

Value Research of Integrating History of Mathematics into Primary School Mathematics Teaching

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Abstract: Jules Henri Poincaré, the great French mathematician, said that if we wanted to see the future of mathematics, the proper way was to study the history and present state of the science. Integrating the History of Mathematics into primary school mathematics classroom teaching can stimulate students' interest in learning, inspire their thinking and enhance their mathematical literacy, while cultivating good qualities, promoting mathematical culture and enhancing cultural confidence. In the paper, the value of integrating the History of Mathematics into students' learning will be specifically explained through the explanation of mathematics knowledge points in primary schools.

Keywords: History of Mathematics; Mathematics Teaching in Primary Schools; Value Research

1. Introduction

History of Mathematics is a science that studies the occurrence and development of mathematical science and its laws. It not only traces the evolution and development process of mathematical content, ideas and methods, but also explores the various factors that affect this process, as well as the influence of the development of mathematical science on human civilization in history. The famous American historian of mathematics M. Kline (1908-1992) stated that "the History of Mathematics is a guide to teaching"^[1].

When we look back at the process of learning mathematics, we were all blank sheets of paper in mathematics as children, full of doubts about what we were learning, but the teacher would hardly tell us where the knowledge came from and in what context it was generated. All we need to do is remember the knowledge and learn to apply it in certain situations. So children slowly turn learning mathematics into rote memorization and mechanical application, and lose their curiosity about mathematics.

In contrast, when we study an ancient poem, the first thing we need to understand is the context of the poem, because knowing the context helps us empathize with the author and helps us enter the author's mindset at that time. This comparison reinforces our belief that it is essential to integrate the History of Mathematics into mathematics teaching. For quite some time, research on the use of the History of Mathematics in the classroom has confirmed a lot of academic benefits for students^[2]. Today, the National Council of Teachers of Mathematics (NCTM), which primarily guides mathematics teaching worldwide, and the other institutions such as the International Study Group on the Relationship between the History and Pedagogy of Mathematics [HPM]) and the Mathematical Association of America are encouraging the use of the history of mathematics in mathematics teaching^[3, 4]. So, teaching children about the history of mathematical ideas as early as possible is a very important and meaningful step in learning mathematics.

Mathematics is a cultural heritage of mankind^[5]. Compared with physics, chemistry and other knowledge, mathematical science has a high historical and cumulative nature. Mathematical science is developed by mathematicians in the process of continuous inheritance and expansion, and it is a non-static, developing, multicultural science^[6]. Its concepts and thoughts are not uprooted and overthrown. It grows slowly in a highly inclusive environment.

The History of Mathematics refers to the history formed in the process of the generation and

development of mathematical knowledge, as well as the resulting mathematics thoughts and methods and spiritual culture. Its specific connotation consists of three parts, the first is the process of the generation and formation of mathematical knowledge, which reflects the History of Mathematics; the second is the mathematics thoughts and methods, reflecting the scientific nature of mathematics; the third is the spirit of mathematics, reflecting the educational nature of the History of Mathematics^[1].

2. The Value of History of Mathematics to Students' Learning

Elementary mathematics teachers pay attention to the History of Mathematics, not to teach History of Mathematics, but to teach mathematics better. What follows is a specific explanation of the value of integrating the History of Mathematics to students' learning through the explanation of primary school mathematics knowledge points.

2.1 *The History of Mathematics can stimulate students' interest in learning and build their confidence*

Students' interest in learning mathematics is likely to have a positive effect on their learning process and outcome^[7]. The effectiveness of learning is closely related to the learning interest especially for primary school students. Florian Cajori once pointed out that the historical knowledge of a subject is the honey that makes the bread and butter more delicious, and believed that through the introduction of the History of Mathematics, teachers can make students understand that mathematics is not a boring subject, but a lively and interesting subject that is constantly progressing^[8]. In teaching, if teachers can provide students with some small allusions and games in the History of Mathematics that are closely related to the teaching content, they can make the mathematics class more colorful and active, and at the same time make students feel the mystery and magic of mathematics, stimulate their interest in learning, and also make students develop the habit of paying attention to the History of Mathematics development and enhance the teaching effectiveness. Thus, The history of mathematics has a significant influence on students' interest in learning mathematics^[9, 10].

