Submarine Position Prediction and Rescue Equipment Optimization Model Based on Dung Beetle Optimizer and Bayes' Theorem

Yuxuan Li†**, Chenxu Tian**†,*

College of Statistics and Applied Mathematics, Anhui University of Finance and Economics, Bengbu, China
†*These authors also contributed equally to this work.*
**Corresponding author: 3394275726@qq.com*

Abstract: *This paper investigates the development of safety procedures during the exploration of underwater shipwrecks. First, the dive path and final position of the submarine were successfully predicted by modeling the submarine's acceleration and position prediction. Secondly, based on literature and methodological analysis, the main factors affecting the submarine's position decision, such as continuous wind direction and equipment accuracy, are identified. In terms of equipment solution selection, the cost-effectiveness of equipment carried by the main and rescue boats was evaluated using the CRITIC and VIKOR methods, pointing out that the thermal salinometer and the new unmanned boat were the best choices. Finally, through the Dung Beetle Optimizer and Bayes' Theorem, a statistical model of rescue equipment deployment points and rescue probability was successfully developed, which provides important support for search and rescue operations.*

Keywords: *Dung Beetle Optimizer, Bayes' Theorem, CTRIC-VIKOR Method*

1. Introduction

The remains of shipwrecks at the bottom of the sea hide a rich biodiversity and mysterious historical stories, making them unique places for explorers to lead adventurous journeys [1]. This study is dedicated to address the development of safety procedures during the exploration of underwater shipwrecks in order to improve the efficiency of search and rescue and ensure the safety of tourists. By building a submarine position prediction model and equipment selection scheme, we explore the key factors for submarine position prediction and optimal equipment configuration to provide important support for search and rescue operations. Through the dung beetle optimizer and Bayes' theorem, we further optimized the rescue equipment deployment scheme to improve the rescue success rate. This study will provide reliable safety procedures and technical support for MCMS Dive Manufacturing's underwater expeditions in the Ionian Sea, making the adventures more exciting and safer.

2. Submarine position prediction

2.1 Idealized force analysis of a submarine

In this paper, the submarine is first regarded as a large particle and the force analysis is carried out on it. The ocean current and density are considered, and the positive direction of the X axis is set to simulate the position change of the submarine in three-dimensional space.

In this paper, the structural model of force analysis during submarine diving was first constructed, as shown in Figure 1. In the process of submarine diving, the forces received in the vertical direction include gravity, buoyancy, friction resistance $F_{z_{\text{friction}}}$, shape resistance Fz_{shape} and its own thrust F_{nz}. The resultant force is shown in formula. Where F_{z_g} is the resultant force of gravity and buoyancy, and F_z is the resultant force in the vertical direction.

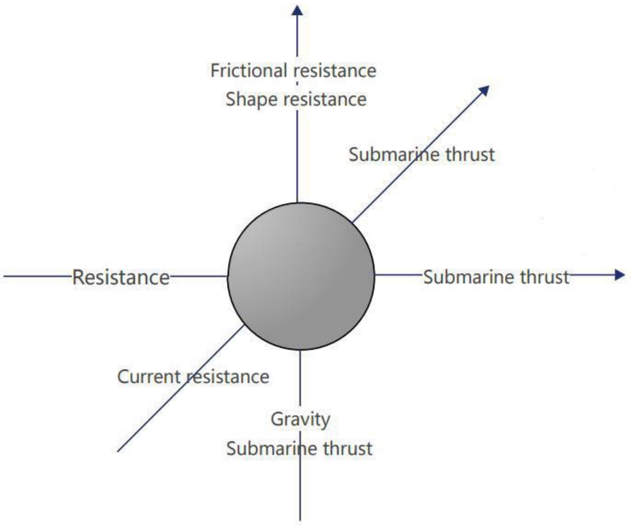

Figure 1: Force analysis

$$F_z = m \cdot a_z = F_{nz} - Fz_{\text{shape}} - F_{z_{\text{friction}}} + F_{z_g} \tag{1}$$

