

Research on emergency distribution problem based on graph theory algorithm

Mengting Liu¹, Yangjie Li², Jiahui Li³, Xiaoqi Tan²

¹Faculty of Accounting, Guangzhou Southern College, Guangzhou, Guangdong, 510000, China

²Faculty of Electrical Engineering, Guangzhou Southern College, Guangzhou, Guangdong, 510000, China

³Faculty of Public Administration, Guangzhou Southern College, Guangzhou, Guangdong, 510000, China

Abstract: When unexpected events such as road block, closure and damage caused by some major events occur, they will have an impact on people's daily life. With the gradual popularization of 5g network, the application of UAV is becoming more and more extensive. This paper mainly studies the "emergency material distribution problem in 5g network environment". It analyzes the two cases of separate distribution of distribution vehicles and the combined distribution of "distribution vehicles + UAVs" respectively. In order to complete the material distribution task as soon as possible, it puts forward the best distribution scheme respectively. First, the optimal time is simplified, that is, the optimal time is converted into the shortest path, and the graph theory model is used for modeling. The path graph is highlighted by MATLAB, and the minimum spanning tree method is used to visualize all paths. After that, the shortest path from the ninth point to any point and the sequence of several traveling paths are obtained. The sequence of several traveling paths is classified into three schemes, and the total kilometers of the three schemes are calculated respectively. The best scheme is the one with the smallest total number of paths. For the best scheme, the path map of the best scheme is obtained by drawing. Secondly, the combined distribution mode of "distribution vehicle + UAV" is adopted for material transportation. Nowadays, 5g network is becoming more and more popular, and UAVs are more and more widely used. In order to complete the material distribution task as soon as possible, it is assumed that UAVs are used to transport materials to the greatest extent to reduce the transportation time and improve the efficiency. Visualize the path of "distribution vehicle + UAV" for subsequent problem solving. The aircraft route is divided into "three consecutive sections" and "two consecutive sections". After a large amount of data processing, the classification scheme is obtained. Combined with the vehicle optimal path, the best scheme of "distribution vehicle + UAV" combination to complete a complete distribution is obtained. Finally, the maximum load capacity of the delivery vehicle is 500 kg, and the other conditions are the same as above. Based on the obtained optimal scheme, the second question is solved by MATLAB for the constraints given in the question, and two feasible schemes are obtained. The frequency of UAV use in both schemes is greater than that of distribution vehicles. Disassemble and analyze scheme 1 and scheme 2 respectively.

Keywords: Minimum spanning tree; Heuristic algorithm; Path visualization; combined distribution

1. Introduction

In the field of logistics and distribution, as a new and novel means of transportation, UAV has many advantages, such as low cost, flight free from the impact of road blocking, damage and closure under emergencies, but it is inferior to traditional means of transportation such as trucks in terms of flight distance and load capacity. Vehicle transportation and UAV distribution have their own advantages and disadvantages. Reasonable cooperation between the two is conducive to reducing costs.

2. Global shortest path analysis

When the vehicle speed V remains unchanged, it belongs to the "global shortest path problem: finding all the shortest paths in the local" in graph theory. Under the condition that the walking path can be repeated, in order to get the shortest overall path, it is necessary to repeat the same path as little as possible. In short, the long path should only be taken once as far as possible. Taking this constraint into

account, when traveling to the forked road at each location, the long forked road will be preferred^[1].

The UAV does not participate in the transportation, and only the distribution vehicles are used for transportation.^[2] Figure 1 shows 14 locations, and the ninth location is the concentration point of emergency materials. Make the minimum spanning tree so that the target result coincides with the path of the spanning tree as much as possible. The distance between two points in the table can be visualized through the distance of each location shown in the first excel table, as shown in Figure 1.

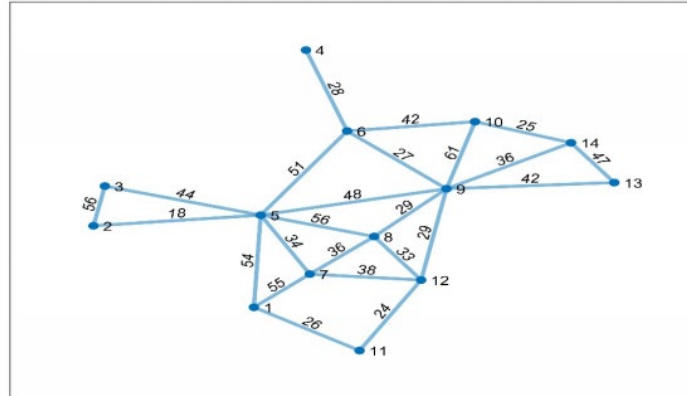


Figure 1: Path distance visualization

2.1 Constructing spanning tree with graph theory knowledge

Graph theory knowledge: starting from any vertex, construct a spanning tree, record the distance from the spanning tree to each vertex, select the nearest non tree vertex (m) from the spanning tree to join the spanning tree, and then update the distance from the spanning tree to each non tree vertex with m as the middle point. Repeat the above steps until the spanning tree has n vertices.