For example, when teaching the section "Understanding of Fractions", it was introduced by ancient Egypt, a world-famous ancient civilization. About 3,000 years ago, ancient Egypt began to use fractions to count, but at that time people can only represent the number whose numerator is 1, so how to divide the number whose numerator is 2? They invented a practice such as dividing 2 apples among 5 children. They first divide an apple into 5 parts and each gets 1 part of it, i.e. $1/5$, and then distribute the other apple in the same way, i.e. $2/5 = 1/5 + 1/5$. It was not until more than 2,000 years ago that China invented the use of Counting Rod to represent fractions, that is, objects similar to wooden sticks to count. In the 12th century, the Arabs found a way to represent fractions, which is what we use today, by dividing an object into parts, one of which is a fraction. In the classroom, teachers explain the History of Mathematics to students by combining it with mathematical knowledge, so that students can experience the wisdom of ancient mathematicians, understand the evolution of knowledge, and have a deeper impression and understanding of what they are learning, and improve their learning efficiency.

When teaching the "Chicken with rabbit cage", the teacher can introduce a famous mathematical problem to introduce the new lesson: let the students challenge an interesting question from Sun Zi's Mathematical Manual. Then show the problem of "chicken with rabbit cage": "Now there are some pheasants and rabbits in the same cage, with 35 heads on the top and 94 feet on the bottom, so how many pheasants and rabbits are there each? Students engage in cooperative inquiry and will come up with a variety of problem-solving strategies such as list method, graphing method, solving equations, and hypothesis method. This interesting but uncomplicated plot can arouse great interest and stimulate great desire for learning, while the solution process is full of challenges and students feel a sense of pleasure and accomplishment in solving the problem in a diverse way.

In the lesson of "Knowing Numbers Up to 10000", you can use short stories from the History of Mathematics to inspire students' thinking. In the classroom introduction, the teacher can tell students that in ancient times there was no pen and paper, let alone advanced computers, and people used knotted ropes to calculate. For example, if a person catches four pheasants, he will tie four knots on a piece of string to indicate that he has caught four pheasants. If he catches two more rabbits, he will tie two knots on the other rope to indicate that he has caught two rabbits. After finishing the short story, the students can give their opinions: "If the prey is different every day, they have to tie a knot on a different rope, which is very troublesome and easy to get confused." Then the teacher can guide the student that because of this, people have gradually found more convenient counting methods, which led to the topic of this lesson: how to represent numbers up to 10000. In this way, a simple story can attract students' attention

and inspire their thinking in a short time.

When teaching "Close Paving Graphics", students are inspired to answer the question: Why does a regular hexagon consume the least amount of material? Under the guidance of the teacher, students can experience the knowledge of a small beehive through group communication. In fact, as early as the 18th century, many famous mathematicians such as Johann Samuel König, Colin Maclaurin have studied the beehive problem. The teacher can tell students some ancient methods that match the knowledge level of elementary school students, and can also introduce the method of Hua Luogeng, a famous mathematician in China, who used elementary school knowledge to study the beehive. Through the comparison of ancient and modern algorithms, we can train students' mathematical thinking and stimulate their interest in learning.

Also, in the lesson of "Knowing the Clock", teachers can show students the evolution of timekeeping tools. In primitive society, people went out to hunt during the day and returned to the cave to rest at night. Therefore, primitive people only knew how to count time by "day" and "night". Later, when people determined the time roughly by observing the length and direction of the shadow throughout the day, that is measuring shadow by setting up a pole. Our understanding of time made great progress. With the development of society, people learned to count time by using sand leakage or water dripping, which is Hourglass timing. Even later, with the gradual development of science and technology, people invented clocks to recognize and calculate time more and more accurately. Students will be impressed by the wisdom and creativity of the ancients, and their interest in learning and enthusiasm for inquiry can be fully stimulated. After that, the teacher can also use multimedia to show a variety of pictures of clocks at home and abroad. Through observation and comparison, students will find that although the shape and size of clocks vary, they all have constituent elements such as hour and minute hands. This gives students an initial impression and understanding of the composition of a clock. In the teaching, the teacher uses multimedia to show some lively historical materials of mathematics and pictures to stimulate students' interest in knowing clocks, adding some fun to the boring and monotonous mathematics classroom, thus stimulating students' interest in learning mathematics and helping them to build up confidence in learning mathematics well.