F_z is the resultant force in the vertical direction, P is the rated power of the submarine generator, and vv is the sum speed of the submarine. The relevant calculation formula is shown in formula:

$$\begin{cases} F = \frac{P}{vv} \\ F_{nz} = F \cdot \frac{v[z]}{vv} \\ Fz_{\text{shape}} = 0.5 \cdot \rho \cdot V^2 \cdot A \cdot Cd \\ F_{z_{\text{friction}}} = \frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot v^2 \end{cases} \tag{2}$$

According to the calculation results, the acceleration a_z of the submarine in the vertical direction can be obtained. According to relevant literature, the diving speed of the submarine is generally $1\ \mathrm{m/s}$. Therefore, when the vertical speed of the submarine exceeds $1\ \mathrm{m/s}$, the change in velocity will be small, basically about $1\ \mathrm{m/s}$. Submarines are not limited by speed thresholds.

$$a_z = \frac{F_z}{m} \tag{3}$$

Where F is the total thrust provided by the submarine engine, P is the submarine engine power, Cd in the formula of Fz_{shape} is the resistance coefficient, the general value is 0.25. In the $F_{z_{\text{friction}}}$ formula, Cd is the friction resistance coefficient, the general value is 0.005, Ais the cross-sectional area of the submarine in the direction of motion, and v is the speed of the submarine relative to the sea water.

It can be seen from the force analysis structure diagram that in the horizontal direction, the submarine is subject to its own thrust F_n and the resistance F_{sf} affected by ocean current factors. The calculation formula of the resultant force on the submarine in the direction of X and Y axis is shown in formula.

$$F = m \cdot a = F_n - F_{sf} \tag{4}$$

Where $m \cdot a$ is the resultant force in all directions north and south of the horizontal, F_n is the thrust force of the submarine itself in all directions north and south of the horizontal, and $F_{z_{sf}}$ is the resistance parallel to the direction of motion caused by the friction force of the submarine in the sea water. The relevant calculation formula is as follows:

$$\begin{cases} F_n = F \cdot \frac{v}{vv} \\ v^2 = (v - vc)^2 \\ F_{sf} = \frac{1}{2} \rho A C_d v^2 \end{cases} \tag{5}$$

Where, v refers to the speed of the submarine relative to the sea water in the horizontal north-south direction, F is the thrust of the submarine itself, vv is the sum speed of the submarine, vc is the speed of the ocean current in all directions in the horizontal plane, v is the sub speed of the submarine in all directions in the horizontal plane. In the F_{sf} formula, Cd is the drag coefficient of the submarine in the horizontal plane, and A is the cross-sectional area of the submarine in the direction of motion.

The acceleration of the submarine in all directions in the horizontal plane can be calculated using the formula. In Euler's formula, the density and ocean current near the position of the submarine are constantly changing, and the acceleration is also constantly changing with the position information.

$$a=\frac{F}{m} \tag{6}$$

2.2 State information updating model based on Euler method

Euler method is a commonly used numerical integration method [2]. In this paper, Euler method is used to update the velocity and position information of submarine in the course of motion. The basic idea is to discretize the time, convert the continuous motion state of the submarine into a discrete motion state, and at the same time use a small-time step to approximate the trajectory of the submarine [3]. The relevant formula is shown in equation:

$$\begin{cases} v_{\text{new}} = v_{\text{old}} + a \cdot \Delta t \\ loc_{\text{new}} = loc_{\text{old}} + v_{\text{new}} \cdot \Delta t \end{cases} \tag{7}$$

Where Δt is the time step, and the smaller the step value, the closer the value is to the true value. v_n ewis the next speed, v_{old} is the current speed, x_n ewis the next position of the submarine, and x_{old} is the position of the current step.

Where the value of acceleration a is affected by the force, the force situation is related to the current and density near the current position, so the force situation of the submarine in all directions is constantly changing according to the position information.

