ARIMA time series based logistics route cargo volume forecasting research

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Abstract: In a commercial logistics network, logistics sites and logistics routes are the key links that make up the logistics transportation process. Therefore, accurate prediction of cargo volume of each logistics site and route is essential to improve logistics operation efficiency, reduce costs, and ensure smooth logistics transportation. In order to predict the cargo volume of logistics routes, the historical cargo volume data of the three routes to be predicted are firstly compiled, and the data are analyzed by using ARIMA time series due to the time-series nature of the data. Since the data are smoothed in the second-order difference, the optimal parameter values are calculated after the second-order difference, and the ARIMA(1,2,3) time series prediction model is established to predict the cargo volume data of the three routes from 2023-1-1 to 2023-1-31.

Keywords: E-commerce logistics; Cargo volume; ARIMA time series

1. Introduction

In the context of today's increasingly dynamic development of e-commerce, logistics network has become one of the key supports for the fast and efficient conduct of e-commerce business and user experience enhancement. In the e-commerce logistics network, logistics sites and logistics routes are the key links that constitute the logistics transportation process[1-2]. Therefore, accurate prediction of cargo volume of each logistics site and route is crucial to improve logistics operation efficiency, reduce costs and ensure smooth logistics transportation[3-4].

However, the e-commerce industry involves many event factors, such as the "Double 11", "618" and other promotional activities brought about by the peak of user orders, special periods of emergencies such as epidemics, natural disasters, etc., will have a direct or indirect impact on the e-commerce logistics network, which will lead to the cargo[5] indirectly on the e-commerce logistics network, resulting in fluctuations in cargo volume or temporary or permanent shutdown of logistics sites.

If we can accurately predict the cargo volume of each logistics site and logistics route, managers can arrange the corresponding transportation and sorting plans in advance, thus maximizing the reduction of logistics operation costs and improving operational efficiency. Therefore, this paper uses historical data to build a prediction model to forecast the daily cargo volume of each route in January 2023.

2. ARIMA modeling

2.1 Preparation of the model

First, the data in the logistics line will be analyzed and processed, and after observing the data, it is found that there are no missing values in the data, and since the data is the cargo data of different lines, then no more outlier analysis will be conducted for its data, and it is considered that all the data can be analyzed, and these data are time-series data, so it is considered to conduct time-series analysis[6-7], and firstly, the historical data graph is drawn and fitting analysis, here the data of this line DC14→DC10 is taken as an example, and the plot is shown in the following 1. The source of the data is Question C of the 13th MathorCup Collegiate Mathematical Modeling Challenge 2023.
From Figure 1, it can be seen that this section of data exists before and after the fluctuation of the change is large, which may be because this line in the early use of the process did not assume a larger cargo capacity, with the expansion of the company's scale and business, the cargo capacity on this line also increases, here we first polynomial fit the data, from the results of the fit, the overall change in the data over time shows an upward trend, here consider the use of ARIMA time series model to predict the analysis of its data.

2.2 Model building

2.2.1 Smoothed time series model

Smooth here means wide smooth, and its characteristic is that the statistical properties of the series do not change with time advection, i.e., the mean and covariance do not change with time advection. Given a random process \( \{X_t, t \in T\} \). \( X^t \) is a random variable. Let its mean value be \( \mu_t \). The variance is noted as

\[
\sigma^2_t = \text{Var}(X_t) = E\left[ (X_t - \mu_t)^2 \right]
\]

Equation (1) is called the variance function of the stochastic process. The square root of the variance function is called the standard deviation function of the stochastic process, which indicates the deviation of the stochastic process from the mean value function.

For stochastic processes \( \{X_t, t \in T\} \), determine \( t, s \in T \), define its autocovariance function as

\[
\gamma_{t,s} = \text{Cov}(X_t, X_s) = E\left[ (X_t - \mu_t)(X_s - \mu_s) \right]
\]

In order to characterize the correlation between the moment \( t \) and the moment \( s \), it is also possible to standardize, \( \gamma_{t,s} \), define the autocorrelation function

\[
\rho_{t,s} = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}} \sqrt{\gamma_{s,s}}} = \frac{\gamma_{t,s}}{\sigma_t \sigma_s}
\]

Therefore, the autocorrelation function is a normalized autocovariance function [8].
Let the random sequence \( \{ X_t, t = 0, \pm 1, \pm 2, \cdots \} \) satisfy 1.