2.2 The History of Mathematics can help students clarify the development of mathematical knowledge and inspire their thinking

The History of Mathematics contains the historical lineage of the emergence and development of mathematics, and is a concentrated embodiment of mathematical concepts and mathematical ideas and methods. There are many typical mathematical problems in elementary school mathematics textbooks, and they all evolved in the long History of Mathematics, from complex to simple, and each solution step has gone through a long and tortuous process. Integrating the History of Mathematics into mathematics classroom and starting to trace the roots of mathematical knowledge can enable students to understand the real process of the emergence and development of mathematical knowledge, help them clarify the basic lineage of the development of mathematical knowledge, establish their own thinking system and form their own thinking skills, thereby deepening their overall understanding of mathematical knowledge, enabling them to appreciate the characteristics of mathematics in different civilizations and the important role of mathematics in the development of human history, and helping them to have a more comprehensive understanding of mathematics^[11].

A survey showed that 97% of the students believed that learning the History of Mathematics was useful for learning mathematics^[12]. Therefore, it is necessary to integrate the History of Mathematics into the classroom, so that the History of Mathematics can play the role of "teacher" and students can gain knowledge and learn spiritual nutrition through the dialogue with the ancient people.

For example, in the section of "Recognition of the circle", the teacher explains to students the history of the circle. The earliest records of circles in Chinese history date back to the Warring States period. Ancient people first knew the circle from the sun. About 6,000 years ago, the first round wooden wheel was born in the world, and after 2,000 years of continuous research, people mounted the round wooden wheel on a wooden frame to form the wheel, and our current wheel is gradually evolved. Through the history of the circle, students can learn about the process of forming a circle. By teaching the History of Mathematics in the class, we can enrich students' understanding of mathematical knowledge, expand their thinking skills, make their knowledge of circles no longer limited to a single object, as well as stimulate their interest in learning, and make the mathematics classroom more exciting!

In the section of "Positive and Negative Numbers", when explaining the Need for Negative Numbers,

teachers can introduce students to the fact that China was the first country to recognize and use negative numbers. In the first century A.D., The Nine Chapters on the Mathematical Art recorded that in the business activities, the ancient people had the idea of Grain in the warehouse is positive, out of the warehouse is negative; Income is positive, expenditure is negative; surplus is positive and the loss is negative. By the second century AD, the Chinese mathematician Liu Hui clearly introduced the concept of positive and negative number and used different colored Counting Rod to represent them. Because of the inconvenience of changing colors when recording, by the thirteenth century AD, mathematicians created the method of drawing slashes over numbers to represent negative numbers. Such learning not only expands students' horizons, but also deepens their understanding of negative numbers and greatly improves the effectiveness of classroom teaching.

When teaching "Circumference of a Circle", teachers can guide students to trace the development of Pi. In ancient times, the wheel maker needed to know how long a piece of wood was needed to make a wheel of a certain diameter to meet the requirements, and by this time it was known that there was some connection between the length of one revolution of the wheel and its diameter. More than 2000 years ago, the ancient Chinese mathematical work Chou Pei Suan Ching recorded "circumference of a circle is proverbially three times its radius". Along with the progress of society, people demand higher and higher precision of circumference. Liu Hui, a mathematician in the Wei and Jin dynasties, invented the cyclotomic method and calculated the circumference to be 3.1416. During the Southern and Northern Dynasties, the famous mathematician Zu Chongzhi calculated that the circumference should be between 3.1415926 and 3.1415927, which was more than 1000 years ahead of other countries. The advent of electronic computers has brought about a huge change in computing, and the number after the decimal point of Pi is known more and more. On September 17, 2010, someone used "cloud computing", using 1000 computers to calculate simultaneously for 23 days, to calculate the circumference of the circle to 20 million decimal places, refreshing people's knowledge of circumference. In teaching, the process of exploring Pi can help students clarify the ins and outs of Pi, deepen the overall understanding of Pi, and add some cultural flavor to the monotonous mathematics class.

2.3 The History of Mathematics can lead students to understand mathematical thinking and methods, improve their problem-solving skills and enhance their mathematical literacy

The purpose of education is not only to make everyone learn more, but also to understand the ins and outs of knowledge, so that it can be translated into the ability to deal with practical problems.

The Mathematics Curriculum Standards point out that students should acquire the basic knowledge, basic skills, basic ideas and basic activity experience of mathematics necessary for social life and further development in the study of mathematics. The History of Mathematics is not only the history of the emergence and development of mathematical knowledge, but also the history of the emergence and development of mathematical thoughts and methods. It permeates the connection between mathematics and practical life, shows the different solutions to the same problem by mathematicians in various periods, and contains a lot of historical materials reflecting innovation, which is conducive to the development of students' mathematical thinking and the enhancement of their practical problem-solving skills. By exploring the mathematical ideas and methods contained in the History of Mathematics, and purposefully integrate them into teaching, teachers can guide students to use them to think about problems, thus deepening students' cognitive depth of mathematical knowledge and improving their ability to analyze and solve problems.

