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Research on "Fast" active Reflector Optimization based on Particle Swarm Optimization algorithm

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Abstract: *This paper mainly studies the related mathematical models of ideal paraboloid and convex lens imaging, and studies the optimization problem of "FAST" active reflector by using particle swarm optimization algorithm. First of all, according to the most basic parabolic equation, through the constraint on the distance between the actuator and the adjacent nodes in the known conditions, the ideal paraboloid problem is transformed into an optimization problem according to the motion range of the center point of the paraboloid. MATLAB is used to program, and particle swarm optimization algorithm is used to obtain the optimal paraboloid. Then, due to the change of the position of the celestial body to be observed, the focus of each cable point reaching the intersection paraboloid of the origin is further calculated, that is, the ideal position of the cable point on the paraboloid, and then the ideal paraboloid is obtained. Then, under the condition that the distance between the adjacent main cable nodes does not exceed 0.07%, and the radial expansion range of the actuator is-0.6 to 0.6 meters, the ideal paraboloid is optimized so that the deviation of the adjusted node position relative to the ideal paraboloid is minimized. Finally, the particle swarm optimization algorithm is used to obtain the optimal solution in MATLAB.*

Keywords: *Active reflector, Ideal paraboloid, Optimization model, Particle swarm optimization algorithm*

1. Introduction

Radio telescope is a basic equipment used to observe and study radio waves from celestial bodies, which can measure the radio intensity, frequency spectrum and polarization of celestial bodies [1]. The 500m aperture spherical radio telescope (Five-hundred-meter Aperture Spherical radio Telescope, referred to as FAST), which is known as one of the "two big eyes" of the earth, is a radio telescope with independent intellectual property rights in China[2], the largest single aperture and the most sensitive radio telescope in the world. The "FAST" project is composed of an active reflector, a signal receiving system (feed cabin) and related control, measurement and support systems[3][4]. Make it close to the ideal paraboloid as far as possible, in order to obtain the best receiving effect of celestial electromagnetic waves reflected by the reflector, has become an important research topic.

2. Model analysis

The parabolic 2x=2pz on the two-dimensional plane is obtained by dividing the three-dimensional rotating paraboloid along the xCz surface (figure 4). The vertex of the parabola coincides with the center of the reference sphere.

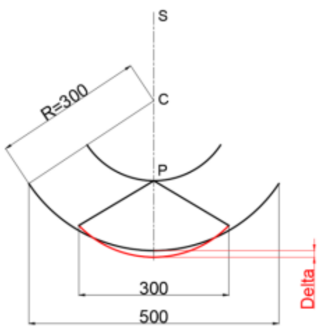

Figure 1: Schematic diagram of section to be optimized

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Since the expansion length of the actuator is limited between -0.6m and 0.6m, that is, the value of delta in the figure, this problem is transformed into finding the most appropriate value of delta, making the ideal paraboloid the most suitable optimization problem

3. Establishment and solution of ideal paraboloid equation

3.1 Establishment of equation

According to the method of solving the rotating surface [5], it is deduced that

$$x^2 + y^2 = x^2(z) + y^2(z) \tag{1}$$

Since the Z coordinate of the vertex of the paraboloid can move delta meters within 0.6m up and down of the center of the reference sphere, for each different vertex, it can be deduced from the half focal length of the parabola vertex from the focus. At this time, the equation of the paraboloid is:

$$z = \frac{x^2 + y^2}{4(F + \Delta)} - \Delta \tag{2}$$

At this time, take any coordinate of the node of the main cable on the datum sphere, and the slope k of the line connected with the center of the datum sphere:

$$k = \frac{R - z_i}{\sqrt{{x_i}^2 + {y_i}^2}} \tag{3}$$

Through the matlab program, the position of each changed main cable node under different delta values can be obtained. Through the connection between each main cable node and its adjacent nodes, the minimum total relative change between all adjacent main cable nodes can be obtained. At this time, the corresponding delta value is an important parameter of the equation for determining the optimal ideal paraboloid.

3.2 Solution of model

The required paraboloid is 300m in diameter. First of all, all the main cable nodes should be screened. Because the center of the paraboloid coincides with the reference sphere, it is only necessary to screen out the points that need $x^2 + y^2 < 150^2$. Through MATLAB drawing, we can directly distinguish whether each main cable node needs to be adjusted or not.

