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The Application of Infinite Series in College Students' Mathematics Competition

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Abstract: *Infinite series is an important part of higher mathematics course, and it is also an important content of college students' mathematics competition. It is an indispensable tool for studying functions. It not only plays an important role in modern teaching methods, but also has a wide range of applications in a large number of applied sciences such as differential equations and numerical calculations. Studying how to use the knowledge points of infinite series to solve problems and its application in college students ' mathematics competitions will help students better understand and master the content, and also have certain reference value for teachers' teaching.*

Keywords: *Infinite Series, Problem-solving Methods, National College Students Mathematics Competition*

1. Introduction

Infinite series is an important part of higher mathematics. It is a tool for representing functions to study the properties of functions and to carry out numerical calculation. In practical applications, infinite series are widely used in physics, engineering, statistics, finance and other fields. In the field of physics, infinite series can be used to describe the distribution of electric field and magnetic field, the propagation of light and the form of wave function. In the field of engineering, infinite series can be used to calculate the strength of the structure, the performance of the material, the sound wave and heat transfer and so on. In statistics, infinite series can be used to establish probability distribution, analyze random variables and estimate probability density function [1-5].

2. Research Content

Through the analysis of the content of the infinite series part in the real questions of the 1-14 th National College Students ' Mathematics Competition from 2009 to 2022, this paper summarizes the knowledge points of the infinite series and its problem-solving methods in detail, and lists the application of its problem-solving methods in the College Students ' Mathematics Competition, as follows [6-10].

1) (The first National College Students' Mathematics Competition in 2009, 15 points) Known $u_n(x)$ meet $u_n'(x) = u_n(x) + x^{n-1}e^x$ (n is a positive integer).farther $u_n(1) = \frac{e}{n}$, find the series of function terms.

Analysis: Find the sum of $\sum_{n=1}^{\infty} u_n(x)$, and the general term $u_n(x)$ is unknown, so first find $u_n(x)$.

Solution: $u_n'(x) - u_n(x) = x^{n-1}e^x$ (first-order non-homogeneous linear differential equation) is obtained from $u_n'(x) = u_n(x) + x^{n-1}e^x$ .general solution formula $u_n(x) = e^{-\int(-1)dx}(\int x^{n-1}e^x e^{\int(-1)dx}$

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$dx+C) \quad = e^x(\int x^{n-1}e^x e^{-x}dx+C) \quad = e^x(\frac{x^n}{n}+C)$. $C=0$ is obtained from $u(1)=e(\frac{1}{n}+C)=\frac{e}{n}$, thus $u_n(x)=\frac{x^n e^x}{n}$, then $\sum_{n=1}^{\infty}u_n(x)=\sum_{n=1}^{\infty}\frac{x^n e^x}{n}=e^x\sum_{n=1}^{\infty}\frac{x^n}{n}$ .That is, the problem becomes the sum function of $\sum_{n=1}^{\infty}\frac{x^n}{n}$ .Note: for $\sum_{n=1}^{\infty}\frac{x^n e^x}{n}=e^x\sum_{n=1}^{\infty}\frac{x^n}{n}$ summation, n is a variable, e^x is equivalent to a constant, which can be mentioned outside the series summation symbol. The first step: find the convergence domain $R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{1/n}{1/(n+1)}=1$ convergence interval $(-1,1)$ .When $x=-1$, the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges.When $x=1$, the series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, so the convergence domain $[-1,1)$. Step 2: Sum function in convergence interval $(-1,1)$ .Let $S(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}$, $S'(x)=(\sum_{n=1}^{\infty}\frac{x^n}{n})' \quad =\sum_{n=1}^{\infty}(\frac{x^n}{n})' \quad =\sum_{n=1}^{\infty}x^{n-1}=\sum_{n=0}^{\infty}x^n=\frac{1}{1-x}$ (arithmetic progression). Step 3: Determine the sum of the series at the convergence endpoint. When $x=-1$, the series is $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$, and the continuity $S(-1)=-\ln 2$ of the power series in the convergence domain is.Then $\sum_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x)$, $x\in[-1,1)$, and thus $\sum_{n=1}^{\infty}u_n(x)=-e^x\ln(1-x)$, $x\in[-1,1)$.

2) (The 2nd National College Students ' Mathematics Competition in 2010, 15 points)Let $a_n>0$, $S_n=\sum_{k=1}^{n}a_k$. Proof: (1) When $\alpha>1$, the series $\sum_{n=1}^{+\infty}\frac{a_n}{S_n^{\alpha}}$ converges; (2) When $\alpha\le 1$ and $S_n\to\infty(n\to\infty)$, the series $\sum_{n=1}^{+\infty}\frac{a_n}{S_n^{\alpha}}$ diverges.

Analysis: the basic idea of judging the convergence and divergence of series by means of comparative method and basic properties of series: (1) comparative discriminant method of positive series; (2) Definition method based on convergence and divergence of partial and sequence. By rewriting the form of the general term, it is transformed into a description of a familiar problem. The general term of $\sum_{n=1}^{+\infty}\frac{a_n}{S_n^{\alpha}}$ is $u_n=\frac{a_n}{S_n^{\alpha}}$, $a_n>0$, $S_n=\sum_{k=1}^{n}a_k$, S_n is the partial sum of a_n, so S_n can be rewritten.

