-centered, highlighting key points and breaking through difficult points by promoting knowledge with action, promoting action with knowledge, and keeping common progress in knowledge and action. The teaching ideas mainly included online previews, classroom teaching, and after-class learning. In the online preview stage, students completed online video learning and the test on time through the class learning group one week in advance. Two days before class, the teachers checked the data from the platform to master the students’ online learning situation, announced the online learning results in the group, and urged the delayed ones to finish online. The day before the class, the teachers should ensure that all students complete the online preview. In the classroom teaching stage we used Rain Classroom (RC) as the interactive teaching platform between teachers and students. 10 minutes before class, the students would scan the RC code to join the class and sign up. The teacher designed some questions and 11 activities to promote students' active participation in learning, organized group discussions, switched roles to set questions and conducted deep thinking and learning to internalize knowledge into cognition. Within one week after class, the teacher would remind students to complete the learning test and discussion on the Superstar Learning App, guided students to discuss and comment on the questions sent in the group, urged students to complete assignments and evaluate each other’s work. Figure 1 was the schematic diagram of the lesson process.

![Figure 1: The Schematic Diagram of the Lesson Process](image)

3.1 Pre-class: promoting knowledge with action and integrating applied teaching resources

The teacher assigned practical activities before class asked students to query the application of IM in Hill cipher and the application of chemical reaction equation equilibrium. The students required to preview the concept and basic properties of IM through MOOCs and to analogize with reciprocals of non-zero numbers. Students' personal experiences captured the resources and basic knowledge. Therefore, the students would be more focused in the classroom and more active in exploring the new knowledge of the lesson to achieve the "promoting knowledge with action" effect.
3.2 In-class: promoting action with knowledge and giving full play to students’ subjective initiative

Lead-in: Students shared the application cases they queried and the problems to be solved.

Activity 1. Posted a single-choice question through the RC: does the matrix

\[
A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \end{pmatrix}
\]

have an IM? Why?

Activity 2. Posted a poll question through the RC: if matrix A is reversible, is A’s inverse matrix unique?

Design intent: To master students’ previews and carry out the framework between mathematical intuition and rigor to pave the way for the analysis of the characteristics of the IM.

Activity 3. Organized students to discuss and explain the connotation of the concept of IM and prove the uniqueness of IM while existing.

Design intent: To enable students to deeply understand and master the concepts and essential characteristics of IM.

E.g. 1. Determine whether the following matrices are reversible. If yes, find its IM.

\[
A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}; \quad A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.
\]

The teacher explained (1) of e.g.1 in detail by indicating the idea being used, i.e., the undetermined coefficient method derived from the definition.

Activity 4. Students did exercise (2) in, e.g.1, and gave the answers.

Design intent: Following the above question, derive the specific calculation method by definition and develop an "abstract" theory into a "specific" calculation.

Reflection 1. Does B calculate from \(AB=E\) in (2) of e.g.1 also satisfy \(BA=E\)? Is this conclusion commonly proper?

Reflection 2. Can people use the undetermined coefficients method to determine whether the third-order matrix is reversible and get IM? Is the operation simple? Are there other ways to determine the reversibility of a matrix and calculate IM?

Design intent: Following the above, the calculation will increase with the order of the square matrix. To guide students to explore other calculation methods of matrix inversion can pave the way for subsequent content.

By reviewing the adjoint matrices and their properties, the teacher could guide students to analyze the decidability theorem of the irreversible matrices.

Theorem: Square matrix A is reversible \(\iff \det(A) \neq 0\), and

\[
A^{-1} = \frac{1}{\det(A)} A^*.
\]

E.g.2. Use the adjoint matrix method to determine whether the following matrices are reversible. If yes, find its IM.

\[
A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}; \quad A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.
\]

Activity 5. Students practiced independently and give answers.

Design intent: Using the fact that the determinant is not 0 to quickly determine the reversibility. To reflect the advantages of the adjoint matrix method.
E.g.3. Given that 
\[
A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -1 & 4 \\ -1 & 2 & -4 \end{pmatrix}
\]

is A reversible? If yes, find its IM.

Design intent: (1) To enable students to practice the learned adjoint matrix method to determine the reversibility of the matrix and find the inverse matrix; (2) To enable students to experience that with the increase of matrix order, the calculation is still extensive, which paves the way for the emergence of the IM method of elementary transformation and provides a basis for the introduction of MATLAB software into course teaching.

Activity 6. Proved that there also stands \( BA = E \) while \( AB = E \).

Design intent: To use the sufficient necessary conditions of matrix reversibility and analyze the simplified form of the IM concept.

Activity 7. Students solved their own cryptological problems they queried or chemical equation equilibrium problems and present them in small groups.

Design intent: Showing the solution process of the cases proposed by students in leading-in: 
\( AB = C \Rightarrow A = CB^{-1} \) or \( BA = C \Rightarrow A = B^{-1}C \). Its process is essentially the solution of matrix equations.

