Prestress modal analysis of the airplane wheel and brake

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Abstract: In this paper, the structural integrated finite element model of aircraft wheel and brake device is established, the modal analysis of the wheel and brake device under prestress is carried out, and the vibration frequencies and modes of the first 12 steps of the wheel working are obtained through prestress modal analysis, based on the macroscopic mode shapes of each mode of the wheel, the influence of the mode shapes on the normal working state of the wheel braking device and the stability of the braking system are analyzed.

Keywords: prestress modal, airplane, wheel and brake, mode shape, vibration

1. Introduction

Modal definition: natural frequency, like inertia, is an inherent property of an object. When an object vibrates at a certain order of natural frequencies, the points on the object will oscillate proportionally away from the equilibrium position, which can be represented simply by a vector.

Modal Analysis: the essence of modal analysis is to find the eigenvalues of the matrix. Modal “order” means the number of eigenvalues of the matrix, however, our computational analysis focuses only on the first modes that dominate the motion.

The mode shapes correspond to the natural frequencies, and each natural frequency corresponds to a mode shape. In experiments, we often observe the displacement of the corresponding point by exciting the structure with a certain frequency, so as to obtain the natural frequency of the object, is not a single mode of vibration display.

Based on the actual problems and phenomena of brake disc vibration in China, the vibration of brake disc is studied systematically. Ku Yuao quantitatively established the relationship between landing gear parameters and brake torque in order to solve the matching problem of wheel and brake as a whole in 1997\cite{1}. Yang Zunshe mainly introduced the new research progress of airplane wheel brake system in brake device, material and so on in 2002\cite{2}. Tao Ke et al studied the vibration frequency and vibration mode of the wheel hub by Ansys, and determined the better structure of the wheel hub in 2006\cite{3}. In 2007, Xiang shangsheng established the equivalent brake moment of inertia model according to the automobile disc brake model\cite{4}. In 2007, Wang Jisen et al. derived the dynamics model of the airframe and landing gear from the D’Alembert’s principle\cite{5}. Shi Xiaoeng et al established a simplified System dynamics model of brake vibration, and obtained measures to avoid resonance by changing the number of slots and increasing the shims vibration frequency in 2011\cite{6}. Zhang Tiejun and his colleagues used ABAQUS software to study the modal analysis and dynamic response of some structures in the simplified wheel and brake system in 2012\cite{7}. Zhang Lijun et al. established the complete finite element model of automobile friction scream through Abaqus Software in 2015 \cite{8}. The research of the above scholars mainly focuses on the analysis of the brake vibration of the automobile and the vibration analysis of the structure of the airplane wheel, such as the wheel hub and the brake disc, there is no finite element modeling involving the structural integration of airplane wheels and brakes. In this paper, the finite element model of structural integration of airplane wheel and brake assembly is established. By modal analysis, the vibration frequencies and modes of the first 30 orders are obtained.
2. Theoretical basis of modal analysis

Free vibration equation: assuming that the structure is free vibration, $F(t) = 0$, and damping matrix $[C]$, generally can be ignored, thus obtained

$$[M][x''] + [K][x] = \{0\} \quad (1)$$

Since the structure is assumed to be linear in the modal analysis, the free vibration is assumed to be harmonic because $[M]/[K]$ are constant:

$$\{x\} = \{\varphi\}_i \sin(w_i + \theta_i) \quad (2)$$

$$\{x''\} = -w_i^2 \{\varphi\}_i \sin(w_i + \theta_i) \quad (3)$$

Plug (2) and (3) into the equation $[M][x''] + [K][x] = \{0\}$, and then obtained:

$$([K] - w_i^2[M])\{\varphi\}_i = \{0\} \quad (4)$$

In equation (4), the root of the equation is $w_i^2$ (the eigenvalue), which is equal to the number of degrees of freedom, the eigenvector of the eigenvalue is, the square root of the eigenvalue is, the natural frequency is, and the eigenvector represents the mode of vibration.

Damping mode setting: in the free vibration state, the equations of motion of the damped single-degree-of-freedom system are as follows:

$$[M][x''] + [C][x'] + [K][u] = 0 \quad (5)$$

$$x'' + 2\xi w_n x' + w_n^2 x = 0 \quad (6)$$

In the equation, $w_n = \sqrt{k/m}$ is the undamped natural frequency; $C = 2\sqrt{k/m}$ is the critical damping; $\xi = C/C_c$ is the damping ratio; $w_d = w_n\sqrt{1 - \xi^2}$ is the oscillation damping rate.

The eigenvalues of damped modal analysis are more complicated. The real part represents natural frequency, the imaginary part represents stability, the negative value represents stability, and the positive value represents instability.

3. Prestress modal analysis of wheel structure

3.1 Establishment of finite element model

The model parameters of hub and tire are selected, and the offset single spoke hub is chosen. The hub is axisymmetric. The tyres were chosen in strict accordance with book 14 of *the airplane design manual*.