Proof: (1) Because $a_n>0$, $S_n=\sum_{k=1}^{n}a_k$, $S_1=a_1$, $S_2=a_1+a_2$, $S_3=a_1+a_2+a_3$,……, $S_n=$ $a_1+a_2+a_3+\cdots+a_n$. It can be seen that S_n is strictly monotonically increasing, $\frac{1}{S_n}$ is strictly monotonically decreasing, then $\frac{1}{S_{n-1}}-\frac{1}{S_n}>0$, and thus $\sum_{k=2}^{n}(\frac{1}{S_{k-1}}-\frac{1}{S_k})=\frac{1}{S_1}-\frac{1}{S_2}+\cdots+\frac{1}{S_{n-1}}$ $-\frac{1}{S_n}=\frac{1}{S_1}-\frac{1}{S_n}$ .That is, $\sum_{k=2}^{+\infty}(\frac{1}{S_{k-1}}-\frac{1}{S_k})=\frac{1}{S_1}-\lim_{n\to+\infty}\frac{1}{S_n}$, Then the series converges. S_n plus

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a α power, $\alpha > 1$, for S_n^α, the same analysis method. S_n^α is strictly monotonically increasing, $\frac{1}{S_n^\alpha}$ is strictly monotonically decreasing, then $\frac{1}{S_{n-1}^\alpha} - \frac{1}{S_n^\alpha} > 0$. Therefore $\sum_{k=2}^{n}(\frac{1}{S_{k-1}^\alpha} - \frac{1}{S_k^\alpha}) = \frac{1}{S_1^\alpha} - \frac{1}{S_2^\alpha}$ $+\cdots+\frac{1}{S_{n-1}^\alpha} - \frac{1}{S_n^\alpha} = \frac{1}{S_1^\alpha} - \frac{1}{S_n^\alpha}$, quasi $\sum_{k=2}^{+\infty}(\frac{1}{S_{k-1}^\alpha} - \frac{1}{S_k^\alpha}) = \frac{1}{S_1^\alpha} - \lim_{n\to+\infty}\frac{1}{S_n^\alpha}$ .When $\alpha > 0$, the subseries converges. Because of $S_n - S_{n-1} = a_n$, $\frac{1}{S_{n-1}^\alpha} - \frac{1}{S_n^\alpha} = S_{n-1}^{-\alpha} - S_n^{-\alpha}$, the difference between the values of two functions, we first think of the Lagrange mean value theorem $f(x) = x^{-\alpha}$, $f'(\xi) = \frac{f(b)-f(a)}{b-a}$. $\frac{1}{S_{n-1}^\alpha} - \frac{1}{S_n^\alpha} = -\alpha\xi^{-\alpha-1}(S_{n-1} - S_n) = \alpha\xi^{-\alpha-1}(S_n - S_{n-1}) = \alpha\xi^{-\alpha-1}a_n$ $(S_{n-1} < \xi < S_n)$ quasi $\frac{1}{S_{n-1}^\alpha} - \frac{1}{S_n^\alpha} = \alpha\frac{a_n}{\xi^{\alpha+1}} > \alpha\frac{a_n}{S_n^{\alpha+1}}$. From $S_{n-1} < \xi < S_n$ to $\frac{1}{\xi} > \frac{1}{S_n}$, for the above formula, replace α with all $\alpha - 1$, $\alpha > 1$, $\alpha - 1 > 0$, then $\frac{1}{S_{n-1}^{\alpha-1}} - \frac{1}{S_n^{\alpha-1}} = (\alpha-1)\frac{a_n}{\xi^\alpha} >$

$(\alpha-1)\frac{a_n}{S_n^\alpha}$ .Because the series $\sum_{n=1}^{+\infty}\frac{a_n}{S_n^\alpha}$ converges, the multiplication of non-zero constant $(\alpha-1)$ does not change the convergence and divergence of the series, so the $\sum_{n=1}^{+\infty}(\alpha-1)\frac{a_n}{S_n^\alpha}$ series converges and the $\sum_{k=2}^{+\infty}(\frac{1}{S_{k-1}^{\alpha-1}} - \frac{1}{S_k^{\alpha-1}})$ converges. According to the comparison criterion of convergence and divergence of positive series, $\sum_{n=1}^{+\infty}(\alpha-1)\frac{a_n}{S_n^\alpha}$ converges, so when $\alpha > 1$, the series $\sum_{n=1}^{+\infty}\frac{a_n}{S_n^\alpha}$ converges.

(2) Proof: the partial sum of the series $\sum_{n=1}^{+\infty}\frac{a_n}{S_n}$ is $\sum_{k=1}^{n}\frac{a_k}{S_k}$, consider narrowing, and take S_k all as S_n, then the whole is reduced. $\sum_{k=1}^{n}\frac{a_k}{S_k} > \sum_{k=1}^{n}\frac{a_k}{S_n} = \frac{1}{S_n}\sum_{k=1}^{n}a_k = \frac{1}{S_n}\cdot S_n = 1$ .Divergence cannot be proved. That is to say, the sum of k from 1 to n is not feasible, and the reduction is not appropriate. Considering the sum of k from n to $n+m$, replacing S_k with the largest item of S_{n+m}, the overall reduction is reduced. Then $\sum_{k=n}^{n+m}\frac{a_k}{S_k} > \sum_{k=n}^{n+m}\frac{a_k}{S_n} = \frac{1}{S_{n+m}}\sum_{k=n}^{n+m}a_k = \frac{1}{S_{n+m}}\cdot(S_{n+m} - S_{n-1}) = 1 - \frac{S_{n-1}}{S_{n+m}}$ .Since $S_n \to \infty(n\to\infty)$, $\frac{S_{n-1}}{S_{n+m}} < \frac{1}{2}$, then $\sum_{k=n}^{n+m}\frac{a_k}{S_k} > 1 - \frac{S_{n-1}}{S_{n+m}} = 1 - \frac{1}{2} = \frac{1}{2}$, thus $\sum_{k=1}^{n}\frac{a_k}{S_k}$ diverges, and then $\sum_{n=1}^{+\infty}\frac{a_n}{S_n}$ diverges, then $\sum_{n=1}^{+\infty}\frac{a_n}{S_n^\alpha}$.