E.g.4. Given that the square matrix A satisfies the equation \( A^2 - A - 2E = 0 \), prove that A and \( A+2E \) are reversible and find their inverse matrices.

Activity 8. The students worked in groups to find the target formula and how to match the equal formula.

Design intent: To guide students to use the analogy to carry out the identity deformation of abstract matrix equations and to strengthen their understanding of the concept of IM.

Activity 9. Posted a single-choice question through RC: given a square matrix A which satisfies the equation \( A^2 - 4A + E = 0 \), prove that \( A - 3E \) is reversible, and its inverse matrix is ( )

\[
\frac{A-E}{2} \quad \text{(A)} \quad \frac{E-A}{2} \quad \text{(B)} \quad \frac{A-E}{2} \quad \text{(C)} \quad \frac{E-A}{2} \quad \text{(D)}
\]

Design intent: To justify students’ learning results and reinforce their cognition.

Activity 10. Imitated e.g. 4 to set a question for one classmate.

Design intent: To strengthen the application of IM concepts and determination methods to internalize knowledge.

Activity 11. Students experienced putting on and taking off shoes and socks, and analyzed the computational properties of the IM:

1. If A is reversible, then \( A^{-1} \) is also reversible and \( (A^{-1})^{-1} = A \).
2. If A is reversible, \( \lambda \neq 0 \), then \( \lambda A \) is also reversible and \( (\lambda A)^{-1} = \frac{1}{\lambda} A^{-1} \).
3. If matrices A and B are reversible with the same order, then AB is also reversible and \( (AB)^{-1} = B^{-1}A^{-1} \).

Extension: \( (ABC)^{-1} = C^{-1}B^{-1}A^{-1} \).

Design intent: To make students master the inverse operation of matrices in different forms, especially the inverse of matrix products, and pay attention to the order of products.

Summary: The session introduced the concept of IM, the existence and method of IM, the operation
nature of IM, the solution of matrix equations, and finally formed the mind map.

3.3 Post-class: keeping common progress in knowledge and action, conducting project-based guidance and modeling training

After class, students were required to use MATLAB to verify the password problems they queried or the chemical equation equilibrium and wrote an essay on "The Application of Inverse Matrices." Students also chose one of the five topics of "Philosophical Thought, Scientific Spirit, Scientific Thinking, Scientific Consciousness, and the Beauty of Mathematics" to write and submit their learning experience. In addition, they also previewed and watched the related videos to think about "how to solve IM for the higher-order matrices."

4. Teaching Feedback and Reflection

Two rounds of questionnaire surveys were conducted before and after the implementation of the teaching reform. The questions were: (1) Do you acknowledge the current teaching content and teaching methods? (2) Do the project cases selected by the teacher help to improve your interest in learning and understanding of knowledge? (3) Do the group activities in the classroom help improve your learning? (4) Is interspersing the class with emotional elements related to the course content beneficial for your learning? The options were: (a) strongly agree, (b) agree, (c) slightly agree, and (d) disagree. A total of 120 students from different teaching classes of the same major in two grades were selected for the survey, and the statistical results of the answers to the four questions are shown in Figure 2.

![Figure 2: Analysis of Questionnaire Results](image)

After several years of teaching reform, students' acknowledgment of the teaching content, methods, cases, and practices of LA improved dramatically. It showed that the project was on the right track and achieved apparent results. From the school management perspective, the LA's student evaluation was high, and the teaching evaluation of the team teachers was excellent. 98% of students believed that case teaching and MATLAB were convenient, and students benefited greatly from mathematical modeling. Undergraduate students have published five pieces of paper related to mathematical modeling. The research has tracked 60 students majoring in materials in 2020; 28 students were admitted to graduate schools, students participating in mathematics-related subject competitions won 23 awards, and more than ten students participated in innovation and entrepreneurship projects.
5. Summary

Based on the concept of “the unity of knowledge and action,” the course explored and practiced the innovative teaching mode of four innovative measures of "integrating teaching content, transforming teaching forms, optimizing teaching modes, and strengthening teaching practice" and “two wings,” and realized the transformation from traditional teaching to information-based teaching, from knowledge teaching to ideological and political teaching, from ability teaching to practical teaching, and from the offline teaching to the blended teaching. In the future, the teaching process should fully adopt modernized information technology to guide inactive and unenthusiastic students. Through the reflection and practice of the pre-class, in-class, and post-class activities, the course took students as the foundation and problems as the core, taught students according to their aptitude, and gave full play to students' learning initiative and enthusiasm. Teaching reform and innovation need to be further refined, explored, and expanded to promote the overall development of students with the cultivation idea of “stimulating interest-improving ability-innovating talents.” What's more, many issues still need to be improved, such as the effectiveness of teaching activities, the construction of a diversified teaching evaluation system, etc., which require educationists to explore.

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