ISSN 2522-6398 Vol. 5, Issue 14: 26-31, DOI: 10.25236/FER.2022.051406

Practical Research on Physical Problem Solving with Mathematical Graphical Models

Fangchun Li

Beijing National Day School, Beijing, 100080, China

Abstract: *There is the tightest relation between mathematics and physics among all disciplines. Specifically, mathematics can serve as an instrument to describe physical phenomena, while physical theory can be used to interpret mathematical laws. As a basic discipline of all natural disciplines, mathematics can provide operational tools, thinking methods, and process simplification approaches for the proposal and development of physical problems. The mathematical graphical model can effectively improve the efficiency of problem solving. This study is conducted based on the transfer theory. Besides, the methods and practice of physical problem solving with mathematical graphical models are analyzed from the perspective of specific examples in physics.*

Keywords: *Mathematics; Graphical model; Physics; Problem solving*

1. Introduction

Peter David Lax, a mathematician, maintained that there is a particularly tight relation between mathematics and physics. The reason is that mathematics is essentially involved with some problems as a discipline, many mathematical problems originate from physics and other fields, and many mathematical theories are developed to solve complicated problems in physics. In addition, mathematics can provide derivation instruments, calculation methods, and logic language for physics. Therefore, mathematics can be recognized as a discipline that is most closely related to physics. Further, the quality of mathematical learning would exert a direct impact on the achievement of physical learning. Based on that, it is of great significance to investigate the application of mathematical thinking methods to solving physical problems.

2. Transfer Theory——A Theoretical Foundation for the Combination of Mathematical Methods and Physics

Transfer refers to "the influence of one kind of learning on another kind of learning". As long as there is learning, there is transfer. Moreover, it is also a kind of transfer to apply the acquired knowledge, skills, and thinking methods to problem solving, which is common in learning physics and mathematics. The fundamental reason is that there is an interrelated, influenced, restricted, and interacted relation between mathematics and physics, both of which are not separated from each other. Students would employ the acquired knowledge, experience, skills, and methods when solving new physical problems. Mathematical thinking methods are the key to solving some physical problems. Thus, the transfer of mathematical thinking methods to physics would occur in this process. During physical problem solving with the aid of mathematical thinking methods, the acquired mathematical cognitive experience and abstract knowledge would be initiatively applied to physical situations. This application can be recognized as the continuation and consolidation of mathematical thinking methods. Besides, it also contributes to perfecting mathematical thinking and improving mathematical methods.[1]

3. Necessity of Solving Physical Problems with Mathematical Graphs

3.1 Mathematical graphs can be used to simplify physical problem solving

The key to applying mathematical thinking to physical teaching in a more scientific and effective manner lies in recognizing the relation between mathematics and physics. Specifically, it is required to understand that mathematics and physics are not two independent disciplines, and they are closely

ISSN 2522-6398 Vol. 5, Issue 14: 26-31, DOI: 10.25236/FER.2022.051406

related to each other. Some theoretical overview of physics and the solution of some problems shall be performed based on mathematical thinking. In other words, mathematics is a tool or means of learning physics, and it plays a significant role in physics teaching.

During the problem solving with mathematical graphs, graphics and graphs are employed to represent known conditions to make the quantity problem become clearer and simpler, which is effective for solving abstract problems. In some physical problems, the known quantitative relationship is too single and abstract, and hence it is difficult to judge the relation between the known conditions and problems. The graphic problem solving methods can be adopted to visualize the known conditions, which significantly reduces the difficulty of problems, thus playing a great role in problem solving. Therefore, graphic problem solving is more effective than digital problem solving. Additionally, it is easier to find solutions with the aid of graphs during solving complex physical problems.[2]

3.2 Mathematical graphs can be used to improve the efficiency and accuracy of physics problem solving

As a practical problem-solving skill, the combination of number and shape plays a crucial role in physical problem solving. During physical learning, problem solving can be simplified by the algebraic expression. However, physics is different from algebra, geometry, and other subjects. It is difficult to clearly describe the shape, position, and state of objects in physical knowledge with numbers and words. These data related to objects can be directly and vividly represented by graphics and graphs, which contributes to solving difficult physical problems. Although numerical problem solving and graphic problem solving are two different problem solving methods, they are interrelated and form an effective complementary relation. If these two methods can be combined during problem solving, physical problems can be simplified and their difficulty can be reduced. This can improve the efficiency and accuracy of problem solving.[3]