3) (The third national college students 'math contest in2011, 6points) find the sum function of power series $\sum_{n=1}^{\infty}\frac{2n-1}{2^n}x^{2n-2}$, and find the sum of series $\sum_{n=1}^{\infty}\frac{2n-1}{2^{2n-1}}$.

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Analysis: the general idea of power series and function calculation: the first step: to find the convergence domain $\lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\frac{2(n+1)-1}{2^{(n+1)}}x^{2(n+1)-2}}{\frac{2n-1}{2^n}x^{2n-2}}\right| = \lim_{n\to\infty}\frac{[2(n+1)-1]2^n}{2^{(n+1)}(2n-1)}x^2 = \frac{x^2}{2}$ .Let $\frac{x^2}{2}<1$, $-\sqrt{2}<x<\sqrt{2}$, the convergence interval of power series is $(-\sqrt{2},\sqrt{2})$ .When $x=\pm\sqrt{2}$, $\sum_{n=1}^{\infty}\frac{2n-1}{2^n}(\pm\sqrt{2})^{2n-2} = \sum_{n=1}^{\infty}\frac{2n-1}{2^n}2^{n-1} = \sum_{n=1}^{\infty}\frac{2n-1}{2}$ diverges, and the convergence domain of the power series is $(-\sqrt{2},\sqrt{2})$. Step 2: Convert the power series to the power series of the standard structure $\sum_{n=1}^{\infty}\frac{2n-1}{2^n}x^{2n-2} = \sum_{n=1}^{\infty}\frac{2n-1}{2\cdot 2^{n-1}}(x^2)^{n-1} = \frac{1}{2}\sum_{n=1}^{\infty}(2n-1)(\frac{x^2}{2})^{n-1} = \frac{1}{2}\sum_{n=1}^{\infty}(2n-1)t^{n-1}$

$$=\sum_{n=1}^{\infty}nt^{n-1}-\frac{1}{2}\sum_{n=1}^{\infty}t^{n-1} = \frac{1}{(1-t)^2}-\frac{1}{2(1-t)} = \frac{t+1}{2(1-t)^2}$$, Then $S(x)=\frac{\frac{x^2}{2}+1}{2(1-\frac{x^2}{2})^2}$

$$=\frac{2+x^2}{(2-x^2)^2}$$

$\sum_{n=1}^{\infty}\frac{2n-1}{2^n}(\pm\frac{1}{\sqrt{2}})^{2n-2} = \sum_{n=1}^{\infty}\frac{2n-1}{2^{2n-1}} = S(\pm\frac{1}{\sqrt{2}}) = \frac{2+\frac{1}{2}}{(2-\frac{1}{2})^2} = \frac{10}{9}$ .for $\sum_{n=1}^{\infty}(2n-1)x^{2n-1}$ When $x=\frac{1}{2}$, get $\sum_{n=1}^{\infty}\frac{2n-1}{2^{2n-1}}$ $x\in(-1,1)$, due to $x=\frac{1}{2}\in(-1,1)$, $\sum_{n=1}^{\infty}(2n-1)x^{2n-1} = x\sum_{n=1}^{\infty}(2n-1)x^{2n-2}$

$$=x\sum_{n=1}^{\infty}(x^{2n-1})' = x(\sum_{n=1}^{\infty}x^{2n-1})' = x(x+x^3+x^5+\cdots)' = x(\frac{x}{1-x^2})' = \frac{x(1+x^2)}{(1-x^2)^2}\quad x\in(-1,1),$$

therefore $\sum_{n=1}^{\infty}\frac{2n-1}{2^{2n-1}} = [\frac{x(1+x^2)}{(1-x^2)^2}]_{x=\frac{1}{2}} = \frac{10}{9}$.

4) (The 4th National College Students ' Mathematics Competition in 2012, 15 points) Let $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}b_n$ be positive series, (1) If $\lim_{n\to\infty}(\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}})>0$, then $\sum_{n=1}^{\infty}a_n$ converges; (2) If $\lim_{n\to\infty}(\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}})<0$ and $\sum_{n=1}^{\infty}b_n$ diverge, then $\sum_{n=1}^{\infty}a_n$ diverge.

Analysis: The convergence of abstract constant series is determined based on comparison method and rewriting condition description. The method of determining the convergence of series can be used to determine the convergence of positive series: ratio method, root value method, comparison method and sequence boundedness.

