Research on the Rube Goldberg Machine Based on the Mechanical Analysis

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ABSTRACT: *This Rube-Goldberg machine includes seven steps that separately show seven different principles of physics during a time span of 42 seconds: conservation of energy, conservation of momentum, inelastic collision, elastic collision, rotation, torque, and 2D projectile motion. Rotation part is the most difficult problem I faced, since I planned to use a pulley to fulfill the requirement but I couldn't build a pulley in Blender. Then I was inspired by a video that teaches people how to make a sling in Blender. Therefore, I changed my plan to make a rotational lever just like a part of a sling to complete the rotation step. I ensured that my machine fulfill all the requirement as it can ran smoothly through all seven required steps with a time span of at least 14 seconds and can also crack an egg successfully in the final step. Each of the seven steps are analyzed mechanically in this article.*

KEYWORDS: *Rube Goldberg machine; conservation of energy; conservation of momentum; projectile; rotation; torque; elastic collision; inelastic collision*

Introduction

My task was to create a Rube-Goldberg Machine in Blender using physic knowledge I've learned in the class and crack an egg in the final step of the machine. Blender was a new software for me and it was a little bit difficult for me to learn how to use it. To study how to build things in Blender, I spent lots of time searching Blender tutorial videos. As I started to build my machine in Blender, there were some technical problems that forced me to change my machine. I didn't find a way to build a pulley so I can only use a lever to build the rotation part. Also, building something that can rotate in Blender is also a difficult task. I watched the tutorial

video again and again and discussed with one of my classmate to find a way to build something rotates. Moreover, some steps of my machine ran too fast at first but the requirement is that each step needed to last at least 2 seconds. To solve this problem, I changed some parts of the machine to make each step last longer. For example, I made the mass of a ball smaller so it can fly longer time in the projectile motion. In order to make the analysis and calculations more simple, I made every step simple and clearly to watch and I also changed objects that I will analyze in Tracker into vivid colors so that Tracker can automatically track the object more easily.

Methods

To begin this project, I decided to use some simple objects in blender such as balls, planes and cubes to build the machine. In order to make the analysis more simple, my machine is just includes some basic motions that show seven required physics principles. Rotation step is the step that I changed most times. I decided to use a pulley to fulfill the requirement at first but I didn't find a way to build it. So I use a lever in stead.

Fig. 1: Visual Representation of Initial Design:
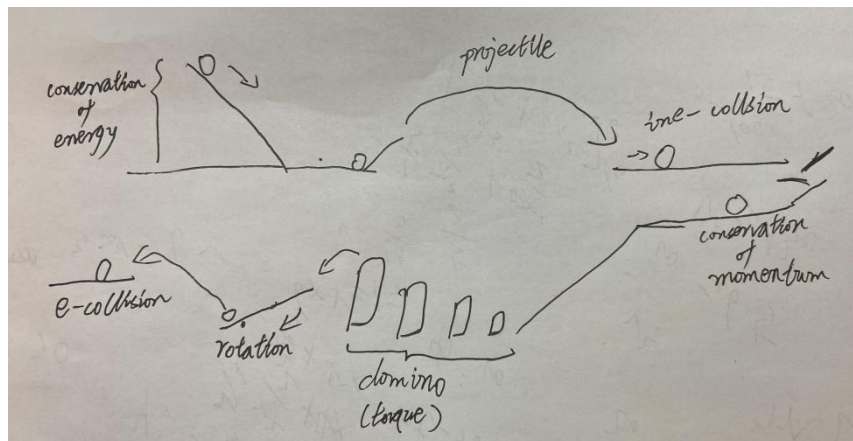
However, I found that it's impossible to let the ball hit the domino right in the middle when I tried to build my machine in Blender, so the domino wasn't a rotational motion and I can't calculate the torque from it. I also found that the rotational part can't last for more than 2 seconds, so I decided to build a foundation for the lever that can raise it up so that the lever can rotate continuously for more than 2 seconds.