For example, in the lesson of "Area of a Circle", students are usually guided to use thought of transformation and limit to derive the area formula of a circle. In fact, the mathematical work Chou Pei Suan Ching has put forward the idea that "circle out of square". Guided by this point of view in teaching, the teacher can guide students to compare a circle to a regular polygon, firstly derive the area formula of polygons with positive even sides, and then use the thought of analogy to derive the area formula of a circle. In the process of exploration, students experience the essence of the idea that "circle out of square", and not only understand the source of the circle, but also apply it to the derivation of the area formula of the circle, deepening the understanding of the nature of the circle. In addition, the typical methods of calculating circle area in history, such as Liu Hui's cyclotomic method, Archimedes' method of exhaustion and Kepler's infinite division method, can also be integrated into teaching, so that students can feel the unique value of limit thought in the process of solving problems in the selection and understanding of methods.

In the lesson of "Three digits times two", the teacher can introduce teaching using an extended question " $1+2+3+4+5+\dots+100=?$ " Maybe some students don't pay attention to the problem and just go

straight to work. At this point, the teacher can interrupt the students and guide them: This is a question that a German mathematician Gauss in elementary school can calculate the answer in a very short time, can you guess if he just adds up all 100 numbers? If not, how do you think he would calculate it? Then ask the students to think carefully before they start to calculate. Soon, some students will find that the problem is regular, that is, the sum of the first number and the last number is 101, the sum of the second number and the ninety-ninth number is 101..... There are 50 101's, so the final result is 5050. In teaching, teachers guide students to explore the "Gauss algorithm", and make students to feel the mystery and charm of mathematical thoughts and methods. This stimulates students' thinking, improves their ability to solve problems, and develops their good mathematical literacy.

2.4 The History of Mathematics can help students get positive emotional experience and cultivate good quality

The process of students learning mathematics is not a purely rational cognitive process, but a warm emotional and cultural cognitive process. History can serve as a motivational tool^[13] so as to enhance affective dispositions such as attitudes towards mathematics and mathematical activity for students. George Sarton (1884-1956), the famous American historian of science, said that the study of the History of Mathematics did not necessarily produce better mathematicians, but it produced more refined mathematicians, whose minds were enriched, whose hearts were soothed, and whose refined quality were cultivated. The famous thinker Bacon said, "Histories make men wise." The life truth, mathematical culture and spirit contained in the History of Mathematics cannot be replaced by any preaching. Only with a deep cultural literacy and heritage will the perception of numbers not deviate from the course.

The subject of mathematics has undergone a long process of development, and each result achieved required the efforts of mathematicians for decades, centuries or even millennia. In the face of the difficulties and problems that arose during this process, the valuable qualities they embodied, such as diligence, perseverance, rigorous learning and overcoming difficulties, are worth learning from. Students' learning difficulties and obstacles in the process of learning may be parallel with those encountered in the evolution of mathematics^[14]. In teaching, teachers can introduce these difficulties and problems into the classroom as materials for contextual reproduction, so that students can experience the spirit of pursuing truth and perseverance of mathematicians in history, thus enabling them to gain positive emotional experiences, stimulating their enthusiasm for learning and inquiry, cultivating the good qualities of being rigorous and conscientious and pursuing the truth, and sharpening their strong will.

As we all know, Pi is the ratio of the circumference of a circle to its diameter, and it is a very important constant in mathematics. In order to find out what this ratio equals, countless mathematicians at home and abroad have persevered and explored. In order to make students appreciate the hardships of scientific exploration and human's persistent spirit of exploration, teachers can have students read the textbook history: History of Pi in class, and then make a short summary: the ancient Chinese mathematician Zu Chongzhi accurately found the value of Pi for 7 decimal places and was a world leader for a thousand years until around 1429 AD when it was broken by mathematicians in Central Asia. But mankind's calculations of Pi did not stop there. In 1610, the German Ludolph calculated Pi to the 35th decimal place according to the classical method, and later generations carved this number on his tombstone in his memory. As time progressed and technology developed, electronic computers appeared and Pi could already be calculated to the billionth decimal place. For thousands of years, no number has been more fascinating than pi. Combined with the teaching content, the teacher asks students to read the History of Mathematics and then summarize it, which can deepen their understanding of what they are learning, and the process of mankind's continuous understanding and exploration of pi can enlighten students and play an important role in training them to establish the quality of facing difficulties and the spirit of dedication to science.