The solution: (1) $\exists N\in z^+$ is obtained by $\lim_{n\to\infty}(\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}})>0$, when $n\ge N$,

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$\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}}>0$, $\frac{a_n}{b_n}>\frac{a_{n+1}}{b_{n+1}}>0$, $\frac{a_n}{a_{n+1}}>\frac{b_n}{b_{n+1}}>0$, $0<\frac{a_{n+1}}{a_n}<\frac{b_{n+1}}{b_n}$, $a_n>\frac{b_n}{b_{n+1}}a_{n+1}>0$, that is $0<a_{n+1}<\frac{b_{n+1}}{b_n}a_n$. It can be seen that the rewriting is invalid and rewrite.Because of $\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}}>0$, $\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}}>0$,That is $\sum_{n=1}^{m}(\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}})=\sum_{n=1}^{N-1}(\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}})+$

$\frac{a_N}{b_N}-\frac{a_{m+1}}{b_{m+1}}$ $(m>N)$ convergence. So let $\lim_{n\to\infty}(\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}})=c>0$. $\exists N\in z^+$, When $n\geq N$,

$\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}}>\frac{c}{2}>0$, $\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}}>\frac{c}{2}a_{n+1}>0$, $a_{n+1}<\frac{2}{c}(\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}})$,namely $\sum_{n=1}^{m}a_n=\sum_{n=1}^{N}a_n+$

$\sum_{n=N}^{m}a_{n+1}<\sum_{n=1}^{N}a_n+\sum_{n=N}^{m}\frac{2}{c}(\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}})$, thus $\sum_{n=N}^{m}\frac{2}{c}(\frac{a_n}{b_n}-\frac{a_{n+1}}{b_{n+1}})=\frac{2}{c}(\frac{a_N}{b_N}-\frac{a_{N+1}}{b_{N+1}}+\cdots+\frac{a_m}{b_m}-$

$\frac{a_{m+1}}{b_{m+1}})=\frac{2}{c}(\frac{a_N}{b_N}-\frac{a_{m+1}}{b_{m+1}})$. By the comparison method, the series $\sum_{n=1}^{\infty}a_n$ converges.

(2) Let $\lim_{n\to\infty}(\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}})<0$, $\exists N\in z^+$, when $n\geq N$, $\frac{a_n}{a_{n+1}b_n}-\frac{1}{b_{n+1}}<0$ $a_{n+1}>\frac{b_{n+1}}{b_n}a_n>0$

$a_n>\frac{b_n}{b_{n-1}}a_{n-1}$, $a_{n-1}>\frac{b_{n-1}}{b_{n-2}}a_{n-2}$, … , $a_{N+2}>\frac{b_{N+2}}{b_{N+1}}a_{N+1}$, $a_{N+1}>\frac{b_{N+1}}{b_N}a_N$, So $a_n>\frac{b_n}{b_{n-1}}a_{n-1}>$

$\frac{b_n}{b_{n-1}}\frac{b_{n-1}}{b_{n-2}}a_{n-2}$ $>\cdots>\frac{b_n}{b_{n-1}}\frac{b_{n-1}}{b_{n-2}}\cdots\frac{b_{N+2}}{b_{N+1}}\frac{b_{N+1}}{b_N}a_N=\frac{b_n}{b_N}a_N=\frac{a_N}{b_N}b_n$ $(n\geq N)$. By the comparison method, the series $\sum_{n=1}^{\infty}a_n$ diverges.

5) (The 5th National College Students ' Mathematics Competition in 2013, 12 points) Let $f(x)$ have a second derivative $f''(0)$ number at $x=0$, and $\lim_{n\to\infty}\frac{f(x)}{x}=0$, prove that the series $\sum_{n=1}^{\infty}\left|f(\frac{1}{n})\right|$ converges.

Solution: $f(x)$ is first order derivable at $x=0$, $f'(x)$ and $f(x)$ are continuous at $x=0$. $f''(0)=$ $\lim_{x\to 0}\frac{f'(x)-f'(0)}{x-0}=\lim_{x\to 0}\frac{f'(x)-f'(0)}{x}=\lim_{x\to 0}\frac{f'(x)}{x}$, $f(0)=0$, $\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=0$, $f'(0)=0$. The second-order McLaughlin formula with Peano remainder of

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$f(x)$. $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+o(x^2)$, $f(\frac{1}{n})=\frac{f''(0)}{2}\frac{1}{n^2}+o(\frac{1}{n^2})$, $\frac{f(\frac{1}{n})}{\frac{1}{n}}=$

$\frac{\frac{f''(0)}{2}\frac{1}{n^2}+o(\frac{1}{n^2})}{\frac{1}{n^2}}\to\frac{f''(0)}{2}(n\to\infty)$, $\lim_{n\to+\infty}\frac{\left|f(\frac{1}{n})\right|}{\frac{1}{n^2}}=\lim_{n\to+\infty}\left|\frac{\frac{f''(0)}{2}\frac{1}{n^2}+o(\frac{1}{n^2})}{\frac{1}{n^2}}\right|=\frac{|f''(0)|}{2}$.

Due to $\sum_{n=1}^{\infty}\frac{1}{n^2}$ convergence, $\sum_{n=1}^{\infty}\left|f(\frac{1}{n})\right|$ convergence can be obtained by the comparative convergence method of positive series.

6) (The 5th National College Students ' Mathematics Competition in 2013, 14 points) Judge the convergence and divergence of the series $\sum_{n=1}^{\infty}\frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(n+1)(n+2)}$.If it converges, find its sum.

Analysis: Because of $\frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(n+1)(n+2)}<\frac{n}{(n+1)(n+2)}\to 0(n\to+\infty)$, the ratio convergence method, or the root value convergence method is not applicable. Comparison discriminant method: The goal is to verify the convergence of the series.