Fig.2: revised machine:
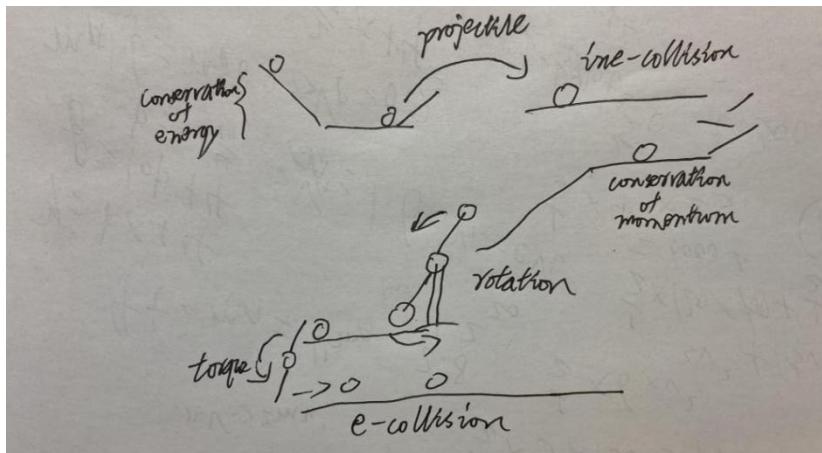
In my redesigned machine, the only part I changed is the rotational and torque parts. I also used the torque part to change the machine from two levels to three levels in order to shorten the length of the machine so that the camera can film the whole machine more easily.

Results

I. Description of Physical steps

Physics Concept	Description of Step

Elastic Collision	Collision between two balls: initially, one ball is moving and the other ball is at rest. I changed the bounciness of the two balls to one in order to make them collide elastically. In the elastic collision, all the kinetic energy of the moving ball would be transferred into the kinetic energy of the other ball and the momentum of the system would be conserved.
Inelastic Collision	Collision between two balls: one ball is in a projectile motion and the other ball is at rest. The collision happens after the projectile ball hit the plane where the collision will happen. I changed the bounciness of the two balls to zero so that the collision would be totally inelastic.
Conservation of Energy	Gravitational potential energy to kinetic energy; To demonstrate the conservation of energy, I make both the ball and the slope totally frictionless. The ball is at rest on the top of the slop at first, so the ball only has potential energy. When the ball reaches the end of the slope, the potential energy would be zero since I set this height as zero and all the potential energy would be transferred to kinetic energy.

Conservation of Momentum	I use a inelastic collision to demonstrate the conservation of momentum. The calculation shows that the total momentum before the collision equals the total momentum after the collision.
2D Projectile Motion	Ball being strike to the air with an angle to the horizon. I used a ball rolling down from a slope and hit a resting ball, the resting ball would be pushed to the air as a projectile along a slope that forms a positive angle with the horizon
Rotation	A rotational lever; a ball falling from a slope will hit one end of the lever. There is a foundation which raise the lever over the ground so that it can rotate continuously.
Torque	Ball pushing one end of a lever; a moving ball will hit one end of the lever, which provide a torque to the lever.

Table 1: Outline of Steps in Rube Goldberg Machine

II. Calculation and Tracker plots

i) Elastic Collision:

t (s)	x (m)	y (m)
4.917	0.265	-2.441E-3
4.958	0.266	-2.313E-3
5.000	0.268	-2.425E-3
5.042	0.269	-2.290E-3
5.083	0.271	-2.401E-3
5.125	0.273	-2.330E-3
5.167	0.275	-2.370E-3
5.208	0.276	-2.481E-3
5.250	0.278	-2.305E-3
5.292	0.279	-2.424E-3
5.333	0.281	-2.486E-3
5.375	0.283	-2.373E-3
5.417	0.284	-2.437E-3
5.458	0.286	-2.327E-3
5.500	0.288	-2.404E-3
5.542	0.289	-2.474E-3
5.583	0.291	-2.305E-3
5.625	0.292	-2.423E-3
5.667	0.294	-2.446E-3
5.708	0.296	-2.353E-3
5.750	0.298	-2.412E-3
5.792	0.299	-2.299E-3
5.833	0.301	-2.370E-3
5.875	0.302	-2.461E-3
5.917	0.304	-2.475E-3
5.958	0.305	-2.366E-3
6.000	0.307	-2.397E-3
6.042	0.308	-2.494E-3
6.083	0.310	-2.513E-3
6.125	0.311	-2.387E-3
6.167	0.313	-2.426E-3

