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Finite-time adaptive-triggered control for nonlinear affine systems with input delay

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Abstract: *In this paper, a new adaptive-trigger control method via adaptive dynamic programming is proposed to study the finite-time optimal control problem of nonlinear systems with input delay. Firstly, the influence of input delay was eliminated through an approximation method. Secondly, in order to obtain the solution of the Hamilton-Jacobi-Bellman equation and design the optimal control scheme, a neural network is constructed to approximate the optimal cost function. In addition, the closed-loop systems have been proven to be finite-time stable. Finally, the feasibility of the proposed method was verified through a numerical simulation.*

Keywords: *Adaptive-triggered Mechanism, Adaptive Dynamic Programming, Optimal Control, Neural Network, Input Delay*

1. Introduction

With the increasing complexity of actual control systems, input delay has become an inevitable factor in control engineering, usually due to control channel modeling errors or external disturbances, which may reduce system performance. Therefore, eliminating the impact of input delay on the stability of nonlinear control systems is of great significance. To address this issue, scholars have proposed the Padé approximation technique[1], which is a design method that integrates time delay into control strategies to ensure satisfactory system performance.

In recent years, the concept of optimal control problem usually involves determining a control strategy that ensures system stability while optimizing performance indicators. However, for nonlinear systems with input delays, the solution typically requires solving the Hamilton-Jacobi- Bellman (HJB) equation. As is well known, the HJB equation is extremely complex and it is almost impossible to obtain its analytical solution. In 1977, Werbos introduced Adaptive Dynamic Programming (ADP) as a means to avoid directly solving the HJB equation[2]. In recent years, ADP has become an effective technique for solving optimal control problems and has been widely applied in practical engineering fields. For example, Zhang et al. [3] proposed a bias strategy iterative method based on ADP. This method utilizes the deviation value function to alleviate the constraints of initial allowable control, while demonstrating outstanding ability to learn the optimal controller in the absence of prior system information, while also retaining the fast convergence characteristics of traditional policy iteration algorithms. Guan et al. [4] conducted research on the ADP optimal tracking control algorithm for multi UAV systems based on zero sum game theory to address the issue of external interference during flight. Compared with sliding mode control, this method has better trajectory tracking performance and robustness.

Extensive research has shown that event-triggered control (ETC) can achieve excellent control performance in various practical fields. Compared with the continuous control method, the ETC method improves energy efficiency and shortens control time while maintaining comparable speed tracking performance[5]. However, the improvement of energy efficiency comes at the cost of convergence time. In many cases, it is expected that dynamical systems have the property of reaching a stable equilibrium state within a finite time, rather than just exponential or asymptotic. After analyzing these findings, it is evident that most systems have achieved asymptotically stable results, and, the lack of new control methods that reduce control frequency and ADP technology to achieve finite time stability of closed-loop systems has stimulated our research.

This paper investigates the optimal control problem of nonlinear systems with input delay using ADP and adaptive-trigger control (ATC) scheme. Firstly, we transform the control problem into an optimal

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feedback control problem for an auxiliary system with a modified cost function. Subsequently, two triggering conditions were derived through the finite time stability criterion to ensure the stability of the nonlinear system under the optimal feedback controller.In this process, in order to obtain the optimal ATC strategy, we use a critical neural network (CNN) scheme to solve the corresponding HJB equation. Finally, a numerical example is provided to verify the effectiveness of the proposed control strategy. In contrast, the main contributions of this study can be summarized as follows:

(1) Compared to the existing control schemes discussed in [6] and [7], this paper addresses additional challenges. Specifically, we consider the existence of input delay. This factor significantly increases the complexity of developing control strategies.

(2) Compared with the existing ETC scheme proposed in [8], we introduce an ADP based ATC scheme. In this scheme, the triggering condition adjusts the control width from the time domain dimension, thereby improving the control performance. This innovative expansion improves control efficiency and provides new possibilities for achieving better system stability and performance.

The rest of this paper is organized as follows: In Section 2, the optimal control problem was explained. Section 3 presents main result. In Section 4, a numerical example is given to demonstrate the effectiveness of the proposed methods, which is followed by the conclusions in Section 5.