Solution: Inference from the sign-preserving property of sequence limit: there exists an $N_1\in Z^+$, when $n>N_1$, $0<\sum_{k=1}^{n}\frac{1}{k}-\ln n<1$, $\ln n<\sum_{k=1}^{n}\frac{1}{k}<\ln n+1$ .When $n>N_1>1$, $\frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(n+1)(n+2)}$ $<\frac{1+\ln n}{(n+1)(n+2)}$ when $\alpha>0$, $\lim_{n\to+\infty}\frac{\ln n}{n^\alpha}=0$. Take $\alpha=\frac{1}{2}$, there is $N_2\in Z^+$.When $n>N_2$,

$\ln n<n^{\frac{1}{2}}$,When $n>\max\{N_1,N_2\}$, $\frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(n+1)(n+2)}<\frac{1+\ln n}{(n+1)(n+2)}<\frac{1+\sqrt{n}}{(n+1)(n+2)}$,

$\lim_{n\to+\infty}\frac{\frac{1+\sqrt{n}}{(n+1)(n+2)}}{\frac{1}{n^{3/2}}}=1$. $\sum_{n=1}^{+\infty}\frac{1}{n^{3/2}}$ convergence, $\sum_{n=1}^{+\infty}\frac{1+\sqrt{n}}{(n+1)(n+2)}$ convergence,that

$\sum_{n=1}^{\infty}\frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(n+1)(n+2)}$ convergence.Sum: $n-1\le x\le n$ $\frac{1}{n}\le\frac{1}{x}\le\frac{1}{n-1}$, $\frac{1}{n}=\int_{n-1}^{n}\frac{1}{n}dx<\int_{n-1}^{n}\frac{1}{x}dx=$

$\ln n-\ln(n-1)$, $\frac{1}{2}+\cdots+\frac{1}{n-1}+\frac{1}{n}<(\ln 2-\ln 1)+\cdots+(\ln n-\ln(n-1))=\ln n$,

$1+\frac{1}{2}+\cdots+\frac{1}{n-1}$

$+\frac{1}{n}<1+\ln n$,record $1+\frac{1}{2}+\cdots+\frac{1}{n-1}+\frac{1}{n}=a_n$, $a_{k+1}-a_k=\frac{1}{k+1}$,

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$\frac{1}{(k+1)(k+2)}=\frac{1}{k+1}-\frac{1}{k+2}$, $\frac{a_k}{(k+1)(k+2)}=\frac{a_k}{k+1}-\frac{a_k}{k+2}$,therefore

$S_n=\sum_{k=1}^{n}\frac{a_k}{(k+1)(k+2)}=\sum_{k=1}^{n}(\frac{a_k}{k+1}-\frac{a_k}{k+2})=(\frac{a_1}{2}-\frac{a_1}{3})+(\frac{a_2}{3}-\frac{a_2}{4})+\cdots+(\frac{a_n}{n+1}-\frac{a_n}{n+2})$,

$\frac{a_{k+1}}{k+2}-\frac{a_k}{k+2}=\frac{a_{k+1}-a_k}{k+2}=\frac{1}{(k+1)(k+2)}$,

therefore $S_n=(\frac{1}{2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n+1)})-\frac{1}{n+2}a_n$ $=[(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+\cdots+(\frac{1}{n-1}-\frac{1}{n})$

$+(\frac{1}{n}-\frac{1}{n+1})]-\frac{1}{n+2}a_n=(1-\frac{1}{n+1})-\frac{1}{n+2}a_n$ .Because of $0<a_n<1+\ln n$, so $0<\frac{a_n}{n+2}$ $<\frac{1+\ln n}{n+2}$, Because of $\lim_{n\to\infty}\frac{1+\ln n}{n+2}=0$, $\lim_{n\to\infty}\frac{a_n}{n+2}=0$, so $S=\lim_{n\to\infty}S_n=1-0-0=1$.

7) (The 7th National College Students ' Mathematics Competition in 2015, 6 points) The value of the Fourier series $x=0$ convergence of the function $f(x)=\begin{cases}3, x\in[-5,0)\\0, x\in[0,5)\end{cases}$ in $[-5,5)$.

Analysis: The problem of finding the sum function of Fourier series of functions.

Solution: Dirichlet convergence theorem: If the function has only a finite number of extreme points or only a finite number of discontinuous points of the first kind on the definition interval, then for any x in the definition interval, there is $S(x)=\frac{f(x+0)+f(x-0)}{2}$, then $\forall x\in(-5,5)$,

$S(x)=\frac{f(x+0)+f(x-0)}{2}$, $0\in(-5,5)$, $S(0)=\frac{f(0+0)+f(0-0)}{2}=\frac{0+3}{2}=\frac{3}{2}$ .So,

when $x\in(-5,5)$, there is $S(x)=\frac{f(x+0)+f(x-0)}{2}=\begin{cases}3, x\in(-5,0)\\\frac{3}{2}, x=0\\0, x\in(0,5)\end{cases}$.

8) (The 7th National College Students ' Mathematics Competition in 2015, 14 points) Find the convergence domain and sum function of the series $\sum_{n=0}^{\infty}\frac{n^3+2}{(n+1)!}(x-1)^n$.

Solution: Step 1: Find the limit $u_n(x)=\frac{n^3+2}{(n+1)!}(x-1)^n$,

$$\lim_{n\to+\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right|=\lim_{n\to\infty}\left|\frac{\frac{(n+1)^3+2}{(n+2)!}(x-1)^{n+1}}{\frac{n^3+2}{(n+1)!}(x-1)^n}\right|=\lim_{n\to+\infty}\left|\frac{(n+1)^3+2}{(n+2)(n^3+2)}(x-1)\right|=0.$$

