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Dynamic Regulation of Great Lakes Water Levels Using Multiple Model Algorithms

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Abstract: *The North American Great Lakes, one of the largest freshwater reservoirs in the world, experience frequent water level fluctuations due to climate change and human activities, posing significant challenges to ecology, navigation, agriculture, and residential life. In this study, based on meteorological and hydrological data of the Great Lakes basin, dynamic differential equations are developed to describe the dynamic changes of water levels over time. The optimal water levels of the Great Lakes are obtained as 183.34, 176.51, 169.27, 174.44, and 75.12 with the objective of benefit maximization. The ARIMA model was used to predict the precipitation and evaporation of the basin based on historical data, and the dynamic network flow regulation mechanism was studied, in order to control the lake water level at the optimal water level of each stakeholder through parameter optimization.*

Keywords: *Dynamic network flows, Differential equation, ARIMA, Parameter optimization*

1. Introduction

The Great Lakes of North America are the largest group of freshwater lakes in the world, not only providing abundant water resources for the surrounding areas but also supporting important ecosystems and economic activities in the region. In recent years, with the impacts of climate change and human activities, water levels in the Great Lakes have fluctuated frequently, posing many challenges to the ecological environment, shipping and transportation, agricultural irrigation, and residential life. It is therefore of great importance to rationally regulate the water level of the Great Lakes system for the benefit of all stakeholders.

In the actual river and lake system, which includes elements such as lakes, rivers connecting between lakes, and artificial water control projects, water flow can be regarded as the momentum of the whole system. Because of the changes in the natural environment and human activities, the water flow at different times has differences and lags. This network flow of flow over time is called Dynamic Network Flow (DNF)[1], and this network flow problem is called Dynamic Network Flow Problem (DNFP), which was first proposed and solved by Ford and Fulkerson[2-3].

In this paper, the Great Lakes system is regarded as a network flow structure. Using the equivalence idea commonly used in physical problems, a cylinder is created to replace the appearance of each lake to establish an idealized lake model, based on which dynamic differential equations of water level change are established to describe the dynamic changes of water flow in the river and lake network[4]. Then, the optimal water level ranges for ecological environment, shipping and transportation, agricultural irrigation, and residential life are analyzed respectively, and the quadratic function relationship between water level and interest is established, and the optimal water level interval under the maximum interest is obtained through parameter optimization. Finally, the natural factors in the differential equation are predicted by ARIMA and combined with the dynamic differential equation of water level change and the optimal water level to derive the mathematical relationship between the inflow and outflow data of the Great Lakes, thus maintaining the optimal water level of the Great Lakes. This study not only helps to optimize the management of water resources in the Great Lakes, but also provides theoretical support and methodological references for future dynamic flow management in similar watershed systems.

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2. Establishment of dynamic flow network model and Determine optimal water level

2.1 Data description and model preparation

The data for this study was obtained from https://www.glerl.noaa.gov/ahps/mnth-hydro.html and the data files used include: subdata_sup_lake.csv, runoff_sup_arm.csv, eri_201110_forecast_ precip.csv, eri_201108_forecast_temp.csv and so on. In this study, the above data were firstly subjected to missing value completion and outlier removal to guarantee the integrity of the data. Then the data are normalized to scale the data to the same range, eliminating the difference in magnitude between different features. The data were analyzed by correlation analysis and visual graphical analysis to observe the data characteristics intuitively.

In the Great Lakes Basin of North America, water flows from west to east from Lake Superior, through Lake Michigan and Lake Huron, Erie, and Ontario, and ultimately into the Atlantic Ocean via the St. Lawrence River. A geographic sketch is shown in the Figure 1:

Figure 1: Geographic sketch of the Great Lakes

In this paper, the lake-river system is regarded as a network flow structure. The lakes are the nodes, the rivers are the edges connecting the nodes, and the water flow is the most important "momentum" of the whole network of rivers and lakes. First, the Great Lakes basin is considered as a purely natural ecological network. That is, the impact of human factors on rivers and lakes is not considered.

It is difficult to explore the variation of lake flow in a short period because river and lake inlet runoff, water storage, shape, and elevation are not the same, and their influencing factors are complex and varied. Therefore, this paper chooses to use the idea of equivalence, which is commonly used in physical problems, to develop an idealized lake model. An equivalent idealized lake model means that when faced with a dynamic, discrete, and complex scientific problem targeting a lake's water volume, the basic feature of the lake, i.e., water storage, is extracted first. All other features are discarded and then transformed into a familiar model for alternative studies. The water storage and surface area of the Great Lakes are known from existing data. Therefore, a cylinder was created to replace the appearance of each lake as shown in Figure 2.