Notation: *$\mathbb{R}^n$ stands for n-dimensional real column vector and $\|\cdot\|$ denotes the Euclidean norm of vectors or matrix. $\mathbb{R}^{n\times m}$ is $n\times m$ real matrices. The superscript $\top$ means the transposition of a matrix or a vector. $\mathbb{N}^+$ represents the set of a positive integer. ∇ is the gradient operator. $\lambda_{\min}(\cdot)$ represent the minimum eigenvalue.*

2. Preliminaries

Consider the nonlinear affine systems with input delay described by

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t-\tau), x(0) = x_0, \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state. $f(x(t))\in\mathbb{R}^n$ and $g(x(t))\in\mathbb{R}^{n\times m}$ are known smooth continuous function. $u(t-\tau)\in\mathbb{R}^m$ is the input and τ denotes the network-induced delay.

Remark 1: *In this paper, we aim at finding an optimal feedback control to stabilize the nonlinear system (1). However, for the control problem of the nonlinear affine systems, the control signal is affected by network-induced delay, which makes it difficult to design the controller.*

Assumption 1: *$f(x(t))$, $g(x(t))$ are Lipschitz continuous on the set $\Omega\subset\mathbb{R}^n (0\in\Omega)$, $f(0)=0$.*

Assumption 2: *$g(x(t))$ is bounded, i.e., $\|g(x(t))\|\le B_g$, where B_g is a constant.*

In order to remove the influence of time delay τ and get the actual control input u, the Padé approximation approach and the delay theorem of the Laplace transform are introduced, then

$$\mathcal{L}\{u(t-\tau)\} = \exp(-\tau\iota)\mathcal{L}\{u(t)\} = \frac{\exp(-\tau\iota/2)}{\exp(\tau\iota/2)}\mathcal{L}\{u(t)\} \approx \frac{1-\tau\iota/2}{1+\tau\iota/2}\mathcal{L}\{u(t)\}, \tag{2}$$

where ι represents the Laplace variable and $\mathcal{L}(u(t))$ denotes the Laplace transform of $u(t)$. Let a new intermediate variable as μ and it satisfies

$$\frac{1-\tau\iota/2}{1+\tau\iota/2}\mathcal{L}\{u(t)\} = \mathcal{L}\{\mu(t)\} - \mathcal{L}\{u(t)\}, \tag{3}$$

so that

$$4\mathcal{L}\{u(t)\} = 2\mathcal{L}\{\mu(t)\} + \tau\iota\mathcal{L}\{\mu(t)\}. \tag{4}$$

Using the inverse Laplace transform, we have

$$\dot{\mu}(t) = \frac{4}{\tau}u(t) - \frac{2}{\tau}\mu(t), \tag{5}$$

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then, we can obtain

$$u(t-\tau) = \mu(t) - u(t), \tag{6}$$

according to the above transformations, system (1) can be further rewritten as

$$\dot{x}(t) = f(x(t)) + g(x(t))(\mu(t) - u(t)). \tag{7}$$

Remark 2: *The new intermediate variable can be considered as an error variable that needs to be addressed. So the problem of input delay can be transformed into eliminating the negative impact of* $u(t)$*. Once there is a deviation in* τ*,* μ *will be a time-varying situation. Therefore, the developed controller should be designed to eliminate this issue. For better analysis, we first define the approximation error as* $\exp(-\tau k) - (1 - \tau k/2 / 1 + \tau k/2)$*, it can be observed that when the time delay is very small, the approximation error approaches zero. Therefore, this delay operation is not suitable for situations with long delays, otherwise it may reduce system performance.*

For simplicity, t is omitted for subsequent time-dependent variables. And the nominal system of system (7) as follows

$$\dot{x} = f(x) + g(x)\upsilon, \tag{8}$$

where the definitions of x, $f(x)$, $g(x)$ are the same as system (1) and $\upsilon \triangleq \mu - u$.

Consider the nominal systems (8), for the optimal control problem of the system (7), we define the cost function as follows

$$J(x) = \int_0^\infty A(x,\upsilon)\mathrm{d}t, \tag{9}$$

where $A(x,u) = x^\top Qx + \upsilon^\top R\upsilon$, $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{m\times m}$ are positive definite matrix.

Further, one can get

$$J^\star(x) = \min_{\upsilon\in\Xi(\Omega)} \int_0^\infty A(x,\upsilon)\mathrm{d}t, \tag{10}$$

where $J^\star(x)$ denotes the optimal cost function, $\Xi(\Omega)$ indicates an admissible control domain.