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Step 2: Calculate the convergence interval. Solve the inequality of $|\rho(x)|<1$ with respect to the variable x, because $|\rho(x)|=0<1$, the convergence interval is $(-\infty,+\infty)$, and the convergence domain is $(-\infty,+\infty)$. $\sum_{n=0}^{\infty}\frac{n^3+2}{(n+1)!}(x-1)^n \xrightarrow{x-1=t} \sum_{n=0}^{\infty}\frac{n^3+2}{(n+1)!}t^n=S(t)$,

$$\frac{n^3+2}{(n+1)!}t^n=\frac{n^3}{(n+1)!}t^n+\frac{2}{(n+1)!}t^n=\frac{2}{t}\sum_{n=0}^{\infty}\frac{t^{n+1}}{(n+1)!}=\frac{2}{t}\sum_{n=0}^{\infty}\frac{t^n}{n!}=\frac{2}{t}(e^t-1),$$

$$\frac{n^3}{(n+1)!}t^n=\frac{n^3+n^2-n^2}{(n+1)!}t^n=\frac{n^2}{n!}t^n-\frac{n^2}{(n+1)!}t^n=\frac{n}{(n-1)!}t^n-\frac{n^2+n-n}{(n+1)!}t^n$$

$$=\frac{n-1+1}{(n-1)!}t^n-\frac{n}{n!}t^n+\frac{n}{(n+1)!}t^n=\frac{1}{(n-2)!}t^n+\frac{1}{(n-1)!}t^n-\frac{1}{(n-1)!}t^n+\frac{n+1-1}{(n+1)!}t^n$$

$$=\frac{1}{(n-2)!}t^n+\frac{1}{n!}t^n-\frac{1}{(n+1)!}t^n$$

therefore $\sum_{n=0}^{\infty}\frac{n^3}{(n+1)!}t^n=\sum_{n=0}^{\infty}\frac{1}{(n-2)!}t^n++\sum_{n=0}^{\infty}\frac{1}{n!}t^n-\sum_{n=0}^{\infty}\frac{1}{(n+1)!}t^n=t^2\sum_{n=0}^{\infty}\frac{t^{n-2}}{(n-2)!}+e^t-\frac{1}{t}\sum_{n=0}^{\infty}\frac{t^{n+1}}{(n+1)!}(t\neq 0)$ $=t^2\sum_{n=0}^{\infty}\frac{t^n}{n!}+e^t-\frac{1}{t}\sum_{n=1}^{\infty}\frac{t^n}{n!}(t\neq 0)$

$=t^2e^t+e^t-\frac{1}{t}(e^t-1)(t\neq 0)$ .therefore

$S(t)=t^2e^t+e^t-\frac{1}{t}(e^t-1)+\frac{2}{t}(e^t-1)=\frac{e^t(t^3+t+1)-1}{t}(t\neq 0)$ When $t=0$, $\sum_{n=0}^{\infty}\frac{n^3+2}{(n+1)!}t^n=$

$2+\frac{1+2}{2!}t+\frac{2^3+2}{3!}t^2+\cdots$, $S(0)=2$, $\sum_{n=0}^{\infty}\frac{n^3+2}{(n+1)!}t^n=\begin{cases}\frac{e^t(t^3+t+1)-1}{t},t\neq 0\\ 2,t=0\end{cases}$,due to

$\lim_{x\to 1}[(x^2-2x+2)e^{x-1}+\frac{1}{x-1}(e^{x-1}-1)]=1+\lim_{x\to 1}\frac{1}{x-1}(e^{x-1}-1)$ $=1+\lim_{x\to 1}\frac{1}{x-1}(x-1)=2$. Therefore, Substitute $t=x-1$ into

$$\sum_{n=0}^{\infty}\frac{n^3+2}{(n+1)!}(x-1)^n=\begin{cases}(x^2-2x+2)e^{x-1}+\frac{1}{x-1}(e^{x-1}-1),x\neq 1\\ 2,x=1\end{cases}.$$

9) (The 8th National College Students ' Mathematics Competition in 2016, 14 points) Let $f(x)$ be derivable on $(-\infty,+\infty)$ and $f(x)=f(x+2)=f(x+\sqrt{3})$, and prove that $f(x)$ is a constant by Fourier series theory.

Analysis: the function is expanded into Fourier series, the key is to calculate the Fourier coefficient: period $f(x)=f(x+2)=f(x+\sqrt{3})$, the period of the function is $T=2,T=\sqrt{3}$,$L=1$. General period Fourier coefficient calculation formula: $a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi}{L}xdx$,

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$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi}{L}xdx$.According to Dirichlet convergence theorem, $f(x)$ can be expanded into Fourier series on $(-\infty,+\infty)$: $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos\frac{n\pi}{L}x+b_n\sin\frac{n\pi}{L}x)$, $x\in(-\infty,+\infty)$.It is proved that $f(x)$ is a constant, and it is verified that all coefficients a_n,b_n $(n=1,2,...)$ are equal to 0.

Proof: $a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi}{L}xdx=\int_{-1}^{1}f(x)\cos n\pi xdx=\int_{-1}^{1}f(x+\sqrt{3})\cos n\pi xdx$

$$=\int_{-1+\sqrt{3}}^{1+\sqrt{3}}f(t)[\cos n\pi t\cos\sqrt{3}n\pi+\sin n\pi t\sin\sqrt{3}n\pi]dt$$

$$=\cos\sqrt{3}n\pi\int_{-1+\sqrt{3}}^{1+\sqrt{3}}f(t)\cos n\pi tdt+\sin\sqrt{3}n\pi\int_{-1+\sqrt{3}}^{1+\sqrt{3}}f(t)\sin n\pi tdt$$

$$=\cos\sqrt{3}n\pi\int_{-1}^{1}f(t)\cos n\pi tdt+\sin\sqrt{3}n\pi\int_{-1}^{1}f(t)\sin n\pi tdt$$

therefore $a_n=a_n\cos\sqrt{3}n\pi+b_n\sin\sqrt{3}n\pi$ (1). quorum $x+\sqrt{3}=t$, $x=t-\sqrt{3}$,

