

Comparative Analysis of the New and Old High School Mathematics Textbooks—Take Version a "Set" as an Example

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Abstract: According to Mathematics Curriculum Standards of Regular High School (2017 edition), People's Education Press published the 2019 Regular High School Mathematics textbook version A (hereinafter referred to as the new textbook). The content of the textbook has been adjusted. The knowledge of set is the foundation of modern mathematics, and also the foundation of high school mathematics. This article will take the set part of People's Education Edition Version A as an example, using literature method, content analysis method, comparison method from the the aspects of textbook system structure, chapter introduction, text content, the summary of this chapter and so on, to analyze and compare the new textbook and the 2004 edition of ordinary high school curriculum standard experimental textbook mathematics A (hereinafter referred to as the old textbook). Therefore, it helps the textbook users to understand the changes and characteristics of the new textbook, and promote the common development of teaching and learning.

Keywords: high school mathematics textbook; set; comparative study

1. Introduction

In high school mathematics textbooks, set is the language and tool to describe things. Learning this part of knowledge well can help students describe mathematical research objects with concise and accurate language, and cultivate students' mathematical abstract ability. Textbooks are the carrier of curriculum implementation, the main basis for teachers to teach, and the main source for students to absorb knowledge.

The 2019 edition of high school mathematics textbooks was compiled according to the Mathematics Curriculum Standards of Regular High Schools (2017 edition), with some adjustments compared with the old textbooks. For front-line teachers, it is crucial to understand the similarities and differences between old and new materials and adjust teaching strategies according to the changes of the new textbook. Based on this, this paper will take the old and new textbooks as the research object to compare and analyze the knowledge of set, so as to provide reference for the textbook users to understand the textbook and teacher to teach[1-2].

2. Comparison of the Arrangement of Old and New Textbook Systems

Table 1: Comparison of the overall framework of the old and new textbooks

The Old Textbook	The New Textbook
Chapter 1: Set and Concept of Function	Chapter 1: Set and Common Logical Terms
1.1 Set	1.1 The Concept of Sets
	1.2 Basic Relationships Between Sets
	1.3 Basic Operations of Sets
Reading and Thinking the Number of Elements in The Set	Reading and Thinking the Number of Elements in The Set

This section of "set" is required in the first chapter of the new textbook, and it is also the first chapter in the old textbook, reflecting the status of the knowledge of set in high school mathematics. Set is the foundation of modern mathematics, but also the foundation of high school mathematics,

laying the foundation for the study of the later chapters.

Comparing the old and new textbooks, it can be found that the knowledge points of the two editions of textbook have not been increased or decreased, which occupies the same space in the textbooks. However, the old textbook puts "Set" and "Concept of Function" in the first chapter, while the new textbook puts "Set" and "Common Logical Terms" in the first chapter. This adjustment is to put the knowledge of junior high school as preparatory knowledge in the first chapter of the textbook, to help students to complete the transition of mathematics knowledge.

As can be seen from Table 1, the old textbook puts "set" in a section, and in the catalogue as "1.1 Set". In the new textbook directory, the content of "set" is subdivided into three sections: "1.1 Concept of Set", "1.2 Basic Relationships Between Sets", "1.3 Basic Operations of Sets", which can make it clear to textbook users quickly when they open the textbook and view the directory. Bruner stressed: "No matter what subject we study, we must make our students understand the basic structure of the subject." The so-called "basic structure of the discipline" refers to the basic concept, principles and its correlation between the discipline. This is also applicable to learning a certain knowledge point. Presenting the basic structure framework of knowledge in the catalogue is helpful for the textbook users to understand the integrity of the knowledge point and the connection between them, and also conducive for students to effectively construct the knowledge structure map when learning the content of the chapter, so as to cultivate students' rational thinking.

3. Comparison of Chapter Import

Before each chapter of the old and new textbooks, there is a situation map and a concise text, called the chapter map and the chapter introduction. As the beginning of each chapter of knowledge, it has the role of mobilizing students' learning enthusiasm, stimulating students' interest in learning, and revealing the content of this chapter.