When the cost function (9) is continuously differentiable, the Hamiltonian of system (8) can be defined as

$$H(x,\psi,\nabla J(x)) = A(x,\upsilon) + \nabla J^\top(x)(f(x) + g(x)\upsilon). \tag{11}$$

According to Bellman's optimality principle, the HJB equation can be formulated as

$$0 = \min_{\upsilon\in\Xi(\Omega)} H(x,\upsilon,\nabla J^\star(x)). \tag{12}$$

Take the partial derivative of $\upsilon^\star$ from the right side of (12), we can get the optimal control policy:

$$\upsilon^\star = -\frac{1}{2}R^{-1}g^\top(x)\nabla J^\star(x), \tag{13}$$

based on (10)-(13), we can rewrite the HJB equation as

$$0 = -\frac{1}{4}\nabla J^{\star\top}(x)g(x)R^{-1}g^\top(x)\nabla J^\star(x) + x^\top Qx + \nabla J^{\star\top}(x)f(x), J^\star(0) = 0. \tag{14}$$

Therefore, in order to obtain the optimal robust controller of the original system, we need to solve $\nabla J^\star(x)$ from (8). Firstly, we introduce the definition of the Semi-global Practical Finite-time stable (SGPFS) as follows.

Definition 1[9]: *The equilibrium* $x \equiv 0$ *of the system (1) is SGPFS, if for* $x(0) = x_0$*, there exist a positive constant* ϵ *and a set time* $t^\#(\epsilon, x_0) < \infty$ *such that* $\|x(t)\| < \epsilon$ *for all* $t > t^\#$.

Lemma 1[10]: *For the system state variables* a_i $(i = 1,2)$*, there exist any positive real numbers*

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b_j $(j=1,2,3)$ *such that*

$$|a_1|^{b_1}|a_2|^{b_2} \le \frac{b_1}{b_1+b_2} b_3 |a_1|^{b_1+b_2} + \frac{b_2}{b_1+b_2} b_3^{-\frac{b_1}{b_2}} |a_2|^{b_1+b_2}. \tag{15}$$

Lemma 2[11]: *Let* $a_i \ge 0$, $i=1,2,...,b$, *and* $0<c\le 1$, *one has*

$$\left(\sum_{i=1}^{b} |a_i|\right)^c \le \sum_{i=1}^{b} |a_i|^c \le b^{1-c}\left(\sum_{i=1}^{b} |a_i|\right)^c. \tag{16}$$

Lemma 3[12]: *For the nonlinear system (1), suppose that there exist a continuously differentiable function* $V(x)$, *real numbers* $\alpha>0$, $\beta\in(0,1)$ *and* $\gamma>0$ *satisfying*

$$\dot{V}(x) \le -\alpha (V(x))^{\beta} + \gamma, t \ge 0, \tag{17}$$

then, the zero solution $x\equiv 0$ *of the system (1) is SGPFS. And for any* $0<\delta<1$, *one has*

$$t^{\#} = \frac{1}{\alpha\delta(1-\beta)}\left((V(x(0)))^{1-\beta} - \left(\frac{\gamma}{\alpha(1-\delta)}\right)^{\frac{1-\beta}{\beta}}\right). \tag{18}$$

Remark 3: *The finite-time stability criterion proposed by Lemma 3 makes it possible to design optimal controllers based on ADP, and analyzes the stability of closed-loop systems with input delay from the perspective of finite-time stability.*

3. Main results

3.1. Adaptive-triggered Control Strategy

In the following, a time-dependent threshold strategy is adopted to design the adaptive-triggering mechanism. Firstly, we set the sampling time as $t_0, t_1, \cdots, t_j, \cdots$, where $t_0 = 0$ represents the first sampling, t_j represents the j th sampling time, $j\in\mathbb{N}$. The adaptive-triggered controller and the adaptive-triggering mechanism are given as

$$\begin{cases} u(t) = u(t_j), \forall t \in [t_j, t_{j+1}), \\ t_{j+1} = \inf\{t \,\|\, t - t_j | > t^{\flat}\}, \end{cases} \tag{19}$$

where $t^{\flat}$ represents the sampling interval between the j th and $j+1$ th times.