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$,

$$b_n=\int_{-1}^{1}f(x)\sin n\pi xdx=\int_{-1}^{1}f(x+\sqrt{3})\sin n\pi xdx=\int_{-1+\sqrt{3}}^{1+\sqrt{3}}f(t)\sin n\pi(t-\sqrt{3})dt$$

$$=\cos\sqrt{3}n\pi\int_{-1+\sqrt{3}}^{1+\sqrt{3}}f(t)\sin n\pi tdt-\sin\sqrt{3}n\pi\int_{-1+\sqrt{3}}^{1+\sqrt{3}}f(t)\cos n\pi tdt$$

therefore $b_n=b_n\cos\sqrt{3}n\pi-a_n\sin\sqrt{3}n\pi$ (2). (1) $\times b_n-$ (2) $\times a_n$ gets $(b_n^2+a_n^2)\sin\sqrt{3}n\pi=0$.

That 's $b_n^2+a_n^2=0$, so $a_n=b_n=0$ $(n=1,2,...)$

10) (The 10th National College Students ' Mathematics Competition in 2018, 14 points) It is known that $\{a_k\},\{b_k\}$ is a positive number sequence and $b_{k+1}-b_k\geq\delta>0,k=1,2,\cdots,\delta$ is a constant. It is proved that if the series $\sum_{k=1}^{+\infty}a_k$ converges, the series $\sum_{k=1}^{+\infty}\frac{k\sqrt[k]{(a_1a_2\cdots a_k)(b_1b_2\cdots b_k)}}{b_{k+1}b_k}$ converges.

Proof: arithmetic-geometric mean inequality $\sqrt[k]{(a_1a_2\cdots a_k)(b_1b_2\cdots b_k)}=\sqrt[k]{(a_1b_1)(a_2b_2)\cdots(a_kb_k)}$

$\leq\frac{a_1b_1+a_2b_2+\cdots+a_kb_k}{k}$, so $k\sqrt[k]{(a_1b_1)(a_2b_2)\cdots(a_kb_k)}\leq a_1b_1+a_2b_2+\cdots+a_kb_k$,

$$\frac{k\sqrt[k]{(a_1a_2\cdots a_k)(b_1b_2\cdots b_k)}}{b_{k+1}b_k}\leq\frac{a_1b_1+a_2b_2+\cdots+a_kb_k}{b_{k+1}b_k}=\frac{A_k}{b_{k+1}b_k}=\frac{1}{b_{k+1}-b_k}\frac{b_{k+1}-b_k}{b_{k+1}b_k}A_k$$

$$=\frac{1}{b_{k+1}-b_k}(\frac{1}{b_k}-\frac{1}{b_{k+1}})A_k\leq\frac{1}{\delta}(\frac{1}{b_k}-\frac{1}{b_{k+1}})A_k,\quad A_k=a_1b_1+a_2b_2+\cdots+a_kb_k,$$

$$\sum_{k=1}^{n}\frac{1}{\delta}(\frac{1}{b_k}-\frac{1}{b_{k+1}})A_k=\frac{1}{\delta}\sum_{k=1}^{n}(\frac{1}{b_k}-\frac{1}{b_{k+1}})A_k=\frac{1}{\delta}[(\frac{1}{b_1}-\frac{1}{b_2})A_1+\cdots+(\frac{1}{b_n}-\frac{1}{b_{n+1}})A_n]$$

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$$=\frac{1}{\delta}(\frac{A_1}{b_1}+\frac{A_2-A_1}{b_2}+\frac{A_3-A_2}{b_3}+\frac{A_4-A_3}{b_4}+\cdots+\frac{A_n-A_{n-1}}{b_n}-\frac{A_n}{b_{n+1}})$$

$$=\frac{1}{\delta}(a_1+a_2+a_3+\cdots+a_n-\frac{A_n}{b_{n+1}})\leq=\frac{1}{\delta}(a_1+a_2+a_3+\cdots+a_n)=\frac{1}{\delta}\sum_{k=1}^{n}a_k.$$

11) (The 13 th National College Students ' Mathematics Competition in 2021, 14 points) Let $\{a_n\}$ and $\{b_n\}$ be positive real sequences, satisfying $a_1=b_1=1$ and $b_n=a_nb_{n-1}-2$, $n=2,3,\cdots$, and let $\{b_n\}$ be a bounded sequence. It is proved that the series $\sum_{n=1}^{\infty}\frac{1}{a_1a_2\cdots a_n}$ converges and the sum of the series is obtained.

Proof: $a_1=1, a_2=\frac{b_2+2}{b_1},\cdots,a_n=\frac{b_n+2}{b_{n-1}}$, is obtained from $b_n=a_nb_{n-1}-2$, $a_n=\frac{b_n+2}{b_{n-1}}$, namely $a_1a_2\cdots a_n=\frac{(b_2+2)(b_3+2)\cdots(b_n+2)}{b_1b_2\cdots b_{n-1}}=\frac{(b_2+2)(b_3+2)\cdots(b_n+2)}{b_1b_2\cdots b_{n-1}b_n}b_n$, thus

$a_1a_2\cdots a_n=(1+\frac{2}{b_2})(1+\frac{2}{b_3})\cdots(1+\frac{2}{b_n})b_n$, Then $\frac{b_n}{a_1a_2\cdots a_n}=\frac{1}{(1+\frac{2}{b_2})(1+\frac{2}{b_3})\cdots(1+\frac{2}{b_n})}$.