4. Combination of Number and Shape - A Method to Solve Physical Problems with Mathematical Graphical Models

4.1 The thinking method of the combination of number and shape

The combination of number and shape refers to a thinking method for solving problems through the transformation and correspondence between number and shape by expressing number with shape and interpreting shape with number to simplify complex problems and concretize abstract problems. However, the combination of number and shape can be employed to solve physical problems based on the thinking method of expressing physics with shape and interpreting shape with physics. In terms of physical problems, graphs are often used to represent the relation between physical quantities. In addition, the words describing physical phenomena, logic, and characteristics can be transformed into mathematical language, including geometric graphs and functional graphs. "Expressing physics with shape" [4]refers to identifying the hidden geometric relationship with the aid of "shape" through the observation, analysis and processing of the given motion trajectory graphics and mechanical graphics. Based on that, a physical problem is transformed into an algebraic problem, and hence the algebraic problem can be solved with the aid of graphics. Further, the connection between physical quantities can be explained clearly due to the intuitiveness and strictness of graphics, which conduces to solving the physical problem more accurately.

4.2 The process of solving physical problems with the combination of number and shape

The combination of number and shape is introduced into physics from mathematics. As an important thinking method for the study of physics, it has the essence to solve problems quickly through the internal relationship between number and shape. However, there are significant differences in its application modes. Abstraction in mathematics is defined as a pure number or shape extracted from concrete objects; while, abstraction in physics bears physical meaning based on abstraction. Besides, abstraction in physics cannot be separated from objects, and is limited by the actual conditions of objects. That is to say, abstraction in physics only describes the essence of objects or highlights some factors, and it is influenced by other factors. These contributors determine that the combination of number and shape has the characteristics of physics in physical application.

Physical problems can be solved by the combination of number and shape, and the process is

ISSN 2522-6398 Vol. 5, Issue 14: 26-31, DOI: 10.25236/FER.2022.051406

presented as follows.

First, the cognitive object, its characteristics, and associated problems are abstracted with the combination of number and shape.

Second, symbolic methods are often used to express the properties, relations, and characteristics of the object as certain symbolic languages to perform formalized calculations when the problems related to the object are involved. For instance, when the problems related to kinematics are involved, the physical quantities of the research object in the movement process and their relations are expressed by symbols or formulas. When the circuit problems are involved, the analytical thinking is often simplified through drawing circuit diagrams.

Third, it is often required to construct a proper model for the research object to abstractly express the quantitative and spatial relations between thinking and cognitive objects as schematic diagrams, flow charts, schematic diagrams and other graphs.

The specific process for model construction can also be expressed as follows.[5]

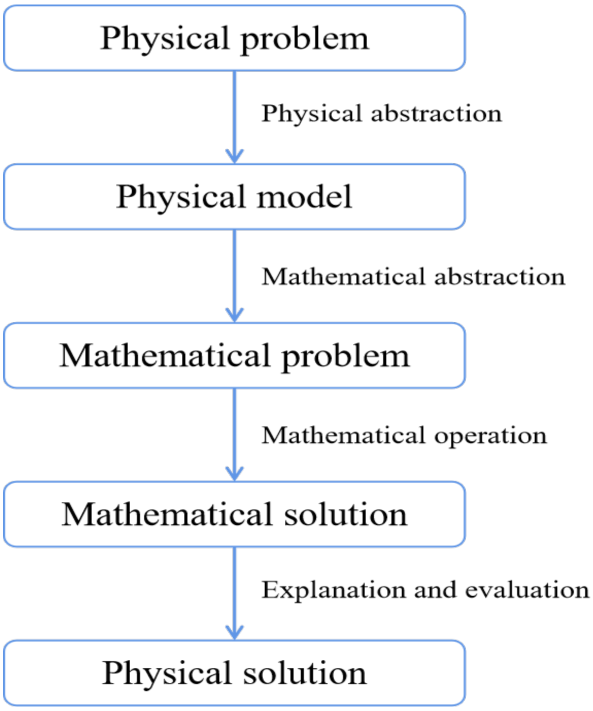


Figure 1: The specific process for mode l construction

5. Practice of Physical Problem Solving with Mathematical Graphical Models

5.1 Physical problems related to kinematics

As an important part of physics in senior high school, motion involves many formulas, and many problems are combined with graphs. It is required to firmly grasp theoretical knowledge, clarify the meaning of various parameters in formulas, and master the skills of interpreting graphs. Many learners are prone to make mistakes in the process of solving problems due to the fact that they cannot fully understand graphs. During kinematical problem solving, mathematical graphical models contribute to consolidating various concepts in kinematics, understanding the approach of related parameters to representing the characteristics of object motion, and thus improve the efficiency of problem solving.