This model uses 2014-T6 aluminum alloy, steel bearing, tyre cord-rubber model, material parameters as shown in Table 1.

In order to reduce the computational memory and time, the model is simplified. The small
components such as air holes, hot-melt plug holes and bolt holes, which have little influence on the calculation accuracy, are neglected, and the rounded corners and chamfered corners, which have little influence on the modal analysis of the brake disc, are simplified:

<table>
<thead>
<tr>
<th>Part</th>
<th>E(Mpa)</th>
<th>( \mu )</th>
<th>( \rho )(kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hub</td>
<td>73000</td>
<td>0.33</td>
<td>2795.67</td>
</tr>
<tr>
<td>Bearing</td>
<td>210000</td>
<td>0.31</td>
<td>7850</td>
</tr>
<tr>
<td>Tire</td>
<td>6894.8</td>
<td>0.49</td>
<td>2768</td>
</tr>
</tbody>
</table>

### 3.2 Modal analysis of engine wheel

The Static Structure module and Modal module in the software of ANSYS Workbench were selected to analyze the prestressed mode of the wheel. The flow chart is as follows:

![Flow chart of prestressed modal analysis](image)

(1) The material properties are shown in table 2 below. Add the following material to Engineering Data.

(2) Mesh generation: in order to improve the accuracy of calculation results, SOLID186 cell is adopted, this cell is a hexahedron cell, the calculation convergence is fast, the geometry shape of brake disc and static disc is regular, the cell is divided by Sweep method, the result of the gridding is shown in Figure 3.

<table>
<thead>
<tr>
<th>Assembly</th>
<th>materials</th>
<th>( \rho )(g/cm(^3))</th>
<th>E(GPa)</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hub</td>
<td>2014-T6 Aluminium alloy</td>
<td>2.80</td>
<td>73</td>
<td>0.33</td>
</tr>
<tr>
<td>Torque tube</td>
<td>ZG30Cr18Mn12Si2N</td>
<td>7.75</td>
<td>194.04</td>
<td>0.3</td>
</tr>
<tr>
<td>Carbon disc</td>
<td>C/C</td>
<td>1.8</td>
<td>90</td>
<td>0.33</td>
</tr>
</tbody>
</table>

![Schematic diagram of meshing results](image)
(3) Because the free mode of the wheel assembly is not analyzed in this chapter, and the mode of the wheel after loading is considered, the prestress should be set. The radial lateral load on the wheel is simplified and applied to the rim of the hub, the equivalent pressure of the tire inflation pressure is applied to the outer surface of the hub, and the pressure of the piston house to the carbon disc under the braking condition of the airplane is applied to the surface of the pressure disc.

(4) Setting of modal analysis. The first 30 order modal results are extracted for analysis.

4. Analysis of prestress modal results

4.1 Mode shape analysis of Prestress mode

The vibration of macroscopic object is the result of linear superposition of all vibration modes, the main factor that determines the vibration characteristics of the wheel is its low-order vibration mode, and the influence of high-order frequency on its vibration is very small, the modal analysis results are shown in Table 2:

<table>
<thead>
<tr>
<th>Order</th>
<th>Frequency</th>
<th>Vibration form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>308</td>
<td>Bending vibration of wheel hub in XY plane</td>
</tr>
<tr>
<td>2</td>
<td>308</td>
<td>Bending vibration of wheel hub in XY plane</td>
</tr>
<tr>
<td>3</td>
<td>459</td>
<td>The axial tension vibration of the wheel hub along the Z axis</td>
</tr>
<tr>
<td>4</td>
<td>577</td>
<td>Torsional vibration of wheel hub around z axis</td>
</tr>
<tr>
<td>5</td>
<td>886</td>
<td>Stretching vibration of the wheel hub in the XY plane</td>
</tr>
<tr>
<td>6</td>
<td>887</td>
<td>Stretching vibration of the wheel hub in the XY plane</td>
</tr>
<tr>
<td>7</td>
<td>1008</td>
<td>Stretching and bending vibration of wheel hub in XY plane</td>
</tr>
<tr>
<td>8</td>
<td>1008</td>
<td>Stretching and bending vibration of wheel hub in XY plane</td>
</tr>
<tr>
<td>9</td>
<td>1275</td>
<td>The axial tension vibration of the wheel hub along the Z axis</td>
</tr>
<tr>
<td>10</td>
<td>1607</td>
<td>The stretching and bending vibration (3-angle vibration) of the wheel hub in the XY plane</td>
</tr>
<tr>
<td>11</td>
<td>1614</td>
<td>The stretching and bending vibration (3-angle vibration) of the wheel hub in the XY plane</td>
</tr>
<tr>
<td>12</td>
<td>1640</td>
<td>The axial tension vibration of the wheel hub along the Z axis</td>
</tr>
</tbody>
</table>

Table 2 shows that the results of the first twelve modes under prestress almost cover all the macroscopic vibration forms of the wheel, the typical vibration problems, such as whirl, chatter and squeal, are caused by the stretching, bending and torsional vibration of the wheel structure.