Note $A_n=\frac{b_n}{a_1a_2\cdots a_n}$, then $A_n-A_{n-1}=\frac{b_n}{a_1a_2\cdots a_n}-\frac{b_{n-1}}{a_1a_2\cdots a_{n-1}}=\frac{b_n-a_nb_{n-1}}{a_1a_2\cdots a_n}$

$=-2\frac{1}{a_1a_2\cdots a_n}$. Thus, there is $\frac{1}{a_1a_2\cdots a_n}=\frac{1}{2}(A_{n-1}-A_n)\ (n\geq 2)$.

So $S_N=\sum_{n=1}^{N}\frac{1}{a_1a_2\cdots a_n}=\frac{1}{a_1}+\frac{1}{2}\sum_{n=2}^{N}(A_{n-1}-A_n)=1+\frac{1}{2}[(A_1-A_2)+\cdots+(A_{N-1}-A_N)]$

$=1+\frac{1}{2}\ (A_1-A_N)=\frac{3}{2}-\frac{1}{2}A_N\ (n\geq 2)$. Among them $a_1=1$, $A_1=\frac{b_1}{a_1}=1$.According to the pinch criterion, $\lim_{n\to\infty}A_n=\lim_{n\to\infty}\frac{b_n}{a_1a_2\cdots a_n}=\lim_{n\to\infty}\frac{1}{(1+\frac{2}{b_2})\cdots(1+\frac{2}{b_n})}=0$ is obtained,

$\sum_{n=1}^{\infty}\frac{1}{a_1a_2\cdots a_n}=\lim_{N\to\infty}S_N=\frac{3}{2}-\frac{1}{2}\lim_{N\to\infty}A_N=\frac{3}{2}$. $\{b_n\}$ is a bounded sequence, then $\exists M>0,\forall n\in Z^+, 0<b_n\leq M$

makes $\frac{1}{b_n}\geq\frac{1}{M}$, $A_n\leq\frac{1}{(1+\frac{2}{M})^{n-1}}\to 0$, $1+\frac{2}{M}>1$, $b_n=a_nb_{n-1}-2$, $\frac{a_nb_{n-1}-b_n}{2}=1$ .Thus

$\frac{1}{a_1a_2\cdots a_n}=\frac{a_nb_{n-1}-b_n}{2a_1a_2\cdots a_n}==\frac{b_{n-1}}{2a_1a_2\cdots a_{n-1}}-\frac{b_n}{2a_1a_2\cdots a_n}=\frac{1}{2}(A_{n-1}-A_n)\ (n\geq 2)$, that is

$\sum_{n=1}^{N}\frac{1}{a_1a_2\cdots a_n}=\frac{1}{a_1}+\frac{1}{2}\sum_{n=1}^{N}(\frac{b_{n-1}}{a_1a_2\cdots a_{n-1}}-\frac{b_n}{a_1a_2\cdots a_n})=\frac{1}{a_1}+\frac{1}{2}\sum_{n=1}^{N}(A_{n-1}-A_n)==\frac{1}{a_1}+\frac{1}{2}$

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$$(A_1 - A_N) = 1 + \frac{1}{2}(A_1 - A_N) = \frac{3}{2} - \frac{1}{2}A_N .$$

12) (The 14 th National College Students ' Mathematics Competition in 2022, 14 points) Let the positive series $\sum_{n=1}^{\infty} a_n$ converge, and prove that there exists a convergent positive series $\sum_{n=1}^{\infty} b_n$, such that $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$.

Proof: Because $\sum_{n=1}^{\infty} a_n$ converges, $\forall \varepsilon > 0$, there is N , such that when $n > N$, $\sum_{k=n}^{\infty} a_k < \varepsilon$ .In particular, for $k = 1,2,\cdots$, take $\varepsilon = \frac{1}{3^k}$, then there is $1 < n_1 < n_2 < \cdots < n_{k-1} < n_k$ such that $\sum_{l=n_k}^{\infty} a_l < \frac{1}{3^k}$.

The construction of $\{b_n\}$ is as follows: when $1 < n < n_1$, $b_n = a_n$; when $n_k < n < n_{k+1}$, $b_n = 2^k a_n$, $k = 1,2,\cdots$ .Obviously, when $n \to \infty$, $k \to \infty$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_n}{2^k a_n} = \lim_{n\to\infty} \frac{1}{2^k} = 0$, there is $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{n_1-1} a_n + \sum_{l=n_1}^{n_2-1} 2a_l + \sum_{l=n_2}^{n_3-1} 2^2 a_l + \cdots$

$$\le \sum_{n=1}^{n_1-1} a_n + 2\cdot\frac{1}{3} + 2^2\cdot(\frac{1}{3})^2 + \cdots = \sum_{n=1}^{n_1-1} a_n + \sum_{k=1}^{\infty} \frac{2^k}{3^k} =$$

$\sum_{n=1}^{n_1-1} a_n + 2 < +\infty$. Therefore, the positive series $\sum_{n=1}^{\infty} b_n$ converges.

3. Conclusion

By summarizing the knowledge points of infinite series and its application in college students ' mathematics competition, it shows a clear knowledge framework and problem-solving ideas, and easy-to-understand problem-solving methods, so that students can better grasp the calculation methods and skills of infinite series. At the same time, it is also conducive to improving students ' ability to analyze problems, stimulating students ' interest in learning, helping to improve teachers ' teaching quality, and also has certain reference value for participating students[11-15].

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