Combined with Zero-order Holder (ZOH), the controller can be described as

$$u(t) = \begin{cases} u(t_j), t \in [t_j, t_{j+1}), \\ u(t_{j+1}), t = t_{j+1}. \end{cases} \tag{20}$$

From (20), one can get

$$\upsilon_j = \mu_j - u_j, \tag{21}$$

where $\upsilon_j = \upsilon(t_j), \mu_j = \mu(t_j), u_j = u(t_j)$. Let $x(t_j) = x_j$, it can be obtained that

$$\upsilon_j^{\star} = -\frac{1}{2} R^{-1} g^{\top}(x_j) \nabla J^{\star}(x_j), \tag{22}$$

according to (22), system (8) can be rewritten as

$$\dot{x} = f(x) + g(x)\upsilon_j^{\star}. \tag{23}$$

Remark 4: *The control process is shown in Figure 1. The adaptive-triggered mechanism can calculate the next triggering moment based on the system's dynamic knowledge and the current*

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triggering moment, and its advantage lies in relieving the real-time detection of triggering conditions by additional hardware. Therefore, this article defines a prediction function $t^{\flat}$ *between two triggering moments as the determining condition for the next triggering moment.*

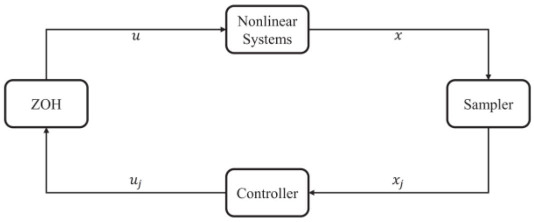

Figure 1: Block diagram of sampling process.

Assumption 3: $\upsilon^\star$ *is local Lipschitz continuous, i.e.* $\|\upsilon^\star(t)-\upsilon^\star(t_j)\|\le l_{\upsilon^\star}\|t-t_j\|$, *where* $l_{\upsilon^\star}$ *is Lipschitz constant.*

Assumption 4: *Suppose there exist two positive constants* ς_1,ς_2 *such that the optimal cost function satisfies* $\varsigma_1\|x\|^2\le J^\star(x)\le\varsigma_2\|x\|^2$.

Theorem 1: *Consider the nominal systems (8) with the optimal cost function (10). The optimal control policy is given by (22), if there are real numbers* $\alpha_1>0$, $\beta\in(0,1)$ *and* $\gamma_1>0$ *such that (17) hold. Then the system (23) is SGPFS if the prediction function is defined as*

$$t^{\flat}=\frac{1}{2\|R\|\,l_{\upsilon^\star}}\lambda_{\min}(R)\|\upsilon^\star\|. \tag{24}$$

Proof. Consider the Lyapunov function candidate $V_1(x)=J^\star(x)$. For $t\in[t_j,t_{j+1})$, we have

$$\dot V_1(x)=\nabla J^{\star\top}(x)(f(x)+g(x)\upsilon_j^\star)=\nabla J^{\star\top}(x)f(x)+\nabla J^{\star\top}(x)g(x)\upsilon_j^\star, \tag{25}$$

according to the HJB equation (12) and the optimal controller (13), we find

$$\nabla J^{\star\top}(x)g(x)=-2\upsilon^{\star\top}R, \tag{26}$$

and

$$\nabla J^{\star\top}(x)f(x)=-x^\top Qx+\upsilon^{\star\top}R\upsilon^\star, \tag{27}$$

Together with (24), (25) and (26), based on Assumption 2, we get

$$\begin{aligned}\dot V_1(x)&=-x^\top Qx+2\upsilon^{\star\top}R\upsilon^\star-2\upsilon^{\star\top}R\upsilon_j^\star-\upsilon^{\star\top}R\upsilon^\star\\&\le -x^\top Qx+2\|\upsilon^\star\|\ \|R\|\ \|\upsilon^\star-\upsilon_j^\star\|-\lambda_{\min}(R)\|\upsilon^\star\|^2\\&\le -x^\top Qx+2\|\upsilon^\star\|\ \|R\|\,l_{\upsilon^\star}\|t-t_j\|-\lambda_{\min}(R)\|\upsilon^\star\|^2,\end{aligned} \tag{28}$$