Example 1:

The acceleration sensor is used to explore the linear motion of an object with a mass (m) of 1kg from rest, and the acceleration-time graph is shown in Figure 2. Please judge which one among the following statement is wrong according to the graph information (   ).

ISSN 2522-6398 Vol. 5, Issue 14: 26-31, DOI: 10.25236/FER.2022.051406

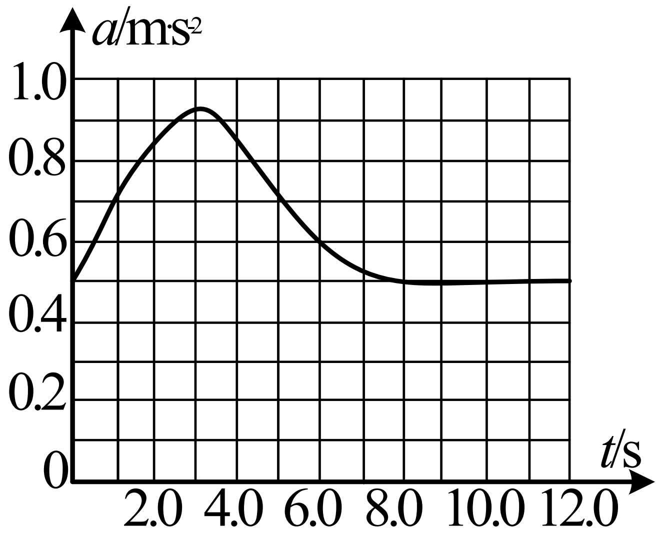


Figure 2: The acceleration-time graph

A. The object moves in a straight line at a uniform speed from t=4.0 to t=6.0s.

B. The speed of this object at t=10.0s is about 6.8m/s.

C. The work done by the combined external force on the object is about 7.3J from t=10.0s to t=12.0s.

D. The combined external force on the object increases first and then decreases from t=2.0s to t=6.0s.

The solution and analysis are presented as follows.

Options	Analysis
A	It can be observed that although the acceleration of the object is decreasing, the speed is still increasing. Thus, A is wrong.
B	The area between the graph and the coordinate axis is the speed change of the object. Thus, B is correct.
C	It can be found in the graph that the speed of the object is 7.8m/s at 12s. Thus, C is correct according to the theorem of kinetic energy.
D	The acceleration of the object in this time range increases first and then decreases, indicating that the combined external force also increases first and then decreases. Thus, D is also correct.

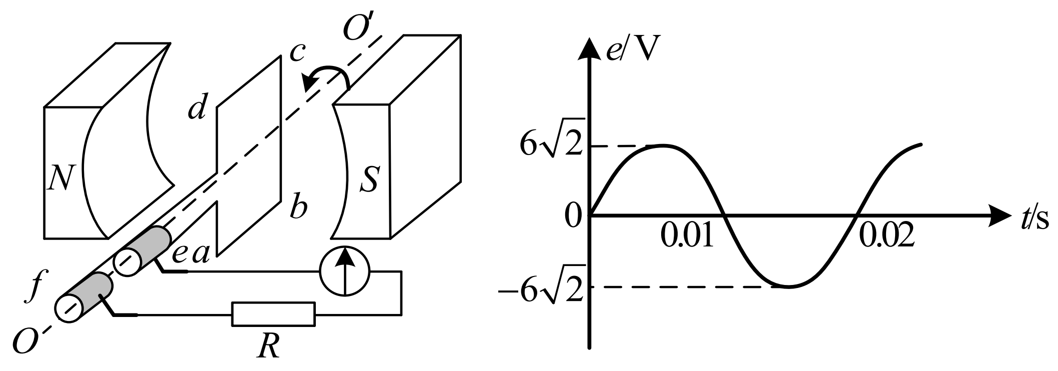
5.2 Physical problems related to magnetic fields

As is known to all, the magnetic field has invisible, intangible, and abstract characteristics. In some exercises, the relation between the parameters related to magnetic fields is represented by graphics. In general, graphics are used to describe magnetic fields, and then learners are required to conduct qualitative analysis or quantitative calculation of parameters. In order to efficiently solve this kind of exercises, it is necessary to extract more useful information from graphs as much as possible, clarify the difference and connection between associated parameters, and construct a clear graphical model in their minds.

