Optimal design of wave energy maximum output power based on equation model

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Abstract: *Wave energy is an important marine renewable energy at present. How to improve the energy conversion efficiency of wave energy conversion devices is one of the key issues to achieve the comprehensive utilization of wave energy. In this paper, the differential equation model is established by analyzing the motion model of the wave energy device during heave and pitch, and the optimization model is established to maximize the average output power of the wave energy device. In this paper, according to the fact that the wave energy device is subjected to gravity and buoyancy in still water and the two forces are balanced, combined with the given parameters of the float and vibrator, it is found that the height of the cylinder part of the float submerged in water is 2m. The float only makes heave motion in the wave. The force of the float and the vibrator is analyzed respectively and the differential equation model is established to solve them. The variation trend of the heave displacement and velocity of the float and the vibrator with time is obtained under the given two conditions. It is found that the initial movement of the whole device under wave excitation is relatively violent, then tends to be stable and presents a periodicity, and fluctuates within 0.8m of the offset balance position.*

Keywords: Differential equation model; Moment of inertia; Optimization model

1. Introduction

Wave energy is an important renewable resource at present, and its application has not been large-scale. The wave energy power generation device presented in this paper mainly includes a float, a vibrator and a PTO. The float separates the outside from the inside, making power generation more stable. Under the excitation of the wave, the float produces heave or swing motion. The relative motion between the internal swing device and the float drives the damper to do work, and the work done is taken as the energy output [1-3]. Improving the energy conversion efficiency of the wave energy device is the only way to realize the large-scale utilization of wave energy. Under these backgrounds and given data, it is considered that the float only makes heave motion in the wave. In the initial state, when the whole device is stationary on the sea surface, calculate the velocity and heave displacement of the float and vibrator with a time interval of 0.2 s in the first 40 wave cycles under wave action when the damping coefficient is 10000 N · s/m and the ratio of the damping coefficient to the arithmetic square root of the absolute value of the relative velocity of the float and vibrator is 10000 [4]. Before solving the problem, it is necessary to understand the energy conversion principle of this type of wave energy power generation device. The external float of the wave energy power generation device isolates the external seawater from the internal power generation part, improving the stability of the device. As the action object of wave force, the float contacts with the sea water. Due to the excitation of the wave, the float produces heave or swing motion. The internal swing device and the float generate relative motion to drive the damper to do work, and the work done is taken as energy output [5]. Wave energy is converted into mechanical energy and finally output electric energy through damper work, realizing indirect conversion from wave energy to electric energy [6]. Considering that the float only makes heave motion in the wave, it is required to solve the heave displacement and velocity of the float and vibrator under the wave excitation force for a given time under a given situation [7]. First, determine the initial position of the float: when the wave energy device is stationary on the water surface, according to the buoyancy and gravity balance of the entire device, combined with the given seawater density and the mass of the float, vibrator and other attribute parameters, the specific position of the device in the sea water, that is, the initial position. Then, according to the motion model of floater and vibrator, the energy conversion and force of wave energy are analyzed to establish differential equations and solve them [8].

2. Model establishment and solution

2.1 Initial state

Determine the initial position of the wave energy device when floating, that is, determine the position of the water surface relative to the device. When the wave energy device is stationary on the water surface, calculate the gravity of the whole device:

$$G = (M + m)g, \tag{1}$$

Buoyancy force on the whole device:

$$F_1 = \rho g h \tag{2}$$

Wherein, refers to the volume of discharged water from the device. From the buoyancy and gravity balance of the whole device, we can get:

$$G = F_1 \tag{3}$$

Get:

$$V = \frac{M+m}{\rho}. \tag{4}$$

Calculate the volume of the cylindrical part and the volume of the conical part:

$$V_1 = \pi R^2 H, \tag{5}$$

$$V_2 = \frac{\pi R^2 h_1}{3}, \tag{6}$$

Calculate the height $x_0 = \frac{V - V_2}{V_1} H$ of the cylinder partially submerged by sea water, and calculate the $x_0 = 2$ m.
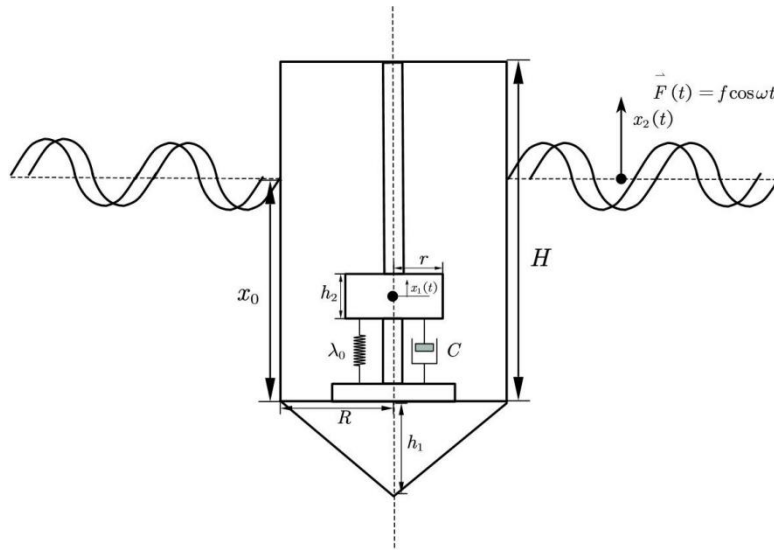