Let the autocovariance function of the smooth series \( \{ \varepsilon_t, t = 0, \pm 1, \pm 2, \cdots \} \) be

\[
\gamma_k = \sigma^2 \delta_{k,0} = \begin{cases} 0, & k \neq 0, \\ \sigma^2, & k = 0, \end{cases}
\]

in equation (4)

\[
\delta_{k,0} = \begin{cases} 1, & k \neq 0, \\ 0, & k = 0, \end{cases}
\]

Then the series is said to be a smooth white noise series.

For a smooth series of samples, the sample mean can be used to estimate the mean of a random series

\[
\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} X_i = \bar{X}
\]

(6)

Let \( \{ \varepsilon_t, t = 0, \pm 1, \pm 2, \cdots \} \) be a zero-mean smooth white noise, \( \text{Var}(\varepsilon_t) = \sigma^2 \) if \( \{ G_k, k = 0, 1, 2, \cdots \} \) is a series satisfying

\[
\sum_{k=0}^{\infty} |G_k| < +\infty, G_0 = 1
\]

(7)

Define random sequence

\[
X_t = \sum_{k=0}^{\infty} G_k \varepsilon_{t-k},
\]

(8)

\( X_t \) is called a stochastic linear series. Under the condition, it is proved that \( X_t \) is a smooth series.

If a zero-mean smooth series \( X_t \) can be expressed in the form , then this form is called the transfer form and \( \{ G_k, k = 0, 1, 2, \cdots \} \) is called the Green function.

Let \( \{ X_t, t = 0, \pm 1, \pm 2, \cdots \} \) be a zero-mean smooth series, and introduce the definition of partial correlation function from the perspective of time series forecasting. If the value of \( \{ X_{t-1}, X_{t-2}, \cdots, X_{t-k} \} \) is known, a forecast is required. In this case, the linear least mean square estimate of the pair \( \{ X_{t-1}, X_{t-2}, \cdots, X_{t-k} \} \) can be considered, i.e., the coefficient of choice \( \varphi_{1,1}, \varphi_{1,2}, \cdots, \varphi_{k,k} \), make

\[
\min \delta = E \left[ \left( X_t - \sum_{j=1}^{k} \varphi_{k,j} X_{t-j} \right)^2 \right].
\]

(9)

Expand \( \delta \), get

\[
\delta = \gamma_0 - 2 \sum_{j=1}^{k} \varphi_{k,j} \gamma_j + \sum_{j=1}^{k} \sum_{i=1}^{k} \varphi_{k,j} \varphi_{k,j} \gamma_{j-i}
\]

(10)

Let \( \frac{\partial \delta}{\partial \varphi_{k,j}} = 0, j = 1, 2, \cdots, k \)
\[-\gamma_j + \sum_{i=1}^{k} \phi_{k,j} \gamma_{j-i} = 0, j = 1, 2, \ldots, k, \]

(11)

Dividing \( \gamma_j \) both ends equally and writing in matrix form

\[
\begin{bmatrix}
1 & \rho_1 & \cdots & \rho_{k-1} \\
\rho_1 & 1 & \cdots & \rho_{k-2} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{k-1} & \rho_{k-2} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\phi_{k,1} \\
\phi_{k,2} \\
\vdots \\
\phi_{k,k}
\end{bmatrix}
= \begin{bmatrix}
\rho_1 \\
\rho_2 \\
\vdots \\
\rho_k
\end{bmatrix}
\]

(12)

Calling the Yule-Walker equation. \( \{ \phi_{k,j}, k = 1, 2, \cdots \} \) is called the partial correlation function of the

If let \( \{ X_t, t = 0, \pm 1, \pm 2, \cdots \} \) be a zero-mean smooth series satisfying the following model:

\[X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \epsilon_t\]

(13)

where: \( \epsilon_t \) is a smooth white noise with zero mean and variance; \( X_t \) is an autoregressive series of order \( p \), abbreviated as AR(\( p \)) series; \( \phi \) is an autoregressive parameter vector, and

\[\phi = [\phi_1, \phi_2, \cdots, \phi_p]^T,\]

(14)

The component is called the autoregressive coefficient.

Introducing the backward shift operator, the operator \( B \) is defined as follows:

\[BX_t = X_{t-1}, B^k X_t = X_{t-k}^c\]

(15)

Notation of arithmetic polynomials

\[\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p,\]

(16)

Then equation (8) can be rewritten as

\[\phi(B) X_t = \epsilon_t\]

(17)

Let \( \{ X_t, t = 0, \pm 1, \pm 2, \cdots \} \) be a zero-mean smooth series that satisfies the following model:

\[X_t = \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q},\]

(18)

where: \( \epsilon_t \) is a smooth white noise with zero mean; \( X_t \) is a moving average series of order \( q \), abbreviated as MA(\( q \)); \( \theta \) is the moving average parameter vector and

\[\theta = [\theta_1, \theta_2, \cdots, \theta_q]^T,\]

(19)

The component \( \theta_j, j = 1, 2, \cdots, q \) is called the moving average coefficient.