In the chapter import of the set part in the new textbook, it puts "set" and "common logical terms" in a chapter, the chapter import has a detailed introduction of the role of set, such as clearing the research object, determining the research scope. In order to concisely and accurately express mathematical objects and the scope of research, it is necessary to use the language of set, which reflects the importance of learning set knowledge. In the old textbook, set and function are put in a chapter to learn, and in the chapter import, there is also a short introduction to set, but mainly introduces the relationship between set and function, describing the concept of function in set language to lay the foundation for the later learning. Compared with the old and new textbooks, the chapter of the new textbook pays more attention to the connection between mathematical knowledge and other disciplines and life, permeates more mathematical ideas, and has a wider range of applications.

4. Comparison of Body Matter

4.1 Comparison of Textbook Column

The content of the column is an important part of the teaching material. Its design and selection not only have a certain auxiliary role in teachers' teaching, so that the teaching content is accurately positioned, but also can leave more space for students to learn actively, promote students to learn to learn and improve their ability of independent learning.

Compared with the column contents of the old and new textbooks, it can be seen from Table 1 that both textbooks have "thinking" in the form of questions, which reflects the important position of mathematical thinking in mathematics learning. The solution of problems in teaching is supported by basic knowledge and basic skills, and the learning of knowledge and skills is inseparable from mathematical thinking. However, there are differences in the design of thinking problems. For example, the question in the first "thinking" column of the new textbook is "Can the above example (3) to the example (6) also form a set? What are their respective elements?" Compared with the old textbook, delete the first "thinking" column in the old textbook, "Summarize these examples, can you tell their common characteristics?", The problems designed in the new textbook are more in line with the development of students' cognitive structure. This "thinking" column is set as the first column at the beginning of the textbook. The real concept has not yet appeared, students have no deep understanding of the collective knowledge, and the cognitive structure has just begun to build. In this case, "tell the common characteristics of examples" these examples include both life and mathematical professional

knowledge, involving a wide range of fields, so this problem is somewhat abstract, and it is not easy for students to find the entry point when thinking about this problem, as shown in Table 2.

Table 2: Comparison of the old and new Textbook

The Old Textbook	The New Textbook
<p>1. The meaning and representation of sets</p> <p>Thinking: Can the above example (3) to the example (8) also form a set? What are their respective elements? Summarize these examples, can you share their common characteristics?</p> <p>Thinking: Judge whether the whole body of the following elements form a set, and explain the reasons: (1) the even number greater than 3 and less than 11; (2) China's small rivers.</p> <p>Thinking: (1) Can you describe the set $\{2,4,6,8\}$ in natural language?(2) Can you use the enumeration method to express the solution set of the inequality $x-7<3$?</p> <p>Thinking: (1) Combined with the above examples, try to compare the characteristics and objects used in natural language, enumeration method and description method. (2) Give several set examples and express them in natural language, enumeration and description respectively.</p>	<p>1. The concept of sets</p> <p>Thinking: Can the above example (3) to the example (6) also form a set? What are their respective elements?</p> <p>Thinking: (1) Can you describe the set $\{0,3,6,9\}$ in natural language? (2) Can you use the enumeration method to express the solution set of the inequality $x-7<3$?</p> <p>Thinking: For example, use natural language, enumeration and description to express the characteristics of the set.</p>
<p>2. The basic relationship between sets</p> <p>Thinking: The real numbers have equal relations, size relations, such as $5=5, 5 < 7, 5 > 3$, and so on. By analogy to the relationship between the real numbers, what do you think of the relationship between sets?</p> <p>Thinking: What is the difference between inclusive relationship and belonging relationship? Try to explain it with some examples.</p>	<p>2.The basic relationship between sets</p> <p>Observation: Looking at the following examples, the equivalence relationship between real numbers, can you find the relationship between the following two sets?</p> <p>Thinking: What is the difference between inclusive relationship and belonging relationship? Try to explain it with some examples.</p>
<p>3. The basic operation of sets</p> <p>Thinking: We know that the real numbers have addition operations, by analogy to the addition operations of the real numbers, whether the set can also be "added"?</p> <p>Thinking: Finding the union of sets is a kind of operation of sets, are there any other operations between sets?</p>	<p>3.The basic operation of sets</p> <p>Observation: Observe the following sets, by analogy to the addition operation of real numbers, can you tell the relationship between sets C and A, B?</p> <p>Thinking: Is the following relationship valid?(1)$A \cup A = A$;</p> <p>Thinking: Observe the following sets. What is the relationship between sets A, B and set C?</p>