note that $t_j\le t<t_{j+1}$, then we get

$$\dot V_1(x)\le -x^\top Qx, \tag{29}$$

now, based on Lemma 1 and 3, let $x_1=1, x_2=\varsigma_2^{-1}\lambda_{\min}(Q)J^\star(x)$ and $y_1=1-\beta$, $y_2=\beta$ and $y_3=\beta^{\beta/(1-\beta)}$, we have

$$\begin{aligned}\dot V_1(x)&\le -\varsigma_2^{-1}\lambda_{\min}(Q)J^\star(x)-(\varsigma_2^{-1}\lambda_{\min}(Q)J^\star(x))^\beta+(\varsigma_2^{-1}\lambda_{\min}(Q)J^\star(x))^\beta\\&\le -(\varsigma_2^{-1}\lambda_{\min}(Q)J^\star(x))^\beta+(1-\beta)\beta^{\frac{\beta}{1-\beta}},\end{aligned} \tag{30}$$

for any $x\in\Omega$, we obtain

$$\dot V_1(x)\le -\alpha_1(V_1(x))^\beta+\gamma_1, \tag{31}$$

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where $\alpha_1 = (\varsigma_2^{-1}\lambda_{\min}(Q))^{\beta}$, $\gamma_1 = (1-\beta)\beta^{\beta/(1-\beta)}$, and for any $0<\delta<1$, the settling time is given by

$$t^{\#} = \frac{1}{\alpha_1\delta(1-\beta)}\left((V(x_0))^{1-\beta} - \left(\frac{\gamma_1}{\alpha_1(1-\delta)}\right)^{\frac{1-\beta}{\beta}}\right), \tag{32}$$

for $t = t_{j+1}$, note that the system state x is continuous, then $\Delta V_1(x) = J^\star(x(t_{j+1}^+)) - J^\star(x(t_{j+1})) < 0$. Therefore, from (24) hold, the system (23) achieves the SGPFS for all $t > t^{\#}$. The proof is completed.

3.2. Neural Network

As we all know, it is extremely difficult to solve $\nabla J^\star(x)$ through the HJB equation, the CNN is used to approximate the optimal cost function $J^\star(x)$ to obtain $\nabla J^\star(x)$ in this section. The structure of the CNN is as follows:

$$J^\star(x) = w^\top e(x) + r(x), \tag{33}$$

where $w \in \mathbb{R}^q, e(x), r(x)$ represent the ideal CNN weight, activation function and CNN approximation error, respectively. q is the number of hidden neurons.

Since the ideal CNN weight w is unknown, its estimated value $\hat{w}$ is used instead

$$\hat{J}^\star(x) = \hat{w}^\top e(x). \tag{34}$$

Based on (34), the control strategy (22) can be written as

$$\hat{\upsilon}_j^\star = -\frac{1}{2}R^{-1}g^\top(x_j)\nabla e^\top(x_j)\hat{w}, \tag{35}$$

and the approximation error of Hamiltonian can be given as

$$\xi = \hat{H}(x,\hat{\upsilon}_j^\star,\nabla\hat{J}^\star(x)) - H(x,\upsilon_j^\star,\nabla J^\star(x)) = \hat{H}(x,\hat{\upsilon}_j^\star,\nabla\hat{J}^\star(x)), \tag{36}$$

where $H(x,\psi_j^\star,\nabla J^\star(x)) = 0$.

From (36), the target function can be given as

$$\zeta = \frac{1}{2}\xi^\top\xi. \tag{37}$$

To keep ζ small enough, we need to train the weight $\hat{w}$ to converge to w by gradient descent method, the adaptive updating law of the weight can be redesigned as

$$\dot{\hat{w}} = -\ell\frac{1}{(1+\sigma^\top\sigma)^2}\hat{H}(x,\hat{\upsilon}_j^\star,\nabla\hat{J}^\star(x))\sigma, \tag{38}$$

where ℓ is learning rate of the network, $1/(1+\sigma^\top\sigma)^2$ is the normalization factor, $\sigma = \nabla\phi(x)(f(x)+g(x)\hat{\upsilon}_j^\star)$.