Minimum weight spanning tree: in a given undirected graph $G = (V, E)$, (m, v) represents the edge connecting vertex m and vertex v, and $w(m, v)$ represents the weight of this edge. If there is a subset of T as E (i.e., an acyclic graph, so that $w(T)$ is minimum, then this T is the minimum spanning tree of G.^[3]

$$\omega(t) = \sum_{(u,v) \in t} w(u, v) \quad (1)$$

In order to further solve the problem, the minimum spanning tree is generated by compiling the code in MATLAB, and the shortest path from the ninth point to any point is obtained in combination with the requirements, as shown in Figure 2 below:

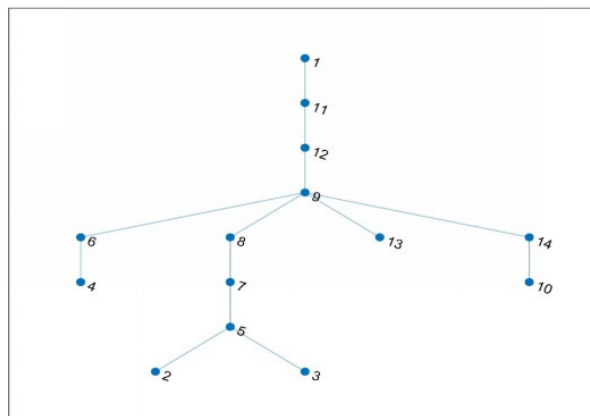


Figure 2: Shortest path spanning tree from the ninth point to any point

The to highlight diagram drawn with matlab compiled code is shown in Figure 3 below: Green is the common route for distribution vehicles and UAVs, blue is the route that the UAV can travel.

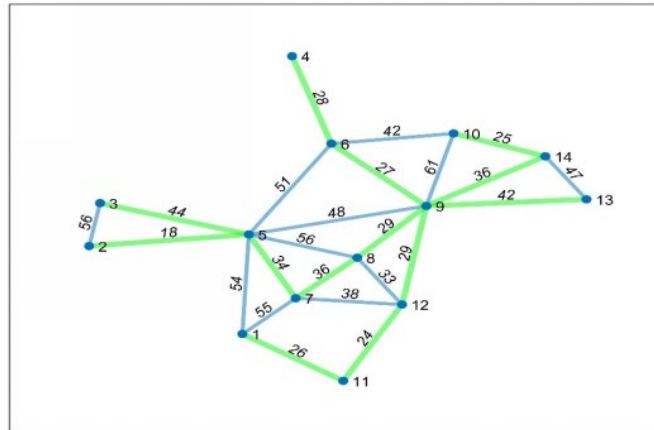


Figure 3: Path highlight

2.2 Design paths using observation and exhaustive methods

According to the shortest path spanning tree in 1.1, all feasible schemes have been screened and multiple feasible schemes can be obtained from the legend. Then, all feasible schemes are listed using the observation method and the exhaustive method.^[4] The path numbers of each scheme are calculated and compared to obtain the shortest path number, so as to determine the best scheme.

The scheme obtained by observation method and exhaustive method is as follows:

Table 1: Path scheme

Path scheme	Path (unit: km)
Scheme I:9-13-14-10-6-4-6-5-3-2-5-7-1-11-12-8-9	582
Scheme II:9-13-14-10-6-4-6-5-3-2-5-7-1-11-12-9	607
Scheme III:9-13-14-10-6-4-6-5-3-2-5-1-11-12-7-8-9	588