Figure 1: Schematic Diagram of Wave Energy Device at Initial Position

As shown in Fig 1, the float is subjected to the wave excitation force $F_2(t) = f\cos\omega t$. As shown in Fig 1, it is the initial position of motion, the displacement of the float at this time is $x_2(0) = 0$, and the displacement of the float with respect to time is $x_2(t)$, taking the vertical upward as the positive direction; At this time, the displacement of the vibrator is $x_1(0) = 0$, and the displacement of the vibrator with respect to time is $x_1(t)$. The vertical upward direction is taken as the positive direction.

2.2 Force analysis

Firstly, the force of the vibrator is analyzed. In the initial state, the vibrator is subjected to gravity and the elastic force of the initial spring, which are balanced by two forces:

$$mg = F_3 \tag{7}$$

At any moment, the vibrator is subjected to gravity mg, the spring force F_N against it and the damper force F_d against it. The spring force $F_{弹}$ against it is the sum of the spring force generated by the later deformation and the spring force in the initial state:

$$F_N = F_1'(x_1(t) - x_2(t)) + F_3 \tag{8}$$

The damping force of the linear damper is proportional to the relative velocity of the float and the vibrator, and the proportional coefficient is the damping coefficient C_1 of the linear damper, then

$$F_d = C_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right) \tag{9}$$

According to Newton's second law:

$$m \frac{d^2 x_1(t)}{dt^2} \tag{10}$$

In the initial state, the vibrator is subjected to gravity equal to the elastic force of the initial spring, and the following results are obtained:

$$m \frac{d^2 x_1(t)}{dt^2} = F_1'(x_1(t) - x_2(t)) + C_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right). \tag{11}$$

Then analyze the force on the float. In the case of motion, the float is subject to buoyancy, gravity, spring force, damping force, wave excitation force and wave making damping force. The buoyancy can be decomposed into the buoyancy F_1 of the float in the initial state and the buoyancy change caused by the displacement $x_2(t)$ of the float, namely

$$\triangle F_1 = \rho g \pi R^2 x_2(t) \tag{12}$$

Hydrostatic restoring force is the force caused by buoyancy change:

$$F_S = \triangle F_1 + Mg - F_1 = \rho g \pi R^2 x_2(t) - mg \tag{13}$$

The wave making damping force is proportional to the speed of the shaking motion, that is

$$F_{d2} = k_2 \frac{dx_2(t)}{dt} \tag{14}$$

According to Newton's second law:

$$(m_1 + M) \frac{d^2 x_2(t)}{dt^2} = f\cos\omega_1 t - F_S - F_N - F_d - F_{d2} \tag{15}$$

Where m_1 is the heave added mass. Simplify the above equation by using equations (13), (14) and (15), and finally obtain:

$$(m_1 + M) \frac{d^2 x_2(t)}{dt^2} = f\cos\omega_1 t - \triangle F_1 - F_1'(x_1(t) - x_2(t)) - F_d - k_2 \frac{dx_2(t)}{dt}, \tag{16}$$

The heave motion model is obtained by simultaneous equation (16) and the above equation:

$$\begin{cases} m \frac{d^2 x_1(t)}{dt^2} = F_1'(x_1(t) - x_2(t)) + C_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right), \\ (m_1 + M) \frac{d^2 x_2(t)}{dt^2} = f\cos\omega_1 t - \triangle F_1 - F_1'(x_1(t) - x_2(t)) - F_d - k_2 \frac{dx_2(t)}{dt}, \end{cases} \tag{17}$$

And initial displacement in initial state:

$$\begin{cases} x_1(0) = 0, \\ x_2(0) = 0. \end{cases} \tag{18}$$

Since the damping coefficient of linear damper is proportional to the power of absolute value of relative velocity of float and vibrator, namely

$$C_1' = \eta_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right)^a, \tag{19}$$

Where η_1 is the scale coefficient and a is the power index. Then the equation becomes:

$$\begin{cases} m \frac{d^2 x_1(t)}{dt^2} = F_1'(x_1(t) - x_2(t)) + \eta_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right)^{a+1}, \\ (m_1 + M) \frac{d^2 x_2(t)}{dt^2} = f\cos\omega_1 t - \triangle F_1 - F_1'(x_1(t) - x_2(t)) - F_d - k_2 \frac{dx_2(t)}{dt}, \end{cases} \tag{20}$$