质量 A (t, x)
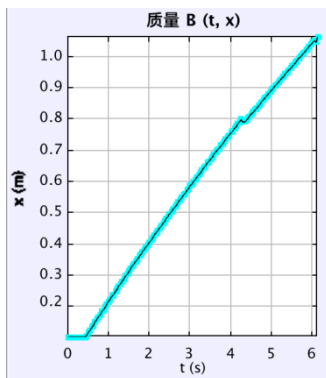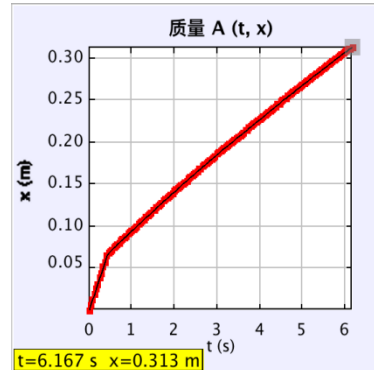
t=6.167 s x=0.313 m

质量 B (t, x)

Fig. i.1, time and corresponding

Fig. i.2, position vs. time for ball 1

Fig. i.3, position vs. time for ball 2 coordinates

Analysis of results

In an elastic collision, energy is conserved so $\frac{1}{2}mv_{before}^2=\frac{1}{2}mv_{after}^2$

Additionally, momentum is conserved so mv_{before}=mv_{after}

$m_1 = 1kg$; m_2=0.5kg

$v_{i,1} = 0.156m/s$; $v_{f,1} = 0.0496m/s$

$v_{i,2} = 0m/s; v_{f,2} = 0.204m/s$

To calculate conservation of momentum

p_{before}=m$v_{i,1}$+ m$v_{i,2}$=1x(0.156)+0=0.15N·s

p_{after}=m$v_{f,1}$+ m$v_{f,2}$=1x(0.0496)+(0.5)(0.204)=0.15N·s
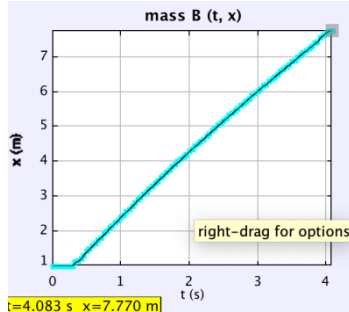
To calculate conservation of KE:

$\frac{1}{2}$x1x(0.156)^2+0=0.012 J

$\frac{1}{2}$x1x(0.0496)^2+$\frac{1}{2}$(0.5)(0.204)^2=0.012 J

ii)Inelastic collision:

t (s)	x (m)	y (m)
3.083	5.193	-2.740E-2
3.125	5.240	-2.732E-2
3.167	5.312	-2.741E-2
3.208	5.361	-2.731E-2
3.250	5.432	-2.721E-2
3.292	5.481	-2.712E-2
3.333	5.552	-2.698E-2
3.375	5.599	-2.691E-2
3.417	5.669	-2.694E-2
3.458	5.718	-2.695E-2
3.500	5.789	-2.698E-2
3.542	5.834	-2.714E-2
3.583	5.906	-2.697E-2
3.625	5.953	-2.712E-2
3.667	6.023	-2.718E-2
3.708	6.070	-2.712E-2
3.750	6.140	-2.702E-2
3.792	6.186	-2.718E-2
3.833	6.255	-2.688E-2
3.875	6.301	-2.674E-2
3.917	6.371	-2.671E-2
3.958	6.416	-2.663E-2
4.000	6.486	-2.661E-2
4.042	6.531	-2.672E-2
4.083	6.600	-2.673E-2

mass A (t, x)

t=4.083 s x=6.600 m

Fig. ii.1, time and   *Fig. i.2, position vs. time for*   *Fig. i.3, position vs.*

time corresponding   *ball 1*   *for ball 2*

coordinates

Analysis of results

In an inelastic collision, momentum is conserved so $mv_{before}=mv_{after}$

*To calculate conservation of momentum:
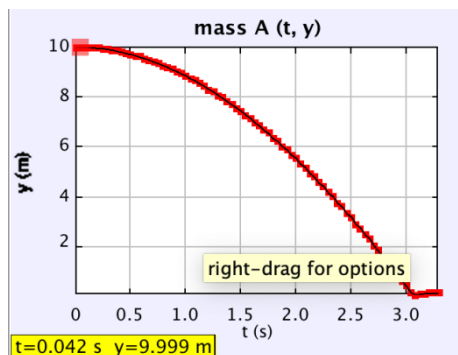
$p_{before}=mv_{i,1}+ mv_{i,2}=2\text{x}(2.807)+0=5.614\text{N}\cdot\text{s}$

$p_{after}=mv_{f,1}+ mv_{f,2}=2\text{x}(1.716)+1\text{x}(2.026)=5.458\text{N}\cdot\text{s}$