In engineering, a smooth white noise generator passes through a linear system, and if its output is a linear superposition of white noise, then this output obeys the MA model.

For the linear backward shift operator there is

\[B \epsilon_t = \epsilon_{t-1}, B^k \epsilon_t = \epsilon_{t-k},\]

(20)

Re-introduce the arithmetic polynomial
Then equation (11) can be rewritten as
\[ X_t = \theta(B)\varepsilon_t \] (22)

Let \( \{X_t, t = 0, \pm 1, \pm 2, \cdots\} \) be a zero-mean smooth series that satisfies the following model:
\[
X_t - \varphi_1 X_{t-1} - \cdots - \varphi_p X_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q},
\] (23)

It is abbreviated as ARIMA \((p,q)\) sequence, which is AR(p) sequence when \(q=0\) and MA(q) sequence when \(p=0\).

Applying the operator polynomial \(\varphi(B), \theta(B)\), Eq. (13) can be written as
\[
\varphi(B)X_t = \theta(B)\varepsilon_t
\] (24)

For a general smooth series \(\{X_t, t = 0, \pm 1, \pm 2, \cdots\}\). Let its mean value satisfy the following model:
\[
(X_t - \mu) - \varphi_1 (X_{t-1} - \mu) - \cdots - \varphi_p (X_{t-p} - \mu) = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}
\] (25)

Using the back-shift operator \(\varphi(B), \theta(B)\), Eq. (15) can be expressed as
\[
\varphi(B)(X_t - \mu) = \theta(B)\varepsilon_t,
\] (26)

The following additional assumptions are usually made about operator polynomials \(\varphi(B), \theta(B)\):

1. \(\varphi(B)\) and \(\theta(B)\) No public factor \(\varphi_p \neq 0, \theta_q \neq 0\).

2. The condition that the roots of \(\varphi(B) = 0\) are all outside the unit circle is called the smoothness condition of the model.

3. The condition that all roots of \(\theta(B) = 0\) are outside the unit circle is called the reversibility condition of the model.

Construction and forecasting of ARIMA model

In practical problem modeling, the first step is to identify and order the model, both to determine the class of AR(p), MA(q) or ARIMA(p,q) models and to estimate the order \(p,q\). In fact, it all boils down to the problem of ordering the model. Once the model is ordered, the model parameters \(\varphi = [\varphi_1, \varphi_2, \cdots, \varphi_p]^T\) and \(\theta = [\theta_1, \theta_2, \cdots, \theta_q]^T\) have to be estimated.

The evaluation is performed. After the order fixing and parameter estimation are completed, the model is also tested, both to see if \(\varepsilon_t\) is a smooth white noise. If the test is passed, the modeling of the ARIMA time series is completed.

3. Model solving

3.1 Solving the prediction model of ARIMA time series

AIC criterion, also known as Akaike information criterion, was proposed by Japanese statistician Akaike in 1974. AIC criterion is an important research result of information theory and statistics, which is of great significance.

The ARIMA \((p,q)\) sequence AIC fixed-order quasi-measure is: choose \(p,q\) such that
\[
\min \quad AIC = n \ln \hat{\sigma}_e^2 + 2(p + q + 1),
\] (27)
When the ARMA \((p,q)\) sequence contains an unknown mean parameter \(\mu\) the model is

\[ \phi(B)(X_i - \mu) = \theta(B)\varepsilon_i, \quad (28) \]

At this point, the number of unknown parameters is \(k = p + q + 2\), The AIC criterion holds that: \(p,q\) is chosen such that

\[ \min \ AIC = n \ln \hat{\sigma}_e^2 + 2(p + q + 2), \quad (29) \]

In fact, when having the same minima \(\hat{\rho}, \hat{\phi}\), for the parameter estimates of the ARMA model if the residual of the fitted model is denoted as \(\hat{\varepsilon}_t\), it is an estimate of \(\varepsilon_t\). For example, for the AR\((p)\) series, let the estimate of the unknown parameter be \(\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_p\), then the residual

\[ \hat{\varepsilon}_t = X_t - \hat{\phi}_1X_{t-1} - \cdots - \hat{\phi}_pX_{t-p}, \quad t = 1, 2, \ldots, n \text{ (let } X_0 = X_{-1} = \cdots = X_{-p} = 0). \quad (30) \]

\[ \eta_k = \frac{\sum_{i=1}^{n-k} \hat{\varepsilon}_i \hat{\varepsilon}_{i+k}}{\sum_{i=1}^{n} \hat{\varepsilon}_i^2}, \quad k = 1, 2, \ldots, L, \quad (31) \]

where: L is the number of trailing tails of the autocorrelation function of \(\hat{\varepsilon}_t\). The \(\chi^2\)-test statistic of the Ljung-Box is

\[ \chi^2 = n(n + 2) \sum_{k=1}^{L} \frac{\eta_k^2}{n - k} \quad (32) \]