Initial displacement:

$$\begin{cases} x_1(0) = 0, \\ x_2(0) = 0. \end{cases} \tag{21}$$

2.3 Model solving

Using the method of degree reduction of ordinary differential equations, $y_1(t) = x_1(t)$, $y_2(t) = \frac{dx_1(t)}{dt}$, $y_3(t) = x_2(t)$, $y_4(t) = \frac{dx_2(t)}{dt}$,

$$\begin{cases} \frac{dy_1(t)}{dt} = y_2(t), \\ \frac{dy_2(t)}{dt} = \frac{F_1'(y_1(t)-y_3(t))+ C_1(y_2(t)- y_4(t))}{m}, \\ \frac{dy_3(t)}{dt} = y_4(t), \\ \frac{dy_4(t)}{dt} = \frac{f\cos\omega_1 t-\triangle F_1-F_1'(y_1(t)-y_3(t))-F_d-k_2\frac{dy_4(t)}{dt}}{m_1+M}, \end{cases} \tag{22}$$

Among

$$\begin{cases} \triangle F_1 = \rho g\pi R^2 x_2(t) = \rho g\pi R^2 y_3(t), \\ F_d = C_1\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right) = C_1(y_2(t) - y_4(t)), \end{cases} \tag{23}$$

The initial conditions are:

$$\begin{cases} y_1(0) = 0, \\ y_2(0) = 0, \\ y_3(0) = 0, \\ y_4(0) = 0. \end{cases} \tag{24}$$

Use MATLAB to solve the equation (24) and bring in the initial conditions, as well as the damping coefficient of the linear damper:

$$C_1 = 10000 \text{ N} \cdot \text{s/m}, \tag{25}$$

The result of solving the heave displacement within the first 40 wave cycles with a time interval of 0.2 s is shown in Fig 2 below:
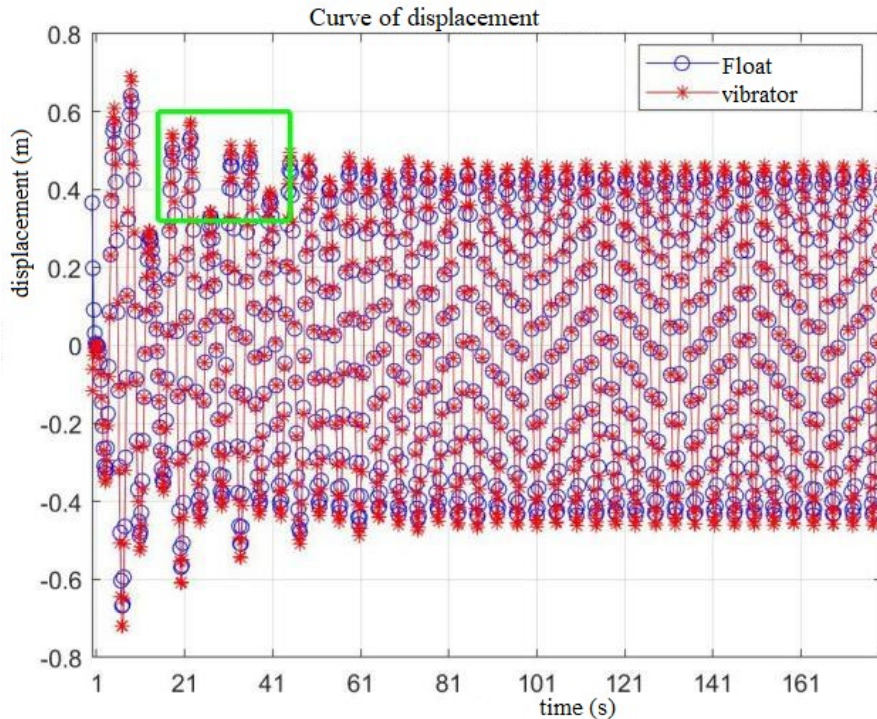

Figure 2: Heave Displacement Diagram

From the solution results, it can be seen that the value of the heave velocity of the float and vibrator

is roughly in the interval [- 1.2,1.2], similar to the trend of displacement change. The initial movement is irregular, and gradually tends to a periodic heave movement, Make $y_1(t) = x_1(t), y_2(t) = \frac{dx_1(t)}{dt} y_3(t) = x_2(t), y_4(t) = \frac{dx_2(t)}{dt}$ equation 11 equivalent to:

$$\begin{cases} \frac{dy_1(t)}{dt} = y_2(t), \\ \frac{dy_2(t)}{dt} = \frac{F_1'(y_1(t)-y_3(t))+\eta_1(y_2(t)-y_4(t))^{a+1}}{m}, \\ \frac{dy_3(t)}{dt} = y_4(t), \\ \frac{dy_4(t)}{dt} = \frac{f\cos\omega_1 t - \triangle F_1 - F_1'(y_1(t)-y_3(t))-F_d - k_2\frac{dy_4(t)}{dt}}{m_1+M}, \end{cases} \qquad (26)$$

The scale coefficient and power index are respectively:

$$\eta_1 = 10000, a = 0.5, \qquad (27)$$

3. Conclusion

Solve the velocity with a time interval of 0.2 s in the first 40 wave cycles, and it can be obtained that when the damping coefficient of the linear damper is proportional to the power of the absolute value of the relative velocity of the float and the vibrator, its motion result is similar to the result of the damping coefficient of the linear damper, which is also that the motion law is not obvious at the beginning, and tends to be stable and more regular with time.

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