*the differences in these values will be explored in the discussion

iii)Conservation of energy: gravitational potential energy to kinetic energy
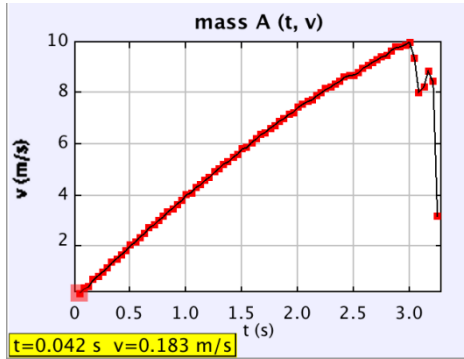
Fig. iii.1, position in the y direction   *Fig. iii.2, velocity vs. timemvs. time*

Analysis of results

According to conservation of energy, the initial potential energy of the ball should be converted to the tangential KE and rotational KE of the ball

m=10kg; h=10m; v=10 m/s

The initial potential energy of the ball:

U=mgh=10x(9.8)x10=980 J

The tangential kinetic energy of the ball when it reaches the end of the slope:
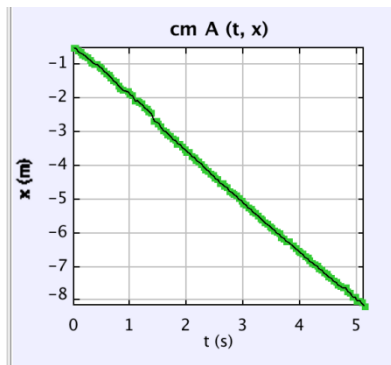
KE=$\frac{1}{2}$mv^2=$\frac{1}{2}$x10x10^2=500 J

The rotational inertia of a hollow ball:

I=$\frac{2}{3}$mr^2

The angular kinetic energy of the ball when it reaches the end of the slope:

$$KE_{angular}=\frac{1}{2}I\omega^2=\frac{1}{2}x\frac{2}{3}mr^2(\frac{v^2}{r^2}) = \frac{1}{3}x10(10^2)=333.33 \text{ J}$$

iv)Conservation of Momentum: inelastic collision

Fig. iv.1, position vs. time for center of mass of the system

Analysis of results

In conservation of momentum, $mv_i=mv_f$

m=1+1=2kg; v_i=1.409m/s; v_f=1.425m/s

p_i=mv_i=2(1.409)=2.82kg·m/s

p_f=mv_f=2(1.425)=2.85kg·m/s
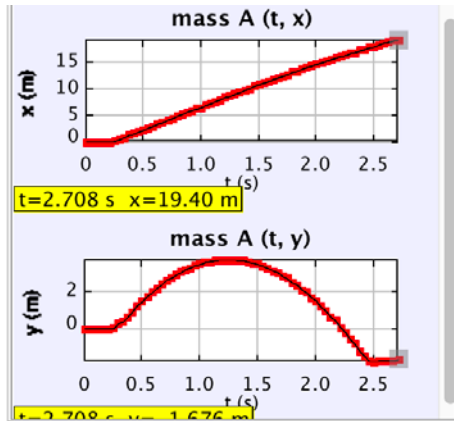
v)Projectile:

Fig. v.1, position in x and y vs. time for projectile ball

To find the initial velocity of the projectile ball:

The projectile ball moves a distance in x direction of Δx=17.92m in 2.25 seconds

So, v_x=17.92/2.25=7.96m/s

Then the projectile ball moves a distance in y direction of Δy=3.787m in 1.042s (from 0.208s to 1.25s)

So, v_y=3.787/1.042=3.63m/s

Using vector addition, the total initial velocity:

v=$\sqrt{(7.96)^2(3.63)^2}$=28.89m/s

The acceleration of the projectile ball is constant and equals 9.8 m/s^2 since the only force exerted on the ball while the ball is in projectile motion is the gravity force.