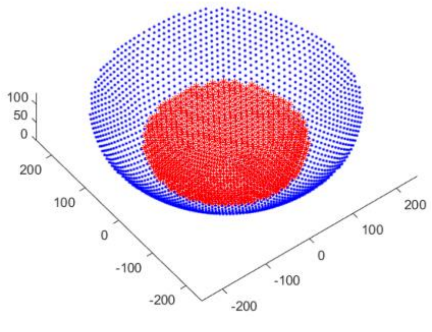

Figure 2: Schematic diagram of the position of all the cable points and the position of the cable points that need to be adjusted

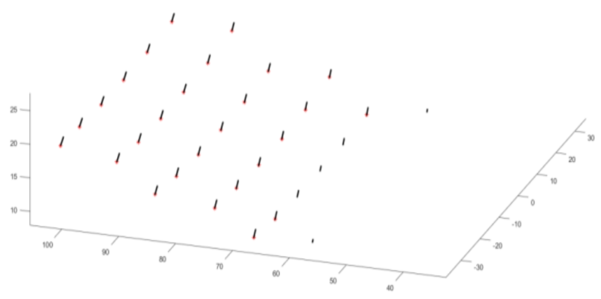

Figure 3: Displacement of some main cable nodes after adjustment

After screening out the nodes that need to be adjusted, the k and r_k of each node are obtained by using the above formula, and the plot function is used to draw different values to adjust the adjustment distance of the node, as shown in the figure2.

Through the iterative cycle, the sum of the relative changes between the adjacent points of all the adjusting nodes is obtained, and the function of the sum of the relative changes is made, which can be obtained intuitively. When the parabola takes 0.48, that is, when the vertex of the parabola is 0.48m

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directly below the center of the reference sphere, it is the optimal ideal paraboloid, and the equation of the parabola is:

$$z=\frac{x^2+y^2}{4(0.466*300+0.48)}-300.48 \tag{4}$$

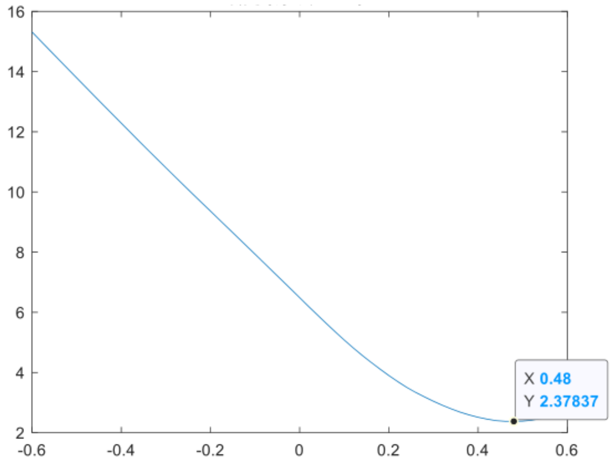

Figure 4: Determine the optimal delta value function

4. Coordinate system transformation

4.1 Coordinate system transformation

The ideal paraboloid can be transformed from a coordinate system to a new coordinate system, which only needs to multiply the original matrix with the azimuth matrix and the elevation matrix.

After transformation, it can be obtained.

$$R_x=R\times R_1\times R_2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha&0\\\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}\begin{bmatrix}\cos\beta&0&\sin\beta\\0&1&0\\-\sin\beta&0&\cos\beta\end{bmatrix} \tag{5}$$

With $\alpha=36.795^\circ$, $\beta=90^\circ-78.169^\circ$, the coordinates of the original reference reflector can be transformed. The position transformed into paraboloid in the active reflector can be screened.

The coordinates of the ideal paraboloid are further obtained, and the focal point of each cable point reaching the intersection paraboloid of the origin is calculated, which is the ideal position of the cable point on the paraboloid. The calculation is as follows:

$$\begin{cases}x^2+y^2=2p(z+c)\\x=kx_0\\y=ky_0\\z=kz_0\end{cases} \tag{6}$$

Among them p=279.67, c=300.035.

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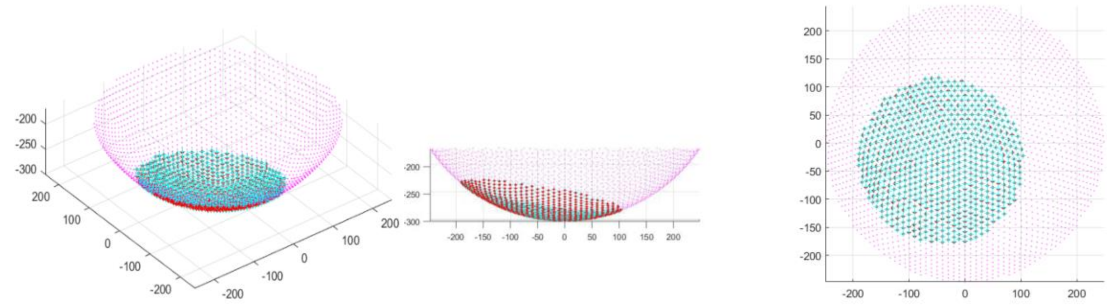