In this paper, by analyzing the force of the submarine at the current position, the acceleration in each direction is obtained, and the speed and position information is constantly updated. At the same time, the position and speed information of the submarine can be accurately predicted as the initial value of the next calculation. The acceleration of the submarine in all directions is shown in formula, where F_{new} is the force exerted on the submarine at its current position.

$$a=\frac{F_{new}}{m} \tag{8}$$

In this paper, the position update path of the submarine was obtained through repeated implementation of Euler formula, and the accurate prediction of the submarine position information was realized. The arcgis tool was used to present the information as shown in Figure 2 and Figure 3. In this paper, the velocity of submarine in the vertical direction is limited by threshold, so its trajectory is basically an inverse parabola shape.

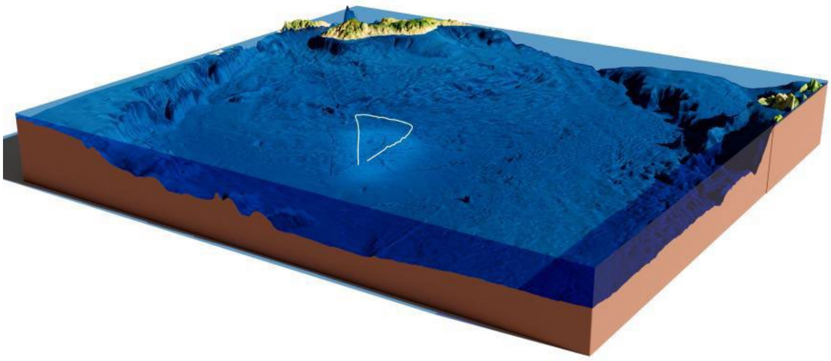

Figure 2: Arcgis1

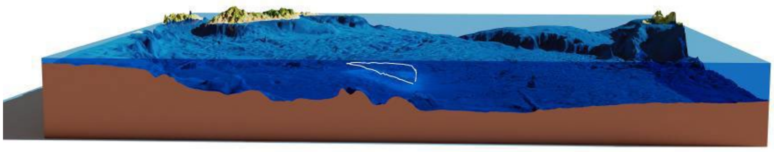

Figure 3: Arcgis2

3. Equipment program selection

3.1 Indicator weight assignment based on CRITIC method

In this paper, Z-Score method is first adopted to normalize the original data, and its formula is shown in formula. Then, the CRITIC weight method is used to weight each indicator [4]. The CRITIC weight method is a weighting method that comprehensively measures the volatility and conflict of evaluation indicators and can provide a relatively objective weight value for the selection of equipment schemes. Where μ is the mean value of the data set and σ is the standard deviation of the data set.

$$z = \frac{(x-\mu)}{\sigma} \tag{9}$$

Based on the normalized data volatility S_j index is calculated main ship equipment, index conflict $A_j, A_j x_o f_{ij}, x_{ik}$ for two groups of data, j, k index respectively x_{ij}, x_a verageofik. The correlation coefficient R is shown in formula:

$$\begin{cases} S_j = \sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij}-\bar{x}_j\right)^2}{n-1}} \\ A_j = \sum_{i=1}^{n}\left(1-r_{ij}\right) \\ R = \frac{\sum_{j,k=1}^{n}(x_{ij}-\bar{x}_j)(x_{ik}-\bar{x}_k)}{\sqrt{\sum_{j=1}^{n}\left(x_{ij}-\bar{x}_j\right)^2 \sum_{k=1}^{n}(x_{ik}-\bar{x}_k)^2}} \end{cases} \tag{10}$$

The information carrying capacity C_j is calculated based on index volatility and index conflict. The larger the information carrying capacity, the more important the corresponding index, and the higher its weight. The relevant formula is shown in equation. In the preceding command, W_j indicates the weight assigned to each indicator.

$$C_j = S_j \times A_j \tag{11}$$

$$W_j = \frac{C_j}{\sum_{j=1}^{n} C_j} \tag{12}$$

The weight assignment results based on the CRITIC method show that the weights of purchase cost, maintenance cost, use cost, range and resolution are 0.184,0.154,0.128,0.314 and 0.220. Among them, the weight of the range index is the largest, and its information bearing capacity is also the largest.