The weight estimation error of the network is defined as $\tilde{w} = w - \hat{w}$, then

$$\dot{\tilde{w}} = -\dot{\hat{w}} = -\ell\frac{\sigma\sigma^\top\tilde{w}}{(1+\sigma^\top\sigma)^2} + \ell\frac{\sigma r_\xi}{(1+\sigma^\top\sigma)^2}, \tag{39}$$

where $r_\xi = -\nabla r(x)(f(x)+g(x)\hat{\upsilon}_j^\star)$.

Remark 5: *Generally, the persistent excitation condition is considered in the weight adaptive update rate of CNN. From the initial time, in order to maintain the system active, a small noise signal is added to the control input, which ensures the convergence of the CNN weight.*

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3.3. Stability Analysis

Before analyzing the threshold and stability, the following necessary assumptions are introduced.

Assumption 5: *$\nabla e(x), \nabla r(x)$ and r_ξ are bounded, i.e., $\|\nabla e(x)\| \le B_e, \|\nabla r(x)\| \le B_r$ and $r_\xi \le B_\xi$, where $B_e > 0, B_r > 0$ and $B_\xi > 0$ are constants.*

Assumption 6: *There exists a constant $\hbar > 0$ that holds for $\| \upsilon^\star - \hat{\upsilon}_j^\star \| \le \hbar \| \upsilon^\star - \upsilon_j^\star \|$.*

Theorem 2: *Supposed Assumptions 5 and 6 hold. Consider the continuous time nonlinear systems (1) with CNN approximator (34) and control input (35). The adaptive update law of CNN weight is tuned by (39). Then, the SGPFS of the system is guaranteed provided that we make the following inequality hold:*

$$t^\flat = \left(\frac{1}{4} \lambda_{\min}(R) \| R\|^{-2} B_g^2 B_e^2 \| \hat{w}\|^2 - \frac{1}{2} \ell B_\xi^2 \right)^{-\frac{1}{2}}. \tag{40}$$

Proof. Consider the Lyapunov function candidate

$$V(x, \tilde{w}) = V_1(x) + V_2(x) + V_3(\tilde{w}), \tag{41}$$

where $V_1(x) = J^\star(x)$, $V_2(x) = J^\star(x_j), V_3(\tilde{w}) = 1/2 \tilde{w}^\top \tilde{w}$.

For $t \in [t_j, t_{j+1}), \dot{V}_2(x) = 0$. Differentiating $V_1(x)$ along the solution of $\dot{x} = f(x) + g(x)\hat{\upsilon}_j^\star$, we have

$$\dot{V}_1(x) = \nabla J^{\star\top}(x)(f(x) + g(x)\hat{\upsilon}_j^\star) = \nabla J^{\star\top}(x) f(x) + \nabla J^{\star\top}(x) g(x) \hat{\upsilon}_j^\star, \tag{42}$$

according to (26) and (27), we get

$$\begin{aligned} \dot{V}_1(x) &= -x^\top Q x + \upsilon^{\star\top} R \upsilon^\star - 2\upsilon^{\star\top} R \hat{\upsilon}_j^\star + \hat{\upsilon}_j^{\star\top} R \hat{\upsilon}_j^\star - \hat{\upsilon}_j^{\star\top} R \hat{\upsilon}_j^\star \\ &\le -x^\top Q x + \lambda_{\max}(R) \| \upsilon^\star - \hat{\upsilon}_j^\star \|^2 - \lambda_{\min}(R) \| \hat{\upsilon}_j^\star \|^2. \end{aligned} \tag{43}$$

Based on (33)-(36), using Assumption 5-6, it can be checked that

$$\begin{aligned} \dot{V}_1(x) &\le -x^\top Q x + \lambda_{\max}(R) \hbar \| \upsilon^\star - \upsilon_j^\star \|^2 - \lambda_{\min}(R) \| \hat{\upsilon}_j^\star \|^2 \\ &\le -x^\top Q x + \lambda_{\max}(R) \hbar \| \upsilon^\star - \upsilon_j^\star \|^2 - \frac{1}{4} \lambda_{\min}(R) \| R\|^{-2} B_g^2 B_e^2 \| \hat{w}\|^2. \end{aligned} \tag{44}$$