At the same time, the new textbook has added a column called "observation", which in mathematics is not limited to simple perceptual activities, but a richer thinking activity. For example, in the "observation" column in the section of "1.2 Basic Relations between Sets" in the new textbook, "Observe the following examples, and by analogy with the equality and size relationship between the real numbers, can you find the relationship between the following two sets?". First, observation, then, analogy and discovery, which is not only to impart knowledge to students, but also to guide students to learn to learn, to help students develop good math learning habits, and cultivate students' mathematical ideas. The textbook reserves more space for students to explore through "observation", and provides a better foothold for the abstraction of mathematical learning. It expects students to find in observation and develop in discovery, so as to cultivate students' habits of being good at observation and diligent in thinking, and cultivate students' core quality of mathematical abstraction[3-4].

4.2 Comparison of Definition

Comparing the old and new textbooks, there are differences in the description of some concepts. The new textbook illustrates that "the elements of a given set must be certain" that "all even numbers

between 1 and 10 constitute a set, 2,4,6,8,10 are the elements of this set.....", using mathematically related examples. In the same knowledge point of the old textbook, the example is that " 'The Capital of Asian countries' constitutes a set, Beijing, Tokyo, New Delhi..... in this set ", using the relevant examples of geographical knowledge, skillfully combining mathematics with the knowledge of other disciplines, fully reflecting the wide applicability of mathematical thinking methods.

There are also differences in the definition of the description method. In the old textbook, the description method is to represent the set with the common features of the elements contained in the set. In the new textbook, the definition of descriptive method is such, in general, let A be a set, we express the set composed of all elements x with common characteristics P (x) in set A as $\{x \in A \mid P(x)\}$. This method of representing set is called descriptive method. The description of the old textbook is all literal language, while the description of the new textbook on the definition of descriptive method is the combination of symbolic language and literal language. In this way, students can cultivate mathematical core literacy, but also contribute to the realization of the new curriculum standards.

4.3 Comparison of math culture integration

The Mathematics Curriculum Standard for Ordinary High Schools (2017 edition) specifically states that mathematics culture refers to the thought, spirits, language, methods, views of mathematics, and their formation and development; it also includes the contribution and significance of mathematics in human life, science and technology, social development, as well as humanistic activities related to mathematics. It is mentioned to pay attention to the infiltration of mathematical culture. The old and new textbooks integrate mathematical culture, application and spirit through examples, exercises and other contents. The new textbook follows the old textbook by setting up "reading and thinking" column to show the application of set in life. For example, "school store" and "school track and field meeting" two life problems, using the basic algorithm of set to solve, reflecting the wide application of set in life. Emphasize the connection between mathematics and life, and improve students' ability to apply mathematics to solve practical problems.

In the section, exercise 1.1 of "The Concept of Sets", the fifth question of the new textbook briefly describes the source of set theory. "Set theory was founded by the German mathematician Cantor at the end of the 19th century", leading to Hilbert and Russell's evaluation of set theory. The history of mathematics is integrated into the textbook, so that students can understand the generation and development process of sets, the main figures and their contributions to human beings, so as to penetrate the mathematical culture. As mentioned in the new curriculum standard, students should be guided to understand the development process of mathematics, understand the role of the mathematics in science and technology, feel the value of mathematics, and enhance students' scientific spirit, application consciousness and humanistic accomplishment. Integrating mathematics culture into teaching is conducive to stimulating students' interest in learning, further understanding mathematics, broadening their horizons, and improving students' core quality of mathematics.