3.2 Main ship equipment selection based on VIKOR method

VIKOR method is a multi-attribute decision making method based on ideal solution [5]. Compared with TOPSIS method, VIKOR method is more reliable and stable. Through the compromise consideration of various schemes, the optimal scheme closer to the ideal scheme is obtained. In this paper, positive and negative ideal solutions are calculated respectively based on normalized data. Positive and negative ideal solutions are the maximum and minimum values of each index data respectively. The calculation formula is shown in equation, where f_{ij} is the standard value of the first j evaluation item on the first i index.

$$\begin{cases} X_i^+ = \max_j f_{ij} \\ X_i^- = \min_j f_{ij} \end{cases} f_{ij} \in [0,1] \tag{13}$$

Secondly, based on the positive and negative ideal solutions, the group utility value S_i and the minimum regret value R_i of the main ship equipment are determined, so that each equipment plan can be considered in the final compromise. X_{ij} is the index value of scheme j on the index of i.

$$\begin{cases} S_i = \sum_{j=1}^{n} \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-} \\ R_i = \max_{1 \le j \le n} \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-} \end{cases} \tag{14}$$

Finally, the median value of Q_i is determined and based on S_i, R_i and Q_i, the compromise is considered for the main ship equipment scheme, and the equipment scheme with the smallest value is selected as the optimal scheme. Where v is the decision mechanism coefficient and $v[0,1]$.

$$Q_i = v \frac{S_i - S^+}{S^- - S^+} + (1-v) \frac{R_i - R^+}{R^- - R^+} \quad i = 1,2,\ldots,m \tag{15}$$

The evaluation results based on the CRITI-CVIKOR method show that the S_i value of five kinds of additional search equipment for the main ship, namely sonar system, magnetic detector, acoustic Doppler current meter, attitude meter and thermosalinity depth meter, is 0.974,1.010,1.226,1.212 and 0.807, respectively. The value of R_i is 0.310,0.307,0.341,0.368,0.196, and the value of Q_i is 0.531,0.567,0.924,0.984,0, respectively. The results are displayed as shown in Figure 4. In Figure 4, $Y(1)$ and $Y(2)$ are respectively the thermohaline depth meter and sonar system. Through inspection, it is found that the value of $Q(Y(2)) - Q(Y(1))$ is 0.531, which is greater than or equal to 1/3. After sorting S_i and R_i, it can be concluded that R_i of the thermohaline depth meter $Y(1)$ is the smallest. Therefore, the optimal solution for the additional search equipment of the main ship is the thermohaline depth meter.

The thermohaline depth meter can accurately measure the temperature and salinity near the submarine position and transmit the data to the main ship to determine the submarine depth and improve the accuracy of the submarine position prediction.

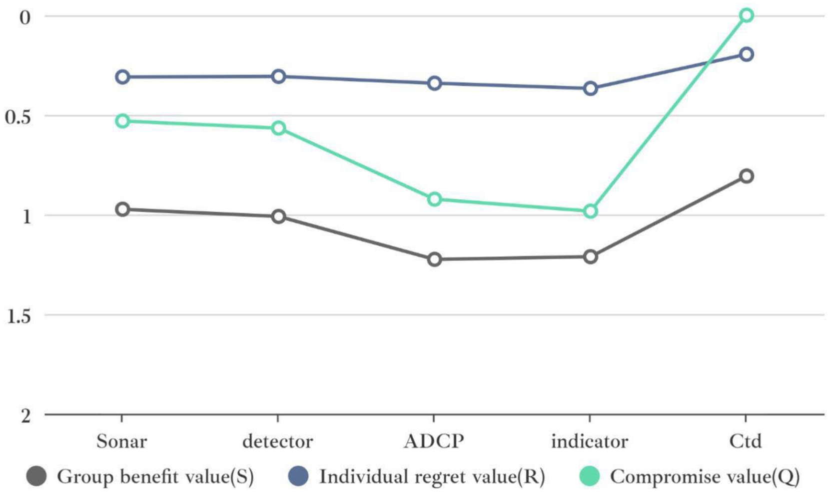

Figure 4: Evaluation results

4. Optimal deployment point for rescue equipment

In this paper, the volume, mass, generator power and other parameters of the rescue equipment are set with reference to the relevant data of SRV rescue deep submarine [6]. The relevant data of the unpowered submarine is set with reference to the data in the section 2, and its movement trajectory is predicted by using the search and rescue simulation model. The results are shown as shown in Figure 5.