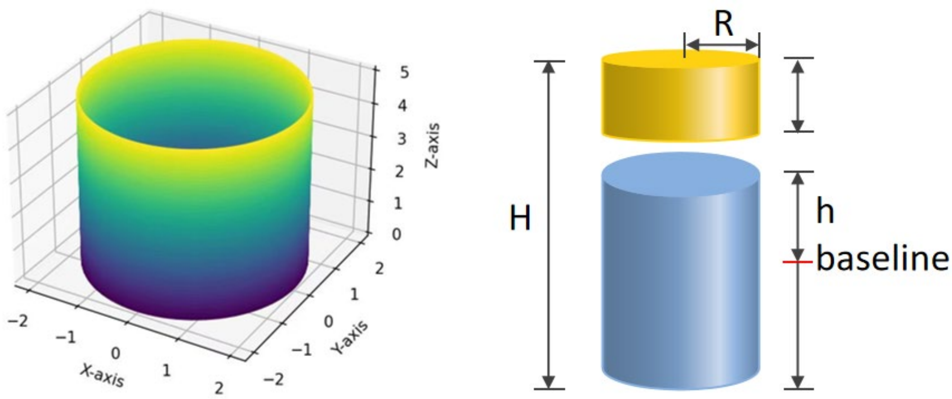

Figure 2: Lake shape equivalent replacement

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The blue color indicates the current amount of water in the reservoir. Yellow indicates the volume of water that has disappeared. The volume of a cylinder can be represented by (R is the radius of the base of the cylinder and H is the height of the cylinder).

$$V_0 = \pi R^2 H \tag{1}$$

To analyze the water flow dynamics of this lake-river network, the following differential equations are established in this paper[5-8]:

$$\frac{dV_i}{dt} = Q_{in,i} - Q_{out,i} + \delta_i \tag{2}$$

Where $Q_{in,\,i}$ inflow of river water into Lake *i*. $Q_{out,\,i}$ denotes the flow of water out of Lake *i*. δ_i denotes the effect of natural environmental factors on the volume of water in the lake, including, but not limited to, evapotranspiration, precipitation, ice volume, and seepage from lake soils.

Assuming that the river and lake water exchange process satisfies the water balance relationship.

$$\pi R^2 (h_{present,i} - h_{origin,i}) = V_i \tag{3}$$

Equation 4 is the model of lake water level change with time. Where V_i represents a period in the water volume of the lake I change in value. $_{present,\,i}$ represents the real-time water level of lake *i*, and $h_{oringin,\,i}$ represents the water level of the lake *i* before the change.

The associative formula (2,3) yields:

$$\frac{dh_{present,i}}{dt} = \frac{Q_{in,i} - Q_{out,i} + \delta_i}{\pi R^2} \tag{4}$$

2.2 Calculation of optimum lake level

2.2.1 Selection of stakeholders

Water resources can be used for water supply, power generation, navigation, and ecological protection. Different stakeholders have different requirements on the water level of the lake. To satisfy the needs of each stakeholder as much as possible, this study selects the three main stakeholders, namely, shipping transportation, ecological protection, and domestic production needs, to investigate the optimal water level of the lake.

For shipping transportation, the water level directly affects the transportation capacity of ships. Lower water levels will cause the narrowing of the channel, the foreshore will be exposed, affecting the safe navigation of the ship. The water level is too high, which will make the reef's rocky shore water beach flooded, and the flow rate increases, reducing the ship control ability to reduce, which will also affect the ship's navigation safety. Therefore, for ship transportation, the lake water level needs to ensure that large ships can freely pass through the lake and connected waterways. In this paper, the water level to meet the 10,000 tons of ships in and out of the lakes is the minimum water level for ship navigation.

$$HN_{low,i} \leqslant h_i \leqslant HB_i - b \tag{5}$$

Where $HN_{low,\,i}$ denotes the minimum navigable water level of Lake *i*. h_i denotes the real-time water level of Lake *i*. HB_i denotes the maximum water level of Lake *i*.

Changes in lake water levels play an important role in the functioning and dynamic balance of lake ecosystems. The ecosystem needs to maintain a relatively stable water level to protect plant and animal habitats around the lake. In the existing studies, there are several methods for calculating the minimum ecological water level of lakes. Here we use the actual operation of the minimum (maximum) ecological water level method.