It can be seen from Figure 3-4 that the bending of the wheel along the XY plane will make the hub asymmetrical to some extent, which will affect the balance of forces and the smooth rotation of the hub structure. on the other hand, it will cause the brake device, especially the carbon disc to bend, and affect the friction contact area between the dynamic disc and the static disc, hydraulic piston appears “Strong can not be used” or “Excessive force” phenomenon.

![Figure. 4 First and second mode diagram](image)

It can be seen that the axial stretching of the wheel along the Z axis will cause the brake to move perpendicular to the surface of the carbon disc from figure.5. It will cause the brake to vibrate and impact
the parts around it, on the other hand, it may affect the normal work of the anti-skid braking system, resulting in the mismatch between the piston pressure and the pressure received by the carbon disc structure, leading to unstable braking efficiency. The structure of the wheel hub is that the movable disc is fixed by the clamping groove and the wheel hub guide rail, and the stationary disc and the torque tube are fixed, and then the brake is realized by the friction of the stationary disc. Fig. 3-6. It can be seen that the torsional vibration of the wheel hub around the z axis will cause additional torque between the brake device and the wheel hub, which will affect the performance of the wheel hub structure. On the other hand, the torsional vibration will change the relative rotation speed of the dynamic and static disks, this can affect the brake pressure to a certain extent and lead to unstable braking efficiency.

Figure 5 Third mode diagram

Figure 7 shows that when the wheel is stretched in the XY plane, the collision between the side of the disk, the wheel hub and the torque tube will result in huge stress and deformation, which will destroy all parts of the wheel, especially the carbon disk structure, as a result, the carbon disc wear is too large and the brake failure is easy to occur.

Figure 6 Fourth mode diagram

Figure 7: Fifth mode diagram
As can be seen from figure 8 and figure 9, the stretching and bending vibration of the wheel hub in the XY plane will cause a large stress concentration and deformation in the hub structure, accelerate the generation and propagation of the cracks in the hub structure, and greatly reduce the service life of the hub, at the same time, it will cause the vibration of the vertical ground in the process of airplane braking and taxiing, affect the passenger’s travel experience and even cause the airplane skidding off the runway.

4.2 Participation Factor and effective quality

Participation Factor: the participation factor is calculated using the following formula.

\[
\gamma_i = [\phi_i^T [M] [D] \phi_i]^{1/2} \tag{7}
\]

In this equation, \(\{D\}\) is the mass of the Cartesian coordinate system in the direction of translation and rotation, the larger the value, the more the mode will be excited by the force in the corresponding direction.

The effective mass is numerically equal to the square of the participation factor. Ideally, the effective mass in each direction is close to or equal to the total mass of the structure, and the ratio of the effective mass to the total mass can be used to check whether a sufficient number of modes have been proposed.

The Solution information in the Workbench module provides the values of the engagement factors and the effective quality. The engagement factors and the effective quality along the X axis for each mode can be calculated. The participation factor of the fifth-order mode is 0.17358, and that of the sixth-order mode is 0.13843, which is obviously larger than that of the other modes, at the same time, the ratio of effective mass to total mass is 0.808041, which is more than 80%, indicating that enough modes have been proposed.

Figure 9 is the ratio of the effective mass to the total mass along the x, y and z axes and the horizontal coordinate is the direction of motion, the longitudinal coordinates are the ratio of the effective mass to the total mass. It can be seen that the ratio of each direction is above 0.8, which indicates that the first twelve modes extracted by the modal analysis in this paper are enough and the modal analysis is effective and reliable.

5. Conclusion

The same structure will exhibit different vibration characteristics under different stress states. By establishing the prestressed mode finite element model of the integrated model of hub structure and brake device, the prestressed modal analysis of the wheel and brake device is carried out, and the first 30 modes of the model are obtained, and the frequency and modes of the first 12 modes are emphatically analyzed, several typical modal vibration modes are analyzed and interpreted according to the actual working conditions of the aircraft, there are three forms of vibration in the working process of the hub: the torsional vibration perpendicular to the surface of the carbon disc, the tensile vibration in the plane of the carbon disc and the flexural vibration in the plane of the carbon disc, the results can be used for reference in the dynamic design of aircraft wheel hub, i. e. structural optimization. At the same time, the ratio of the effective mass to the participation factor is introduced to judge whether there are enough modes to be extracted. The ratio of the effective mass to the total mass of the extracted modes is more than 80%,
and the number of extracted modes is enough, the results of the analysis are reliable.

![Image: Participation factor and effective mass in all direction](image_url)

**Figure 10:** Participation factor and effective mass in all direction

### References