To calculate the displacement of the projectile ball:

$x_x = v_x t_x$=7.96x2.25=17.92m

x_y=$x_{y,f} - x_{y,i}$=-1.796-0=-1.796m

vi)Rotation: rotating lever
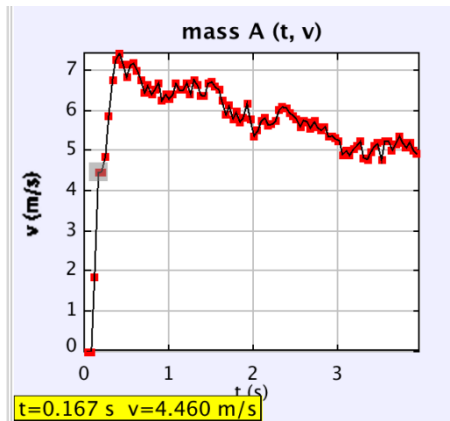
Fig. vi.1,   tangential speed vs. Time

To calculate the centripetal acceleration:

$a_c = \dfrac{v_{tangential}^2}{r} = \dfrac{(7.456)^2}{1}$=55.59m/$s^2$

To calculate the tangential acceleration (from 0.417s to 3.625s):

$$a_{tangential} = \frac{v_f - v_i}{\Delta t} = \frac{5.026 - 7.456}{3.208} = -0.8\text{m}/s^2$$

Then the angular acceleration can be calculated:

$$\alpha = \frac{a_{tangential}}{r} = \frac{-0.8}{1} = -0.8\text{rad}/s^2$$
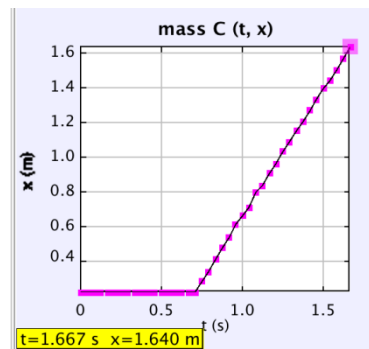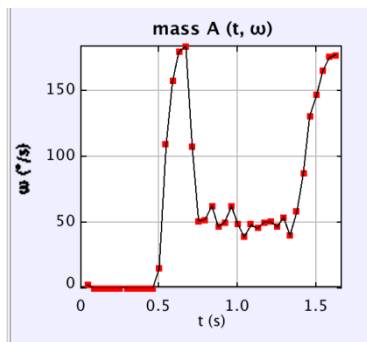
vii)Torque:lever

Fig. vii.1, angular velocity vs. time   *Fig. vii.2, position vs. time for the ball* for the lever

Assuming all rotational kinetic energy of the lever is converted into the KE of the ball

So, $\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$

ω=6.44rad/s; m=1kg; v=1.513m/s

We have: $\frac{1}{2}I(6.44)^2 = \frac{1}{2}(1.513)^2$

So, I=0.055 kg·m^2

To calculate the angular acceleration of the lever:

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{6.44}{0.209} = 30.81$ rad/s^2

To calculate the torque:

$\tau = I\alpha = (0.055)(30.81) = 1.69$ N·m

Discussion

I. Elastic collision

The elastic collision in my Rube Goldberg machine is a collision between two balls. In real world, a collision can't be elastic since collision will create sound and heat. However, one benefit of using Blender is that I can easily make everything frictionless and change the bounciness of the two balls to zero so that I can creat an elastic collision. In this step, one of the two balls is pushed by a lever and collide with the ball at rest. The graphs start from the moment that the moving ball is pushed. The velocity of the two balls stay constant after the collision, which could be showed by the slope of the graphs. I chose the same moment for two graphs and use the slopes at that moment to calculate the velocity of the two balls.

In an elastic collision, the kinetic energy and the momentum should be conserved. According to my calculation, the kinetic energy and the momentum before the collision is equal to the KE and momentum after the collision. Hence, it's a perfectly elastic collision.

II. Inelastic collision

The inelastic collision in my machine is still a collision between two balls. One of them is the ball in projectile motion. I put the other ball on a plane and the projectile ball will collide with it after landing on the plane. To make a inelastic collision, I change the bounciness of the two balls to zero, which means that some energy will lose from the system when the collision happens.

In an inelastic collision, the momentum should be conserved. However, the results of my calculations are not equal. I think that may because of the camera angle. In order to record the whole scene, I need to put my camera in an angle that is not totally horizontal with that part of the machine. As a result, the size of the object may looks different with reality in the video. As I tracking the object in Tracker, the changed size may also make the data inaccurate.

III. Conservation of energy

In my machine, I use a slope and a ball to demonstrate the conservation of energy. I set the height of the slope as 10 m in Tracker so the initial potential energy of the ball is 980J, which is also the energy we need to put the machine in motion. Ideally, when the ball reaches the end of the slope, the initial potential energy should be converted to both tangential kinetic energy and angular kinetic energy. However, the sum of KE is equal to 833.33 J, which is not equal to the initial potential energy. I make every thing in the system frictionless, there is no other way that energy can lose. Hence, I think the inaccuracy may still because of the camera angle like I said above.