Further, the differential of $V_3(\tilde{w})$ with respect to $\tilde{w}$ satisfies

$$\dot{V}_3(\tilde{w}) = \ell \tilde{w}^\top \dot{\tilde{w}} = -\ell \frac{\tilde{w}^\top \sigma \sigma^\top \tilde{w}}{(1 + \sigma^\top \sigma)^2} + \ell \frac{\tilde{w}^\top \sigma r_\xi}{(1 + \sigma^\top \sigma)^2}, \tag{45}$$

and note that $1 + \sigma^\top \sigma \ge 1$, based on Young's inequality, one can obtain

$$\dot{V}_3(\tilde{w}) \le -\frac{1}{2} \ell \frac{\tilde{w}^\top \sigma \sigma^\top \tilde{w}}{(1 + \sigma^\top \sigma)^2} + \frac{1}{2} \ell r_\xi^\top r_\xi \le -\frac{1}{2} \ell \lambda_{\min}(\Pi_\sigma) \| \tilde{w}\|^2 + \frac{1}{2} \ell B_\xi^2. \tag{46}$$

Together with (40), (44) and (46), using Assumption 3 we have

$$\begin{aligned} \dot{V}(x, \tilde{w}) &\le -x^\top Q x + \lambda_{\max}(R) \hbar \| \upsilon^\star - \upsilon_j^\star \|^2 - \frac{1}{4} \lambda_{\min}(R) \| R\|^{-2} B_g^2 B_e^2 \| \hat{w}\|^2 - \frac{1}{2} \ell \lambda_{\min}(\Pi_\sigma) \| \tilde{w}\|^2 + \frac{1}{2} \ell B_\xi^2 \\ &\le -x^\top Q x - \frac{1}{2} \ell \lambda_{\min}(\Pi_\sigma) \| \tilde{w}\|^2, \end{aligned} \tag{47}$$

where $\Pi_\sigma = \sigma \sigma^\top / (1 + \sigma^\top \sigma)^2$, based on Lemma 1-2 and Assumption 4, we get

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$$\begin{aligned}\dot{V}(x,\tilde{w}) &\le -\varsigma_2^{-1}\lambda_{\min}(Q)J^\star(x)+\frac{1}{2}\ell\lambda_{\min}(\Pi_\sigma)\left(\frac{1}{2}\tilde{w}^\top\tilde{w}\right)-(\varsigma_2^{-1}\lambda_{\min}(Q))^\beta(J^\star(x))^\beta \\ &+(\varsigma_2^{-1}\lambda_{\min}(Q))^\beta(J^\star(x))^\beta-\left(\frac{1}{2}\ell\lambda_{\min}(\Pi_\sigma)\right)^\beta\left(\frac{1}{2}\tilde{w}^\top\tilde{w}\right)^\beta+\left(\frac{1}{2}\ell\lambda_{\min}(\Pi_\sigma)\right)^\beta\left(\frac{1}{2}\tilde{w}^\top\tilde{w}\right)^\beta \\ &\le -(\varsigma_2^{-1}\lambda_{\min}(Q))^\beta(J^\star(x))^\beta-\left(\frac{1}{2}\ell\lambda_{\min}(\Pi_\sigma)\right)^\beta\left(\frac{1}{2}\tilde{w}^\top\tilde{w}\right)^\beta+2(1-\beta)\beta^{\frac{\beta}{1-\beta}},\end{aligned} \tag{48}$$

using Lemma 3, one can obtain

$$\dot{V}(x,\tilde{w}) \le -\alpha_2(V(x,\tilde{w}))^\beta+\gamma_2, \tag{49}$$

where $\alpha_2=\min\{(\varsigma_2^{-1}\lambda_{\min}(Q))^\beta,(1/2\,\ell\lambda_{\min}(\Pi_\sigma))^\beta\}$, $\gamma_2=2(1-\beta)\beta^{\frac{\beta}{1-\beta}}$.

For $t=t_{j+1}$, the differential form of (41) is

$$\Delta V(x,\tilde{w})=\Delta V_1(x)+\Delta V_2(x)+\Delta V_3(\tilde{w}), \tag{50}$$

where $\Delta V_1(x)=J^\star(x(t_{j+1}^+))-J^\star(x(t_{j+1}))<0$, $\Delta V_2(x)=J^\star(x_{j+1}))-J^\star(x_j)\le\kappa(\| t_{j+1}-t_j\|)$, $\kappa(\cdot)$ denotes a class-κ functionm $\Delta V_3(\tilde{w})=1/2\tilde{w}^\top(t_{j+1}^+)\tilde{w}(t_{j+1}^+)-1/2\tilde{w}^\top(t_{j+1})\tilde{w}(t_{j+1})<0$, $t_{j+1}^+\triangleq\lim_{\tilde{t}\to 0}(t_{j+1}+\tilde{t})$.