After that, the optimal route is plotted, and Figure 4 can be obtained

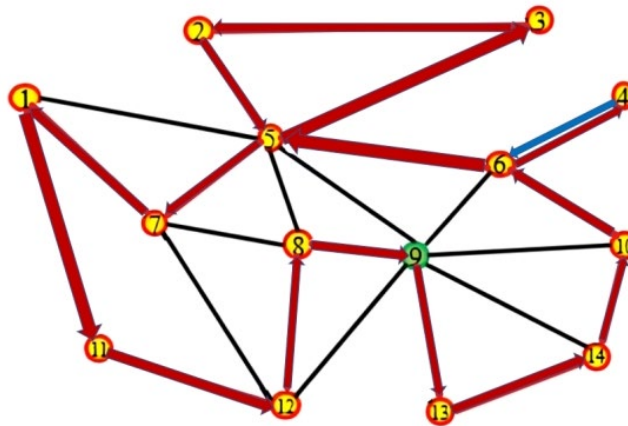


Figure 4: Optimal path of distribution mode

3. Distribution vehicle and UAV distribution mode

The distribution mode of "distribution vehicle + UAV" means that the UAV needs to pick up goods from the vehicle. After the distribution, it needs to return to the vehicle to reload materials and replace batteries. The distribution vehicle can release the UAV at a certain place and then go to other places to distribute materials. It can also synchronously distribute goods at other customer points after the UAV picks up goods. This distribution mode can greatly improve the distribution efficiency of emergency materials, It can also solve material distribution under complex road conditions and avoid secondary injury to personnel caused by secondary disasters. At the same time of distribution, the UAV can also distribute to the surrounding feasible places, and return to the distribution vehicle after distribution to

reload materials and replace batteries.

For the above analysis, from the original distribution mode with only distribution vehicles to the combined distribution mode of "distribution vehicles + UAVs", the dotted line path is added, and the other conditions are the same.

Considering that the cost of using UAV to distribute materials is relatively low, 5g network is gradually popularized, and UAV is more and more widely used, UAV is used to the greatest extent for distribution.

3.1 Visual analysis

To facilitate the analysis, first visualize the figure in question, as shown in Figure 5 below: "Red path" refers to the route that distribution vehicles and UAVs can travel, "blue path" is the driveable route of UAV

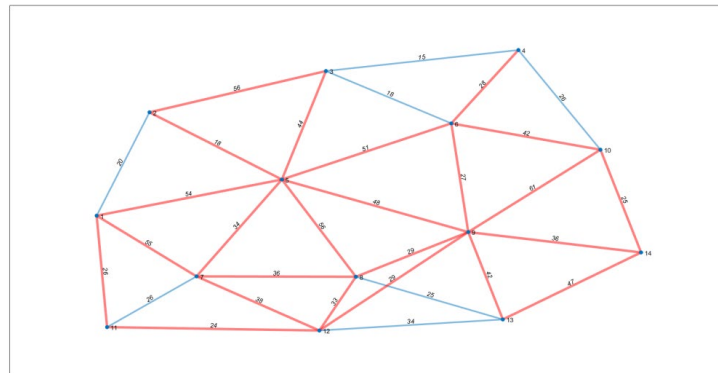


Figure 5: Route visualization of "distribution vehicle + UAV"

3.2 list the paths passing through two or three continuous sections respectively

Table 2: UAV operation path

UAV operation path					
Through three consecutive sections		Passing through two consecutive sections			
2-1-11-7	1-11-7	5-9-12	8-13-12	12-9-6	
2-1-11-12	1-11-12	5-9-14	8-13-14	12-9-8	
3-4-6-9	2-1-5	6-4-3	9-6-4	12-9-13	
3-4-6-10	2-1-7	6-4-10	9-6-5	12-9-14	
3-6-4-10	2-1-11	6-9-5	9-6-10	12-11-1	
3-6-9-8	2-3-4	6-9-8	9-12-7	12-11-7	
3-6-9-12	2-3-6	6-9-12	9-12-8	12-13-8	
3-6-9-13	3-4-6	6-9-13	9-12-11	12-13-9	
3-6-9-14	3-4-10	6-9-14	9-12-13	12-13-14	
4-3-6-5	3-6-4	7-1-2	9-13-8	13-9-6	
4-3-6-9	3-6-5	7-1-11	9-13-12	13-9-8	
4-3-6-10	3-6-9	7-11-1	9-14-10	13-9-12	
4-6-9-8	3-6-10	7-11-12	9-14-13	13-9-14	
4-6-9-12	4-3-2	7-12-8	10-4-3	13-12-7	
5-3-4-6	4-3-5	7-12-9	10-4-6	13-12-8	
5-3-4-10	4-3-6	7-12-11	10-6-4	13-12-9	
6-4-3-5	4-6-5	7-12-13	10-6-9	13-12-11	
7-11-1-2	4-6-9	8-9-5	10-14-9	13-14-9	
8-9-6-4	4-6-10	8-9-6	10-14-13	13-14-10	
9-6-4-3	5-1-2	8-9-12	11-1-2	14-9-5	
9-6-4-10	5-1-11	8-9-13	11-1-5	14-9-6	
10-4-3-5	5-3-4	8-9-14	11-1-7	14-9-8	
10-4-3-6	5-3-6	8-12-7	11-12-7	14-9-12	
10-4-6-9	5-6-4	8-12-9	11-12-8	14-9-13	
10-6-4-3	5-6-9	8-12-11	11-12-9	14-13-8	
12-9-6-4	5-9-6	8-12-13	11-12-13	14-13-12	
12-11-1-2	5-9-8	8-13-9	12-9-5		