The hypotheses tested were

\[ H_0 : \rho_k = 0, \text{ when } k \leq L; H_1 : \text{ for } k \leq L, \rho_k \neq 0. \]

When \(H_0\) holds, if the sample size \(n\) is sufficiently large, \(\chi^2\) approximates the \(\chi^2(L - r)\) distribution, where \(r\) is the number of estimated model parameters.

\(\chi^2\) test: given the significance level \(\alpha\), check the table to obtain the upper \(\alpha\) quantile \(\chi^2_{\alpha}(L - r)\) then when \(\chi^2 > \chi^2_{\alpha}(L)\) rejects \(H_0\) that \(\varepsilon_t\) is not white noise, the model test fails: and when \(\chi^2 < \chi^2_{\alpha}(L - r)\), accepts \(H_0\) that \(\varepsilon_t\) is white noise, the model passes the test[9-10].

The m-step forecast of a time series is an estimate of the random variable \(X_{k+m}(m > 0)\) at the future \(k + m\) moments based on the values taken by \(\{X_k, X_{k-1}, \ldots\}\). The estimate is denoted as \(\hat{X}_k(m)\). The estimated quantity is denoted as \(c\). It is a linear combination of \(X_k, X_{k-1}, \ldots\). The forecast of the AR\((p)\) series, the recursive formula for the forecast of the AR\((p)\) series is

\[
\begin{align*}
\hat{X}_k (1) &= \varphi_1 X_k + \varphi_2 X_{k-1} + \cdots + \varphi_p X_{k-p+1}, \\
\hat{X}_k (2) &= \varphi_1 \hat{X}_k (1) + \varphi_2 X_k + \cdots + \varphi_p X_{k-p+2}, \\
\vdots \\
\hat{X}_k (p) &= \varphi_1 \hat{X}_k (p-1) + \varphi_2 \hat{X}_k (p-2) + \cdots + \varphi_{p-1} \hat{X}_k (1) + \varphi_p X_k, \\
\hat{X}_k (m) &= \varphi_1 \hat{X}_k (m-1) + \varphi_2 \hat{X}_k (m-2) + \cdots + \varphi_p \hat{X}_k (m-p), \quad m > p, \quad (33)
\end{align*}
\]
It follows that \( \hat{X}_k(m) \) depends only on the value of \( X_t \) for the \( k \) moments before the \( k \) moments \( X_k, X_{k-1}, \ldots, X_{k-p+1} \).

This is a characteristic of AR(p) sequence forecasts. For the forecast of the MA(q) sequence \( \{X_t, t = 0, \pm 1, \pm 2, \cdots\} \), we have

\[
\hat{X}_k(m) = 0, m > q.
\] (34)

Therefore, it is only necessary to discuss \( \hat{X}_k(m), m = 1, 2, \cdots, q \). For this purpose, define the forecast vector

\[
\hat{X}_k^{(q)} = \left[ \hat{X}_k(1), \hat{X}_k(2), \ldots, \hat{X}_k(q) \right]^T.
\] (35)

The so-called recursive forecast is to find the recurrence relation between \( \hat{X}_k^{(q)} \) and \( \hat{X}_{k+1}^{(q)} \). For the MA(q) sequence, there are

\[
\begin{align*}
\hat{X}_{k+1}(1) &= \theta_1 \hat{X}_k(1) + \hat{X}_k(2) - \theta_1 X_{k+1}, \\
\hat{X}_{k+1}(2) &= \theta_2 \hat{X}_k(1) + \hat{X}_k(3) - \theta_2 X_{k+1}, \\
&\vdots \\
\hat{X}_{k+1}(q-1) &= \theta_{q-1} \hat{X}_k(1) + \hat{X}_k(q) - \theta_{q-1} X_{k+1}, \\
\hat{X}_{k+1}(q) &= \theta_q \hat{X}_k(1) - \theta_q X_{k+1}
\end{align*}
\] (36)

thereby obtaining

\[
\hat{X}_{k+1}^{(q)} = \begin{bmatrix}
\theta_1 & 1 & 0 & \cdots & 0 \\
\theta_2 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\theta_{q-1} & 0 & 0 & \cdots & 1 \\
\theta_q & 0 & 0 & \cdots & 0
\end{bmatrix} \hat{X}_k^{(q)} \quad \text{or}
\]

\[
X_{k+1} = \begin{bmatrix}
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_{q-1} \\
\theta_q
\end{bmatrix} \hat{X}_k^{(q)}.
\] (37)