4.4 Comparison of sample questions and exercises

Table 3: Comparison of the number of sample questions and exercises

Textbook version	Sample questions		practice		exercises	
	New Textbook	Old Textbook	New Textbook	Old Textbook	New Textbook	Old Textbook
The concept of set	2	2	3	2	5	16
The basic relationship between the sets	2	1	3	3	5	
The basic operations of the sets	6	6	7	5	6	
grand total	10	9	13	10	16	16

The old and new two editions of the teaching material are set up with sample questions, practice, exercises, which are helpful to consolidate, improve and expand the content of the teaching material. Therefore, the sample questions, practice and exercises of the two textbooks are compared and analyzed to grasp the similarities and differences of the teaching materials.

As can be seen from Table 3, the number of sample questions and practice in the old and new versions of the textbooks has increased. Appropriately increasing the number of questions can help

students consolidate and master knowledge. The number of exercises has not changed, but the arrangement form of the exercises has changed. The old textbook divided the review questions into groups A and B according to the difficulty of the questions. Formally, it focuses on identifying the strength of students' math learning ability through exercises of different difficulty. To a certain extent, this will cause psychological burden to students and affect students' enthusiasm for learning. The new teaching material divides the exercises into three modules: "review and consolidation", "comprehensive application" and "expansion and exploration". The difficulty of each module is from shallow to deep, layer by layer. This division makes the exercises more distinct, more suitable for students who have more skills. In this way, the basic concept of the new curriculum standards is reflected, based on the development of students, and everyone can get good mathematics education, and different people can get different development in mathematics[5-6].

Compared with the old textbook, the new textbook reflects the emphasis on the problem solving process and the investigation of the search for the basis for solving the problem in the problem setting, and the key words such as "explain the reasons" appear in the exercises. For example, in the exercise 1 of the "Concept of Set" section of the new textbook, "judge whether all of the following elements form a set and explain the reasons", it is important to guide students to find evidence to explain in the process of problem solving. This practice improves the shortage of common closed questions in the old textbooks, reflects the investigation of students' mastery of the new knowledge points, and attaches great importance to the process of students seeking evidence to support the conclusion.

5. Comparison of summaries

5.1 Comparison of knowledge structure

The knowledge structure of the old and new textbook is presented in the form of structure block diagram, mind map is used to connect the each knowledge point of set, and clearly show the connection between them. The difference between the two versions is that the mind map of the old version is relatively simple. There are three branches at the next level of set, namely "meaning", "basic relationship between sets" and "operation of sets", while these three branches have not expanded at the next level. However, in the new textbook, there are three branches under set, namely "meaning of set", "basic relationship between sets", "operation of set". Two branches "contain" and "equal" under "basic relationships between sets", and under "operation of set", there are three branches "intersection", "union" and "complement". The content of the knowledge structure integration of the new teaching material is more detailed and specific, making the presentation of knowledge points more three-dimensional.

5.2 Comparison of review and thinking

The new textbook has been greatly adjusted in the part of review and thinking. The new textbook first summarizes the knowledge points learned in the part of set, and then reviews the knowledge of set again with the research path of "meaning and representation of set—relationship between sets—operation of sets". When reviewing the operation of sets, it is mentioned that the operation in mathematics is not limited to the operation of numbers, which is very meaningful to improve our mathematical operation quality. It is proposed that students should pay attention to the improvement of mathematical operation ability.

In the review part, the old textbook mainly talks about the function and significance of the set language, and directly raises questions without summarizing and integrating the knowledge of the set part.

Both the new and old textbooks review the learning content of this chapter in the form of question string, which drive students recall and review the whole chapter core knowledge, but the problems in the new teaching material not only include "illustration by example", "thinking by analogy", and "talk about experience", "what's the meaning". By putting forward specific problems with thinking strength on key points, and difficult points, students can deepen their understanding of the core content of this chapter and the mathematical thoughts and methods, so that students can have understanding on the basis of thinking.

In short, the new textbook more fully reflects the basic concept and target requirements of the new curriculum standards, and thoroughly implement the educational concept of "cultivating people by

virtue". Compared with the old textbook, the structure, content, exercises, summary and other aspects have been greatly improved. We should pay attention to the changes, grasp the new textbook on the basis of inheriting the old one, and naturally complete the transition from the old textbook to the new textbook.

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