The 6-4-3-5, 5-1-2, 7-11-12, 8-13-9, 10-4-6 contents in the above table are the best path for the

aircraft to travel.

3.3 "Distribution vehicle + UAV" is the best scheme to complete one-time overall distribution

Table 3: Best combination distribution scheme of "distribution vehicle + UAV"

The best scheme of "distribution vehicle + UAV" combined distribution	
Delivery vehicle route	9-14-10-6-5-2-5-7-12-8-9
	10-4-6
	Waiting for delivery vehicles at location 6
	6-3-5
	Waiting for delivery vehicles at location 5
UAV route	5-1-2
	After receiving the UAV at point 2, the delivery vehicle will follow the UAV to point 7
	7-11-12
	Waiting for delivery vehicles at point 12
	8-13-9

3.4 Formula application

According to the above problems, this paper defines S_k as the station of the K UAV, t_i as the time when point i is accessed, and defines 0-1 variables x_{xjv} and y_{xjk} as follows^[5]:

$$x_{xjv} = \begin{cases} 1, & \text{The delivery vehicle travels from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_{xjk} = \begin{cases} 1, & \text{UAV } k \text{ flies from } i \text{ to } S_k \text{ and then to } j \\ 0, & \text{otherwise} \end{cases}$$

Based on the description and variable definition, the following models can be built^[6]

$$\min Z = \sum_{i \in N} \sum_{j \in N} c_{ij} \sum_{i \in V} x_{ijv} + 2 \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} c_{skj} y_{ijk} + a \sum_{j \in N} \sum_{i \in V} x_{ojk} + b \sum_{k \in K} \sum_{j \in N_{ab}} y_{s_k k} \quad (2)$$

$$\sum_{i \in N} \sum_{v \in V} x_{ijv} \leq \sum_{i \in N} \sum_{k \in K} \sum_{k \in K} y_{ijk}, \Lambda j \in N_s \quad (3)$$

$$\sum_{i \in N} \sum_{v \in V} x_{ijv} = 1 \Lambda j \in N_{cr} \quad (4)$$

$$s.t. \sum_{i \in N} \sum_{v \in V} x_{ijv} = \sum_{i \in N} \sum_{k \in K} y_{ijk} \Lambda j \in N_{CD} \quad (5)$$

$$\sum_{i \in N} \sum_{v \in V} x_{ijv} \leq \sum_{i \in N} \sum_{k \in K} y_{ijk} \Lambda j \in N_s \quad (6)$$

$$\sum_{k \in N} x_{ijv} = \sum_{k \in N} x_{jiv} \leq 1, \Lambda j \in N, \Lambda v \in V, i \neq j \quad (7)$$

$$\sum_{i \in N_s \cup N_{CD}} y_{ijk} = \sum_{i \in N_s \cup N_{CD}} y_{jik} \leq 1, \Lambda j \in N_{CD}, \Lambda k \in K, i \neq j \quad (8)$$

$$\sum_{i \in N_s \cup N_{CD}} y_{ijk} = \sum_{i \in N_s \cup N_{CD}} y_{jik} = 0, \Lambda j \in N_{CT}, \Lambda k \in K \quad (9)$$

$$\sum_{i \in N} \sum_{j \in N} x_{ijv} \cdot d_i \leq Q_T, \Lambda v \in V \quad (10)$$

$$2 \cdot y_{ijk} \cdot c_{S_{kj}} \leq C_D, \Lambda i \in N, \Lambda j \in N, \Lambda k \in K \quad (11)$$

The objective function (1) is to minimize the total cost, which includes the running cost of the

distribution vehicle, the flight cost of the UAV, the fixed cost of the distribution vehicle and the fixed cost of the UAV; Formula (2) indicates that each station can only be visited once by the distribution vehicle or UAV; Formula (3) indicates that the stations that can not be accessed by UAVs are accessed by distribution vehicles; Formula (4) indicates that only after the distribution vehicle visits the UAV site, the UAV in the site can carry out the distribution service; Formula (5) indicates that the access degree of the distribution vehicle for each point is the same; Formula (7) indicates that the UAV has the same access to the station; Formula (8) indicates that the point beyond the UAV load formula and distance formula cannot be accessed by UAV; Formula (9) indicates that the distance between the station distributed by the UAV and the UAV station is within the range of UAV mileage; Formula (10) represents the capacity formula of the distribution vehicle.