IV. Conservation of momentum

This step is shown by another inelastic collision between two balls in my machine. Since the collision is inelastic, only momentum is conserved. To make the calculation more simple, I track the center of mass of the system in stead of tracking two balls separately. Then I can calculate the total momentum of the system using center of mass and the total momentum before the collision should be equal to the one after the collision. There is only a very little difference in the momentum I've calculated.

As I analyzed the collision, Tracker always failed to track one of the balls because the balls were very close to each other when collision happened so Tracker couldn't recognize the ball I was tracking. As a result, I can only track the balls manually which may cause the data inaccuracy.

V.Projectile

The projectile motion step is a ball being pushed to the air alone a ramp that has a positive slope. I use the horizontal distance between the initial position and the point the ball lands on the plane to represent the displacement in x direction. Then use the time it takes to cover the distance to divide the x-displacement, which is the velocity in x direction. Since the direction of the velocity in y direction will change during the motion, I use the vertical distance between initial position and the highest point of the projectile to show the y-displacement. Let the displacement divided by the corresponding time period and I can get the velocity in y direction. Finally, the displacement of the projectile motion can be calculated by using the final position to minus the initial position.

VI. Rotation

My rotation step is a rotating lever which is raised by a foundation above the ground in order to rotate continuously. In Tracker, I set the length of the lever as 2m, which means the rotating radius is 1m since the lever rotates around its center. Then I chose a point from the graph and use the corresponding tangential speed of that point to calculate the centripetal acceleration by the formula $a=\frac{v^2}{r}$. To calculate the tangential acceleration, I use the speed at 0.417s and 3.625s to be the initial speed and the final speed because the speed over this time period stays most constant. Let this time period divide the change in speed and the result is the tangential acceleration. Since the relationship between tangential and angular acceleration can be calculate by the formula $\alpha=\frac{a_{tangential}}{r}$, I can easily get the angular acceleration.

VII.Torque

In this step, a ball rolling down from a slope will hit one end of the lever and provide a torque on the lever. Then the lever rotates and the other end of it will hit another ball, thus provide KE to the ball. The ball which hit the lever first is a projectile, so it's a little bit difficult to calculate the KE of that ball. I chose to use the ball pushed by the lever to calculate the rotational kinetic energy of the lever. Assuming all KE from the lever is converted to the KE of the ball, I track the ball's velocity and calculate it's KE which is also equal to the rotational KE of the lever. Using the equation $\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$, I got the rotational inertia of the lever. Then I use the same way I've used above to calculate the angular acceleration, $\frac{\Delta v}{t}$, and let α time I to get the torque.

Conclusion

In order to design my Rube Goldberg Machine and meet all the constraints, I revised my machine three times. The changes were all about the rotation and torque parts. At first, I planed to use the pulley to be the rotation part. However, the biggest problem was I couldn't figure out how to build a pulley in Blender and I didn't find any pulley model on the Internet that I can input into Blender. So I used a lever instead to build the rotation part but I found the lever was like a seesaw which can't rotate continuously. Finally, I put this "seesaw" in torque part and built another lever which was raised by a foundation so that it can rotates continuously. Moreover, the other problem I met is the time limits. Some steps in my machine can't last for more than 2 seconds at first. I changed the slopes and planes longer so that balls will move for longer time on the slopes or planes. Also, I changed the mass of the projectile ball smaller so that it can reaches a greater height, which also means the projectile motion will last for longer time.

I acknowledge that there are many inaccuracy in my data and analysis because of the camera angle and the Tracker's problem. Also, when I calculated the torque part, I assumed that all angular KE of the lever was converted into KE of the ball, which is impossible. The truth is the lever was still moving after it hit the ball, so the lever still had angular KE. If I build this machine in real world, it may be impossible to eliminate the friction, air resistance and the heat or sound produced by collision.

Fortunately, I can create a ideal machine in Blender. However, the data I collected from my machine was not perfect ideal because there are many other reasons that can cause the inaccuracy. I learned that theoretical and experimental data is often more different than expected.

In future, if I have another opportunity to build a machine like this, I believe I can do better than this time. Also, I will try to make a pulley to be the rotation part next time. I gained a lot of experience of building model in Blender so next time I will spend much less time on figure out how to build machine in Blender and put more effort on how to make my machine perfect.

References

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