The minimum (maximum) ecological level is the lowest (highest) level at which a lake can maintain its basic ecological functions, and is calculated as follows:

$$\begin{cases} HE_{ij} = \min(HE_{jk}) \\ HE_{low,j} = \min(HE_{ij}) \end{cases} \quad \begin{cases} HE'_{ij} = \max(HE_{jk}) \\ HE_{high,j} = \max(HE_{jk}) \end{cases} \tag{6}$$

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$$HE_{low,i} \leqslant h_i \leqslant HE_{high,i} \tag{7}$$

Where HE_{ij} indicates the lowest ecological water level of Lake *i* in the month *j*. HE_{jk} denotes the actual operational minimum ecological water level of Lake *i*. HE'_{ij} denotes the maximum ecological water level of Lake *i* in month *j*. HE_{jk} denotes the natural monthly average water level in month *j* of year *k* of the water level data series. $HE_{low,\ i}$ denotes the monthly minimum ecological water level. $HE_{high,\ i}$ indicates the monthly maximum ecological water level of the lake *i*.

For domestic production needs, three main areas were considered: agriculture, industry, and residential water use. For agriculture, data on the area of farmland in each lake basin is collected. Assuming that the water demand of each piece of farmland is fixed, the daily agricultural water demand $D_{a,\ i}$ can be easily estimated based on the available data by multiplying the farmland area $S_{a,\ i}$ with the average farmland water demand a_1.

$$D_{a,i} = \frac{S_{a,i} \times a_1}{365} \tag{8}$$

In the case of industry, the average annual gross industrial product p_i and the amount of water required per unit of production value a_2 were collected for each lake basin. The daily industrial water demand can be estimated from this calculation:

$$D_{c,i} = \frac{\left(\sum \bar{p}_i\right) \times a_2}{365} \tag{9}$$

Finally, regarding residential water consumption, data on per capita water consumption was collected from the Internet. The per capita daily water consumption is a_3. The total daily water use for each lake basin can be obtained as $D_{r,\ i}$.

$$D_{r,i} = \left(\sum Pop_i\right) \times a_3 \tag{10}$$

Therefore, in terms of production and domestic demand, the total amount of water required per day, through the equivalent conversion, this part of the water caused by the change in the water level is:

$$HC_{low,i} = \frac{(D_{a,i} + D_{c,i} + D_{r,i})}{\pi R^2} \tag{11}$$

Then, to satisfy the basic water demand for production and domestic use, the basic range of the lake level, using the water level datum as a reference, is:

$$HC_{low,i} \leqslant h_i \leqslant HB_i - b' \tag{12}$$

2.2.2 Optimum water level intervals

From the above analysis, combined with the relevant data, a reasonable range of water levels required by the various stakeholders in each lake basin can be obtained. This is shown in the Table.1.

Table 1: Range of water levels required for each stakeholder

	Ship transportation	Ecological conservation	Domestic production requirements
Lake Superior	(179.79, 197.33)	(167.32, 193.88)	(172.54, 216.26)
Lake Michigan	(175.57, 183.42)	(124.35, 177.45)	(154.32, 189.24)
Lake Huron	(167.22, 179.25)	(118.37, 172.49)	(148.32,184.35)
Lake Erie	(173.73, 178.24)	(121.34, 175.14)	(143.27,186.54)
Lake Ontario	(74.28, 88.32)	(68.76, 75.91)	(72.54,92.53)

With the above-obtained range of reasonable water levels to satisfy each stakeholder, the intersection between them can be found.

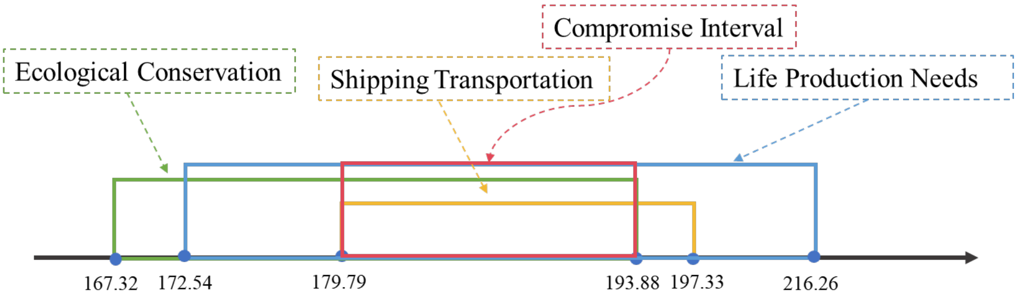

Figure 3: Lake Superior Water Level Discount Interval

From the Figure 3, the water level intersection is (179.79, 193.88), and this intersection is taken as the fluctuation interval of the optimal water level. Then, to maximize the benefits for all parties, the fluctuation interval of the water level is narrowed down to find the optimal water level of each lake.