In the lesson of "The area of parallelograms", students' conjectures about the area of parallelograms can be elicited from the origins of geometry - the distribution of land after the flooding of the Nile in ancient Egypt. Many students are influenced by the negative migration of the area of rectangles and believe that the area formula is the product of two adjacent sides. Next, through the Exploration and Discovery, Cooperation and Communication sessions, students are guided to test their conjectures, and teachers and students discuss the results of students' tests to obtain the formula for the area of parallelograms. Finally, the ancient Egyptian formula for the area of a parallelogram is presented: the product of two adjacent sides, which is the same as the students' conjecture. In this way, students are guided to understand that mathematicians also make mistakes, that it is not terrible to make mistakes, and that mathematics is a process of progress in making and correcting mistakes.

For example, in the lesson of "Statistics and Probability", the teacher organizes a coin toss in pairs, with one student tossing a coin and the other counting the number of times the coin has gone up heads and tails, and the two working together to find the pattern. Some groups toss the coin 10 times, some toss it 20 times, and some toss it 30 times. By analyzing the data, students may conclude, "The number of times a coin goes heads up and tails up is similar." At this point, the teacher asks, "What if the coin is tossed 100 times, 1,000 times, or 10,000 times?" The teacher then shows the students the experimental data table of the five scientists who tossed the coins. The students analyze the table and conclude that "the more experiments, the closer the number of heads up and tails up".

In class, students play a coin toss game and initially learn that the probability of a coin going heads up and tails up is equal. On the basis of this, the teacher shows students the experimental data of thousands of coin tosses conducted by five scientists, so that students can feel the scientists' spirit of hard work and rigorous research on the basis of mathematical conclusions, thus enabling them to gain positive emotional experiences.

2.5 The History of Mathematics can promote the culture of mathematics and enhance cultural confidence

The history of all peoples is broad and profound, and the mathematical cultures in their histories are also dazzling. Much of the knowledge in the history of Chinese mathematics has had an important influence on the history of the development of mathematics in the world, such as pi and the Pythagorean theorem. For a long time, however, the History of Mathematics has not been given much attention by teachers, who have missed the importance of integrating it into class to facilitate teaching and learning. Teachers who can incorporate appropriate mathematical history into class will not only provide students with a deeper understanding of knowledge, but also allow them to learn more about the historical value of mathematics, and implicitly pass on and promote the culture of mathematics to a wider audience.

For example, in the lesson of "Circumference", teachers can explain to students the origins of Pi, first created by Liu Hui during the Wei and Jin dynasties who created the cyclotomic method, and then by Zu Chongzhi in 460 A.D., when he made Pi accurate to 7 decimal places, i.e. 3.1415926, which was more than 1000 years ahead of the West. It was only in 1596 that Ludolph van Ceulen calculated Pi to the 35th decimal place. For ease of calculation, the Pi we use in mathematics today is generally kept to two decimal places. In the classroom, through the history of Pi, students can feel that the knowledge we learn today is the result of the wisdom of the ancients, and that learning knowledge also opens up students' horizons and improves their learning efficiency.

During the study of "Year, Month and Day", in addition to the study of the unit of time, the year, month and day, we can also extend the study of the unique Chinese calendar, the lunar calendar, and learn about the mathematics behind the lunar calendar and the origins of it. Externally, students will learn about the Chinese calendar and the 24 solar terms, which are closely related to the lunar calendar, and their role in understanding China's contribution to the astronomical calendar, as well as the links between the lunar calendar and literature, and between the lunar calendar and customs, to enhance our cultural confidence.

3. Conclusion

The History of Mathematics is an important part of the history of human civilization. We should inherit and carry forward the scientific spirit and cultural qualities contained in the History of Mathematics, and appropriately integrate mathematical history materials related to mathematical knowledge into teaching, so that the History of Mathematics can play the role as a teacher by showing the ins and outs of mathematical knowledge, presenting the process of mathematicians' search for truth, and allowing students to obtain sources of knowledge and draw spiritual nourishment from the dialogue with the ancients.

Jules Henri Poincaré, the great French mathematician, said that if we want to see the future of mathematics, the proper way is to study the history and present state of the science. It is evident that only with a proper understanding of the past and present of mathematics can we better look into the future of it. Therefore, in teaching primary school mathematics, teachers should continue to explore more ways and means to integrate the History of Mathematics into our teaching in a scientific and rational way, so that our mathematics classroom can burst out a different kind of excitement.

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