4. Analysis of two path schemes

On the basis of the optimal path in the second section, figure 5 is divided into two parts to get the two-part path scheme, and the end points of the last two schemes are collected to the ninth site.

4.1 "Distribution vehicle + UAV" route map scheme I

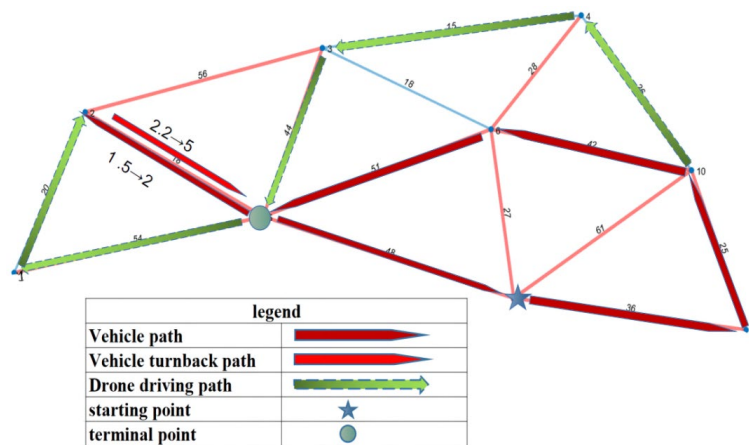


Figure 6: Path scheme I

The travel route of distribution vehicles is: 9-10-6-5-2-5-9 and traveling route of UAV is: 9-14-10-4-3-5-1-2.

The starting point of this scheme is point 9, and the ending point is point 5 (the end point of receiving the aircraft). (1) the distribution vehicle carries the UAV from station 9, passes through station 10, and the UAV separately transports materials from point 9 to station 14, and then flies to station 10 to supplement power supply. (2) The distribution vehicle starts from the 10th station and passes through the 6th and 5th stations. The UAV delivers materials from the 10th station to the 4th and 3rd stations and then flies to the 5th station. The distribution vehicle receives the UAV here. The UAV charges and changes its battery and starts again. (3) when the UAV delivers materials to the 1st and 2nd stations, the aircraft stops at the 2nd station, When the distribution vehicle travels to the fifth station, it will return to the ninth station with the UAV.

4.2 Route map scheme II of "distribution vehicle + UAV"

The traveling route of the distribution vehicle is 9-8-7-8-9, and that of the UAV is 9-13-8-12-7-11-12-9.

The scheme starts from point 9 and ends at this point. (1) The distribution vehicle starts from point 9 to point 8, and the UAV starts from point 9, passes through points 12 and 8, then stops at point 8. The distribution vehicle receives the UAV at point 8 and changes the battery. (2) The delivery vehicle starts from point 8 to point 7, and the UAV drives from point 8 to point 12 and then flies to point 7 for charging. (3) The delivery vehicle starts from point 8 to point 9, and the UAV starts from point 7, passes through points 11 and 12, and then flies to point 9, that is, it does not return to the destination with the delivery vehicle.

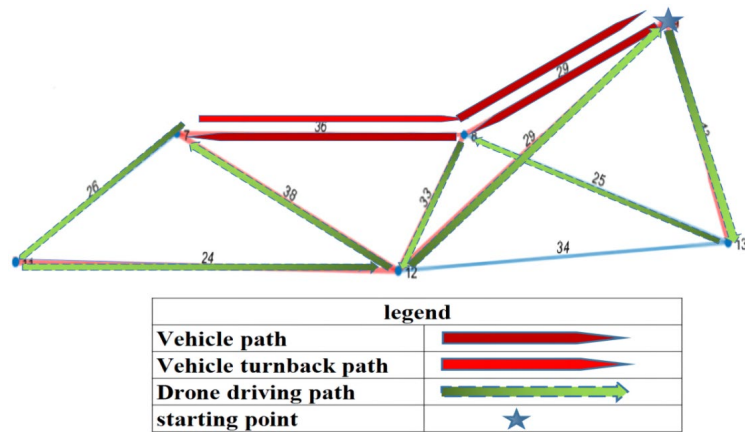


Figure 7: Path scheme II

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