The initial value of the recurrence can be taken \( \hat{X}_k^{(q)} = 0 \) because the reversibility of the model ensures that the recurrence is asymptotically stable, i.e., the effect of the initial error can gradually disappear when \( n \) is sufficiently large. For the ARIMA(p,q) sequence, there are

\[
\hat{X}_k(m) = \phi_1 \hat{X}_k(m-1) + \phi_2 \hat{X}_k(m-2) + \cdots + \phi_p \hat{X}_k(m-p), m > p.
\] (38)

Therefore, it is only necessary to know \( \hat{X}_k(1), \hat{X}_k(2), \cdots, \hat{X}_k(p) \) to recursively calculate that \( \hat{X}_k(m), m > p \) still defines the forecast vector. Let

\[
\phi_j = \begin{cases}
\phi_j, & j = 1, 2, \cdots, p, \\
0, & j > p
\end{cases}
\] (39)

The following recursive forecast equation can be obtained
\[
\hat{X}_{k+1}^{(q)} = \begin{bmatrix}
-G_1 & 1 & 0 & \cdots & 0 \\
-G_2 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-G_{q-1} & 0 & 0 & \cdots & 1 \\
-G_q + \phi_q^* & \phi_{q-1}^* & \phi_{q-2}^* & \cdots & \phi_1^*
\end{bmatrix}
\hat{X}_{k+1}^{(q)} + \begin{bmatrix}
G_1 \\
G_2 \\
\vdots \\
G_{q-1} \\
G_q
\end{bmatrix}
X_{k+1} + \sum_{j=q+1}^{p} \phi_j^* X_{k+q+1-j}
\]

(40)

where: \(G_j\) satisfies \(X_t = \sum_{j=0}^{\infty} G_j \varepsilon_{t-j}\). The third term in Eq is 0 when \(p \leq q\). The reversibility condition ensures that when \(k_0\) is small, the initial value \(\hat{X}_{k_0}^{(q)} = 0\) can be made.

In practice, the model parameters are unknown. If the time series has been modeled, the unknown parameters in the theoretical model are replaced by their estimates and then forecasted using the method described above.

First, the data are tested for smoothness and randomness, which are used to ensure that the data used in modeling are smooth, non-white noise time series, and the data are transformed into smooth series using first-order difference and second-order difference, followed by correlation analysis. The time series obtained after first-order difference and second-order difference of the series are shown in the following figure:

![Figure 2: Trend of global temperature](image)

From Figure 2, we can see that the series of the first-order difference is still not smooth, so we continue to carry out the second-order difference, we can find that the second-order difference compared to the first-order difference, the series is much smoother, the original data will be second-order difference, using the ADF unit root smooth type test, to test, after passing the test using the differential data to continue the time series modeling. The results of the second-order difference calculation are shown in Figure 3.
Figure 3: Second-order differential calculation

The following is the need to determine the parameters p, q, d of ARIMA. The common method is to determine them by the autocorrelation function (ACF) and the partial autocorrelation function (PACF). Here p and q are the orders of the coefficients in the following equation, and d is the difference order.

\[ X_t = c + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \ldots + \beta_q \varepsilon_{t-q} \]  \hspace{1cm} (41)

Using the ARIMA time series model to solve for the line DC14→DC10 from January 1, 2023 to January 31, 2023, the data are shown in Figure 4 below.

Figure 4: DC14→DC10 line predicted material flow

4. Conclusions

In this paper, for the problem of emergency transportation and structure optimization of parcels in e-commerce logistics networks, an ARIMA time series prediction model is established based on the historical data of the cargo volume of different routes, and the prediction data of the cargo volume of three routes are given; specifically. Firstly, the historical cargo volume data of the three routes need to be predicted, and since the data are time-series, ARIMA time series is used to analyze the data, and since the data are smoothed in the second-order difference, the optimal parameter values are calculated after the second-order difference, and the ARIMA(1,2,3) time series prediction model is established, and finally the prediction of the three routes from 2023-1-1 to 2023-31 is made. 1-31 time period of the three routes.
References