Figure 5: Ideal paraboloid

4.2 Optimization process

The goal of optimization is to keep the blue cable point as close to the red cable point as possible under the condition that the distance between the adjacent main cable nodes does not exceed 0.07%, and the radial expansion range of the 11 actuator is-0.6 to 0.6 meters. Since the final cable length change is needed in the actual adjustment, the load increment acting on the mesh node in the adjustment process is taken as the design variable, which is recorded as follows:

$$\Delta F=\left\{\Delta F_1^T,\Delta F_2^T,\cdots,\Delta F_{\mathrm{n}}^T\right\}^T \tag{7}$$

Then, through the setting of the position of the adjusted main cable node and the analysis of the lighting part of the feed at a certain time, it is assumed that T is the coordinate transformation matrix from the global coordinate system to the follow-up coordinate system $OX_0Y_0Z_0$. If the node initial position vector x_0, the node displacement vector u, and the adjusted node position vector x are converted to the follow-up coordinate system, the deviation degree between the position of any node j and the design paraboloid can be expressed as follows:

$$\delta_j\left(x_j^r\right)=2x_{j0}^r\Delta x_j^r+2y_{j0}^r\Delta y_j^r-4f\Delta z_j^r+\left(x_{j0}^r\right)^2+\left(y_{j0}^r\right)^2-4f\left(z_{j0}^r+h_0^r\right) \tag{8}$$

The displacement component of node j in the follow-up coordinate system, including $\Delta \mathrm{u}_j^r=\left\{\Delta x_j^r,\Delta y_j^r,\Delta z_j^r\right\}^T$.

$$\begin{aligned}A_j&=\left[2x_{j0}^r,2y_{j0}^r,-4f\right]\\ \mathbf{b}_j&=\left(x_{j0}^r\right)^2+\left(y_{j0}^r\right)^2-4f\left(z_{j0}^r+h_0^r\right)\end{aligned} \tag{9}$$

When the nodes of the lighting part are located on the design paraboloid, the residue of any node j (J ∈n) should be zero, so the following objective function is taken to minimize the sum of squares of the residues of each node.

$$\begin{aligned}f\left(\Delta F\right)&=\sum\nolimits_{j\in C}\left[\delta_j\left(x_j^r\right)\right]^2=\sum\nolimits_{j\in C}\left[A_j\Delta u_j^r+b_j\right]^2\\ &=\left(\Delta u_c^r\right)^T A^T A\Delta u_c^r+2\left(\Delta u_c^r\right)^T A^T B+B^T B\end{aligned} \tag{10}$$

The corresponding node displacement cannot be too large. If the maximum cable length adjustment that can be realized in the project is $\Delta L_{\max}$, the following node displacement constraints can be introduced:

$$\Delta u_i\in\left[-\Delta L_{\max},\Delta L_{\max}\right],i=1,2,\ldots,n \tag{11}$$

At the same time, the distance between adjacent nodes cannot change too much. The difference of the maximum distance between adjacent nodes that can be realized in the project shall comply with:

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$$\left|\frac{\left(\mathrm{x}_{i0}+\Delta \mathrm{u}_i\right)-\left(\mathrm{x}_{(i+1)0}+\Delta \mathrm{u}_{i+1}\right)}{\mathrm{x}_{i0}-\mathrm{x}_{(i+1)0}}\right| \leq 0.07\% \tag{12}$$

It is assumed that under the reference state, the radial expansion amount of the top of the actuator is 0, and the radial expansion range is -0.6 ~ 0.6m. This constraint condition is expressed as:

$$\Delta z_j^r \in \left[-0.6, 0.6\right],\ j=1,2,\ldots,n \tag{13}$$

4.3 Find the optimal solution

The optimal model can be obtained:

$$find\,\Delta F=\left\{\Delta F_1^T, \Delta F_2^T, \cdots, \Delta F_{\mathrm{n}}^T\right\}^T$$

$$\min \quad f\left(\Delta F\right)=\sum\nolimits_{j\in C}\left[\delta_j\left(x_j^r\right)\right]^2$$

$$s.t.\Delta u_i \in \left[0, \Delta L_{\max}\right], i=1,2,\ldots,n$$

5. Conclusion

This paper mainly studies the related mathematical models such as ideal paraboloid and convex lens imaging, and uses the particle swarm optimization algorithm to solve the most basic parabolic equation. Through the constraint on the distance between the actuator and the adjacent nodes in the known condition, it is obtained that the paraboloid $z=\frac{\left(x^2+y^2\right)}{559.2}-300.48$ is the optimal solution of all feasible paraboloids where the vertex coordinate is 0.48m downward extending from the positive center of the reference sphere. Then the focus of each cable point reaching the origin intersection paraboloid is further calculated, that is, the ideal position of the cable point on the paraboloid is obtained, and then the ideal paraboloid is obtained. Finally, the optimal solution is obtained by using the particle swarm optimization algorithm in MATLAB.

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