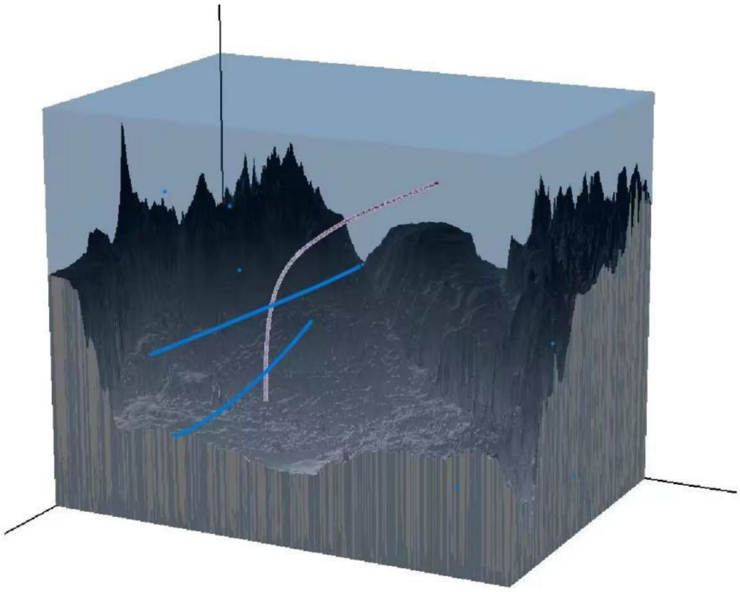

Figure 5: Rescue paths

In this paper, the Dung Beetle Optimizer (DBO) algorithm is used to explore the optimal initial deployment point. DBO algorithm is a swarm intelligence optimization algorithm with strong exploration and development ability for complex problems [7]. In this paper, the fitness function is set as the linear distance between the equipment and the rescue submarine, and the variable range of longitude, latitude and depth is set as the Ionian sea area. The fitness function takes longitude, latitude and depth as variables, and its distance calculation formula is shown in formula:

$$d = 2r\arcsin\left(\sqrt{(\varphi_2 - \varphi_1) + \cos(\varphi_1)\cos(\varphi_2)(\lambda_2 - \lambda_1)}\right) \tag{16}$$

Firstly, a 3D initial population is generated, and the upper and lower bound vectors are set as the boundary values of the Ionian sea area, where the upper bound vector is [21.125574, 40.209567, -170.0], the lower bound vector is [14.988225,35.991576, -180.0], and the initial population number is set as 1000. The maximum number of iterations is 1000. Secondly, five iterative methods of DBO algorithm are used to update the location of the initial population, so as to find the global optimal solution, that is, the best initial deployment point.

The final result shows that the coordinate of the optimal deployment point is [17.42,35.99,-180.0], the diving time of the search and rescue equipment is 56.2min, and the linear distance between the search and rescue equipment and the submarine is 523m.

The results show that the optimal deployment point has the potential to find the unpowered submarine and achieve rescue as soon as possible, and the model can accurately predict the position of the submarine and explore the optimal deployment point by taking into account the influence of ocean currents. This method greatly increases the probability of successful rescue.

In this paper, Bayes-based theorem is used to calculate the probability of successful search and rescue. Based on the data set, the success criteria are divided, and the conditional probability formula is used to calculate the probability of search and rescue success under the condition that the rescue time is met.

In this paper, the optimal solution data set in the case of multiple iterations is first established. The proportion diagram of time data and linear distance data is shown in Figure 6 and Figure 7.