Example 2:

Figure 3a presents a schematic diagram of the engine structure. The coil rotates counterclockwise, and the generated electromotive force (e) changes with time (t), with the changing law shown in Figure 3b. Under the condition that the internal resistance of the generator coil is ignored, and the resistance value of the external bulb resistance is 12Ω, please identify which one among the following statement is wrong (   ).

ISSN 2522-6398 Vol. 5, Issue 14: 26-31, DOI: 10.25236/FER.2022.051406



3a: Schematic Diagram of the Engine Structure *3b: Changing law of electromotive force (e) with time (t)*

Figure 3: Schematic diagram

A. The magnetic flux through the coil is 0 at t=0.01s.

B. The voltmeter shows the value of $6\sqrt{2}\mathrm{V}$.

C. The electric power consumed by the bulb is 3W.

D. If other conditions remain unchanged, the coil speed is only increased by one time, and the electromotive force of the coil can be expressed as $e = 12\sin 100\pi t\mathrm{V}$.

The solution and analysis are presented as follows.

Options	Analysis
A	According to the combination of number and shape, when t=0.01s, the electromotive force is 0, and the magnetic flux through the coil reaches the largest. Thus, A is wrong.
B	As shown in the figure, it can be observed that the maximum value of electromotive force is $6\sqrt{2}\mathrm{V}$ and the value measured by the voltmeter is effective, and the indicated value shall be $\frac{6\sqrt{2}\mathrm{V}}{\sqrt{2}} = 6\mathrm{V}$. Thus, B is wrong.
C	The power consumed by the bulb is $P = \frac{E^2}{R} = \frac{36}{12} = 3\mathrm{W}$. Thus, C is correct.
D	After the coil speed is increased by one time, the electromotive force can be expressed as $e = 12\sqrt{2}\sin 200\pi tV$. Thus, D is wrong.

6. Classification of Problems Suitable for Mathematical Graphical Models during Physics Problem Solving

6.1 Multiple choice problems related to graphs

In some physical problems, students are required to choose the graph that meets the requirement in the problem. The exclusion method is the most practical method to solve this kind of problem. This method can be implemented by performing checking one by one according to the known conditions in the problem until the most proper graph is identified. It shall be noted that the key to solving this kind of problem is to find out the differences between these graphs. The correct answer can be obtained by accurately understanding the characteristics of these graphs.

6.2 Problems related to graph drawing

When the problems related to physical variables and time are involved, the graph drawing method is effective in solving these problems. This method can establish the relation between physical variables and time, so as to accurately draw the changing law of physical variables with time. This method is useful in solving this kind of problems.

ISSN 2522-6398 Vol. 5, Issue 14: 26-31, DOI: 10.25236/FER.2022.051406

6.3 Problems related to graphic conversion

It seems difficult to identify the law in some physical exercises. However, if physical graphs are into function graphs to express this quantitative problem with geometric graphs, the essence of these problems can be intuitively presented. On that basis, the solution can be easily obtained and the problem can be solved quickly.

7. Conclusion

The mathematical graphical model is an important instrument for solving physical problems at the senior high school level. In order to let learners understand the importance of the combination of number and shape, recognize and interpret the graph accurately, grasp the characteristics and laws of different graphs, and solve problems quickly, it is required for them to integrate the combination of number and shape with physical knowledge, and master the solving thinking of associated problems, thus effectively improving the efficiency and accuracy of problem solving.

References

[1] Yan Baohua. The application of the idea of combining numbers and shapes in high school physics problem solving [J]. Mathematical and Physical Problem Solving Research, 2020(10):61-62.
[2] Wang Huayin. A preliminary investigation of the mathematical "inflection point" problem in physics images[J]. Physics Teacher, 2013,34(08):61-62.
[3] Yang Zichan. A few thoughts on the use of mathematical knowledge in high school physics problem solving[J]. Science and Technology Innovation Herald, 2016,13(30):148-149.
[4] Liu Kunyi. On the application and status of mathematics in the development of physics[J]. Low Carbon World, 2016(30):224-225.
[5] Zhai Jibin, Zheng Fu. Reflections on the integration of mathematics and physics in secondary schools[J]. Science Consulting (Educational Research), 2021(03):10-12.