Because the range of water levels required for shipping transportation and ecological protection fluctuates very little, the water demand aspect is considered. The function between water level and benefits is established to maximize the benefits of production and life. The data collected on water use in each lake basin was used to determine the functional relationship between them.

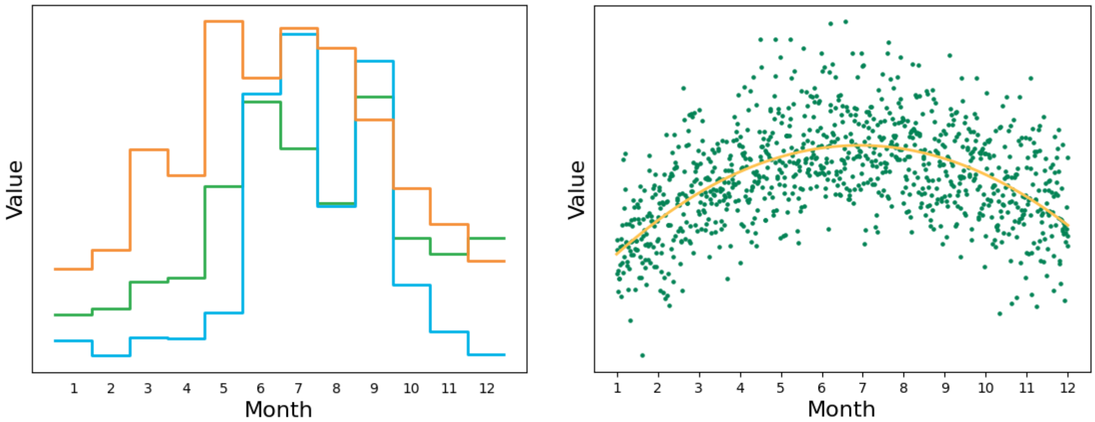

Figure 4: Water Level-Time Variation and Water Use-Time Variation

With the historical water consumption and water level data, a graph of water consumption-time and water level-time variation can be obtained as Figure 4. From the graphical trend, it can be roughly determined that there is a quadratic relationship between the two. So the functional relationship between them is established by quadratic fitting of the trend graph.

$$\begin{cases} W_i = \lambda D_i \\ D_i = a_{i1} t^2 + a_{i2} t + \varepsilon_i^{(1)} \\ h_i = b_{i1} t^2 + b_{i2} t + \varepsilon_i^{(2)} \end{cases} \tag{13}$$

The system of joint equations gives a functional relationship between the water level and the benefits in terms of productive life:

$$h_i = F(W_i) \tag{14}$$

Where W_i represents the value of production and livelihood benefits of lake *i* basin. D_i denotes the total water use in the watershed of Lake *i*. λdenotes the rate of change of production and living benefits caused by water use. h_i denotes the water level of Lake *i* at time t.

$$\max F(h_i),\ h_{down,i} \leqslant h_i \leqslant h_{up,i} \tag{15}$$

Where $h_{down,\ i}$ and $h_{up,\ i}$ denote the compromise intervals of the required water levels for each stakeholder, respectively.

Maximize the function to get the optimal water level of each lake as 183.34, 176.51, 169.27, 174.44, and 75.12, respectively.

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3. Predictive Modeling for Stabilizing Water Levels in the Great Lakes Network

The optimum water level for each lake at any given time has been determined in the previous section, and this level can then be controlled so that it is always maintained at the optimum level. Changes in this dynamic system of inputs and outputs as a whole are regulated by predicting changes in natural environmental factors. Since precipitation and evapotranspiration data are based on time variations, this paper uses a time series model to predict future trends.

In this paper, we construct the ARIMA (p, d, q) model, where p is the autoregressive term; q is the moving average term, and d is the number of differences made when the time series is smooth. The model is a combination of autoregressive (AR) and moving average (MA), which can predict future data based on historical data, transform non-stationary time series into stationary time series, and then regress the lagged value of the dependent variable, the current value of the random error term, and the lagged value into the established model[9-12].

The ARIMA model building flowchart is shown in Figure 5.