Then, from Lemma 3, for any $0<\varsigma<1$, one has

$$t^{\#}=\frac{1}{\alpha_2\delta(1-\beta)}\left((V(x_0,\tilde{w}(0)))^{1-\beta}-\left(\frac{\gamma_2}{\alpha_2(1-\delta)}\right)^{\frac{1-\beta}{\beta}}\right). \tag{51}$$

Hence, for $\forall t>t^{\#}$, the trajectory of the closed-loop system is SGPFS. The proof is completed.

4. Numerical simulations

In this section, an example will be performed on a single-link manipulator arm to verify the correctness and effectiveness of the proposed method. The dynamic equation is described as

$$\ddot{\Theta}(t)=-\frac{MGL}{F}\sin(\Theta(t))-\frac{D}{F}\dot{\Theta}(t)+\frac{1}{F}u(t-\tau), \tag{52}$$

where, M is the mass of the payload, G is acceleration of gravity, L is the length of the robotic arm, D is friction, F is the moment of inertia. In the experiment, take $M=10kg$, $G=9.81m/s^2$, $L=1m, D=5N, F=10kg\cdot m^2$.

Consider Θ as state x_1, $\dot{\Theta}$ as state x_2, then the formula (52) can be written as

$$\dot{x}=f(x)+g(x)u(t-\tau), \tag{53}$$

where $x=(x_1,x_2)^\top, f(x)=(x_2,-9.81\sin x_1-0.5x_2)^\top, g(x)=(0,0.1)^\top, u$ is control input.

In the simulation, set $Q=\mathrm{diag}\{1,1\}, R=1, l_{\upsilon^\star}=1$, the CNN learning rate $\ell=0.5$, the network-induced delay $\tau=0.01s$. Select the activation function $e(x)=[x_1^2,x_1x_2,x_2^2]$. In order to meet the PE condition, a small signal $\eta(t)=0.01e^{-0.1t}sin^2(t)+cos^2(t)$ is added to the control u in the first 3 seconds and turn off after 3 seconds.

Set the initial value of the system state as $x(0)=(2,1)^\top$, as can be seen in Figure 2(a), the system converges to 0 after 10 seconds under the action of the ATC input. The convergence of CNN weights are shown in Figure 2(b), which show that the CNN weight parameters reach stable values, that is $\hat{w}=[0.0247,-0.044,-0.105]$. The change of the control input is shown in Figure 2(c). From Figure 2(d), it can be seen that the cost function eventually converges to a stable value.

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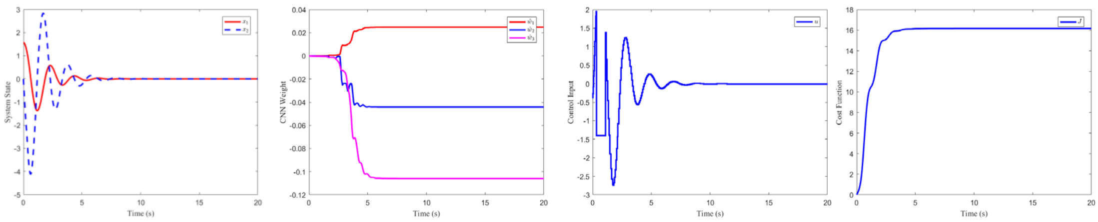

(a) (b) (c) (d)

Figure 2: The numerical simulation results.

5. Conclusions

In this article, an ATC scheme was designed based on the ADP method, which effectively reduced the control frequency of the controller, alleviated the communication burden to a certain extent, and ensured the stability of the system with input delay. Specifically, the impact of input delay was eliminated through approximate methods. Secondly, in order to obtain the solution of the HJB equation and design the optimal control scheme, a neural network was constructed to approximate the optimal cost function. In addition, closed-loop systems have been proven to be SGPFS. Finally, the effectiveness of the proposed method was verified through system control simulation of a single link-robotic-arm. In the following research, it is expected to extend the ATC method to more complex nonlinear systems.

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