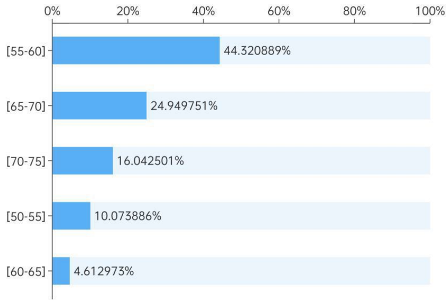

Figure 6: Time

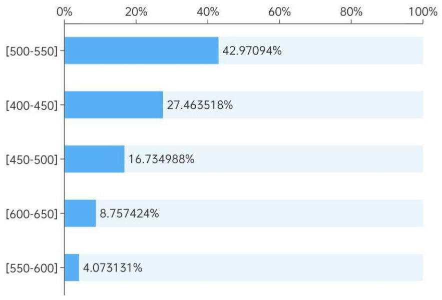

Figure 7: Distance

The results show that the proportion of search and rescue time between 5560 is the largest. This paper sets 60 as the standard of search and rescue time, and the probability of search and rescue time meeting the standard is 54.39. The linear distance between the equipment and the submarine occupies the largest proportion between 500 550. In this paper, 550 is set as the standard of the region as the linear distance, and the probability of the linear distance reaching the standard at the final moment is 87.17. Assuming that the straight-line distance attainment at the last moment is event A and the search and rescue time is event B, the probability calculation formula of the straight-line distance attainment in space is expressed as formula 47.42:

$$P(A \mid B)=\frac{P(A \cap B)}{P(B)} \tag{17}$$

The final results show that when the search and rescue time is up to standard, the probability of the space linear distance reaching the standard is 0.8614, indicating that the optimization algorithm proposed in this paper has strong exploration and development ability in search and rescue optimization, and the application of this method can provide great help for submarine search and rescue in practical rescue.

5. Conclusions

Through this study, we have successfully established safety procedures for exploring underwater shipwrecks, providing significant support for MCMS Dive Manufacturing's underwater expeditions in the Ionian Sea. Our research on submarine position prediction modeling and equipment selection schemes has improved search and rescue efficiency and ensured the safety of tourists. Our findings show that accurate prediction of submarine position is influenced by factors such as continuous wind direction, sea salt density and equipment data accuracy; in terms of equipment selection, thermal salinometers and a new unmanned boat proved to be the best solution. In addition, the dung beetle optimizer and Bayes' theorem were used to optimize the rescue equipment deployment scheme and improve the rescue success rate. In summary, our study provides reliable safety procedures and technical support for underwater expeditions, making the adventure more exciting and safer.

References

[1] Kingsley S A. Environmental Reviews and Case Studies: Impacts on Maritime Cultural Resources: Assessing the Invisible [J]. Environmental Practice, 2016, 18(3): 184-191.

[2] Kamruzzaman M, Nath M C. A comparative study on numerical solution of initial value problem by using Euler's method, modified Euler's method and runge–kutta method [J]. Journal of computer and mathematical sciences, 2018, 9(5): 493-500.

[3] Guiying Z, Ruidi A. Application of Euler Method in Discrete Dynamic Systems [J]. Curriculum and Teaching Methodology, 2023, 6(12): 21-27.

[4] Mukhametzyanov I. Specific character of objective methods for determining weights of criteria in MCDM problems: Entropy, CRITIC and SD [J]. Decision Making: Applications in Management and Engineering, 2021, 4(2): 76-105.

[5] Shumaiza, Akram M, Al-Kenani A N, et al. Group decision-making based on the VIKOR method with trapezoidal bipolar fuzzy information [J]. Symmetry, 2019, 11(10): 1313.

[6] Poupkou A, Zanis P, Nastos P, et al. Present climate trend analysis of the Etesian winds in the Aegean Sea [J]. Theoretical and applied climatology, 2011, 106: 459-472.

[7] Xue J, Shen B. Dung beetle optimizer: A new meta-heuristic algorithm for global optimization [J]. The Journal of Supercomputing, 2023, 79(7): 7305-7336.