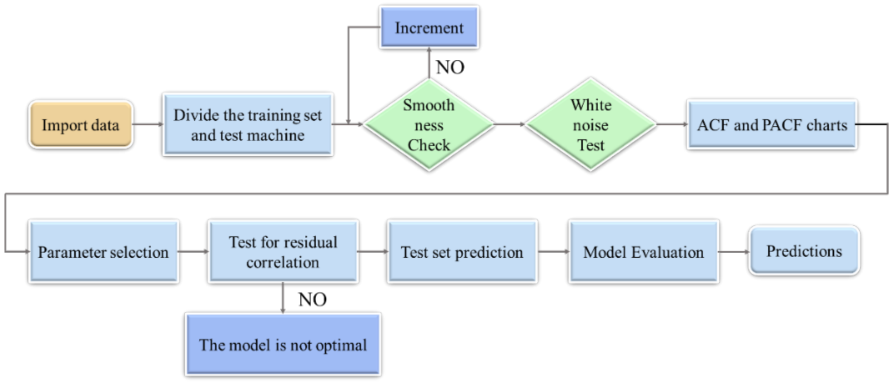

Figure 5: Flowchart of the ARIMA model

Through the above process, we forecast precipitation and evapotranspiration for each lake basin, and in this paper, only show the forecasting process for the Lake Ontario basin. The time-series prediction model for precipitation in the watershed of the Lake Ontario Stream is ARIMA (1,0,0) and for evapotranspiration is ARIMA (2,0,2). The predicted time series plot is shown in Figure 6.

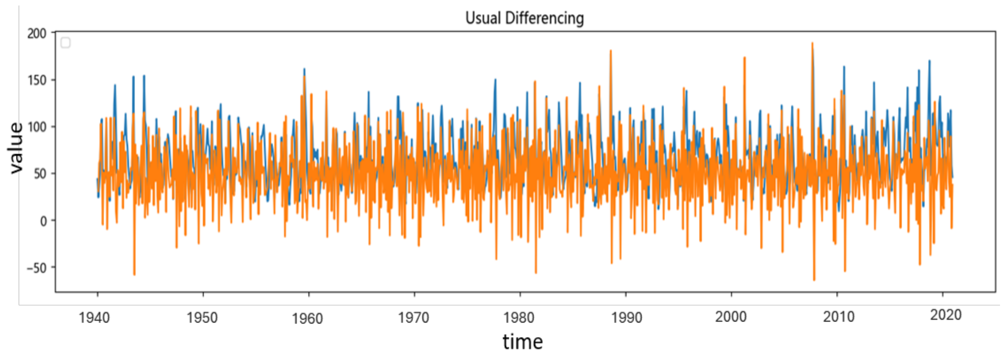

Figure 6: Projected Precipitation in the Lake Ontario Basin

Applying the same procedure, ARIMA prediction models of precipitation and evaporation for each lake basin can be obtained as Table2.

Table 2: ARIMA Model for Precipitation and Evaporation in the Great Lakes Basin

	Precipitation	Evaporation
Lake Superior	ARIMA (1, 0, 0)	ARIMA (2, 0, 2)
Lake Michigan	ARIMA (2, 0, 2)	ARIMA (3, 0, 2)
Lake Huron	ARIMA (1, 0, 0)	ARIMA (2, 0, 3)
Lake Erie	ARIMA (1, 0, 0)	ARIMA (2, 0, 3)
Lake Ontario	ARIMA (1, 0, 0)	ARIMA (2, 0, 2)

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4. Conclusions

In this paper, the Great Lakes are regarded as nodes, and the connected rivers are regarded as line-constructed network flow systems between the nodes. Based on the collection of natural climate and hydrological data of the Great Lakes from 1940 to 2020, precipitation, evaporation, and inflow and outflow of river runoff and lakes are selected to establish dynamic differential equations for water level changes. The historical data were subjected to step and scatter plot visualization and primary and secondary relationship simulation; the secondary relationship was found to be r = 0.87427 and the primary relationship to be r = 0.47961. The values of optimal water levels for the Great Lakes at any given time were derived as 183.34, 176.51, 169.27, 174.44, and 75.12.

The time series visualization of the natural influence factors was found to have the characteristic of seasonal change over time. An autoregressive integrated moving average model was constructed to predict the natural factors in the differential equations. Taking Lake Ontario as an example, the time series prediction model for precipitation in its watershed is ARIMA(1,0,0) and the prediction model for evapotranspiration is ARIMA(2,0,2).

In this paper, the network flow model of the Great Lakes is developed and the ARIMA model is utilized to predict the trend of natural factors' influence on the lake levels, which provides a basic idea for further exploring more complex models and water level control algorithms to deal with more complicated dynamic network flow problems in lake systems. In subsequent studies, the temporal and spatial scopes of the data can be further expanded to include more detailed historical data and a wider range of geographic areas to enhance the accuracy and generalizability of the model. By continuously deepening the research and exploration of the dynamic management of the lake system, more scientific and sustainable solutions can be provided for the protection and effective utilization of the water resources of the Great Lakes to meet the challenges and opportunities faced in the future.

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