# Research on pure azimuth passive localization in UAV formation flight

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***Abstract:*** *In order to avoid external interference, the UAV cluster should keep electromagnetic silence as much as possible and send less electromagnetic wave signals to the outside world when conducting formation flight. In order to maintain formation flight, azimuth-only passive positioning method is proposed to adjust the position of UAVs, that is, some UAVs in the formation send signals, and other UAVs passively receive signals, from which orientation information is extracted for positioning. In this context, this paper studies the problem of pure azimuth passive positioning in UAV formation flight, and discusses the necessity of using pure azimuth passive positioning in UAV formation flight to cope with the limitation of electromagnetic interference on traditional positioning methods. The specific problem includes how to use the three-point positioning method to determine the position of the signal receiving UAV, whose receiving direction is determined by the Angle formed by the two UAV sending signals in the formation. Consider a circular formation containing 10 Uavs, where one UAV is located at the center of the circle and the remaining nine are evenly distributed on the circumference. By establishing mathematical models and geometric relationships, solutions are proposed to ensure positional accuracy under different measurement errors. This study provides a theoretical basis and practical method for improving the accuracy and stability of UAV formation flight.*

***Keywords:*** *Three point positioning method, plane geometry, pure azimuth passive positioning*

## 1. Introduction

With the rapid development of science and technology, UAV technology is increasingly widely used in military, civil and commercial fields. In modern warfare and disaster rescue[1], UAV formation can effectively improve the efficiency and safety of task execution. However, the electromagnetic interference problem faced by UAV formation flight poses a challenge to traditional positioning systems, especially when the formation integrity needs to be maintained. In order to cope with the influence of electromagnetic interference[2], pure azimuth passive positioning becomes an effective solution. By receiving the direction information of signals sent by other Uavs in the formation, the method can determine its own position without actively sending signals, thus reducing the risk of electromagnetic signal exposure. In this context[3], this paper investigates the feasibility and effectiveness of using a pure azimuth passive positioning method in UAV formation flight. The core problem of this paper is how to establish the positioning model of receiving signal UAV by using three-point positioning method[4]. Specifically, we will analyze the Angle formed by the two Uavs that send signals in the formation and the geometric relationship between the Uavs that receive signals and these two[5]. By establishing a mathematical model, we will explore solutions under different measurement errors to ensure the accuracy and stability of position positioning during UAV formation flight. Through this study, we aim to provide theoretical support and practical methods for optimizing the positioning technology in UAV formation flight, so as to promote the further application and development of UAV technology in various fields[6].

## 2. Modeling and implementation

According to the three-point location method, the center position of the circle formed by the unknown point and any two known points is found first [7]. As shown in Figure 1.

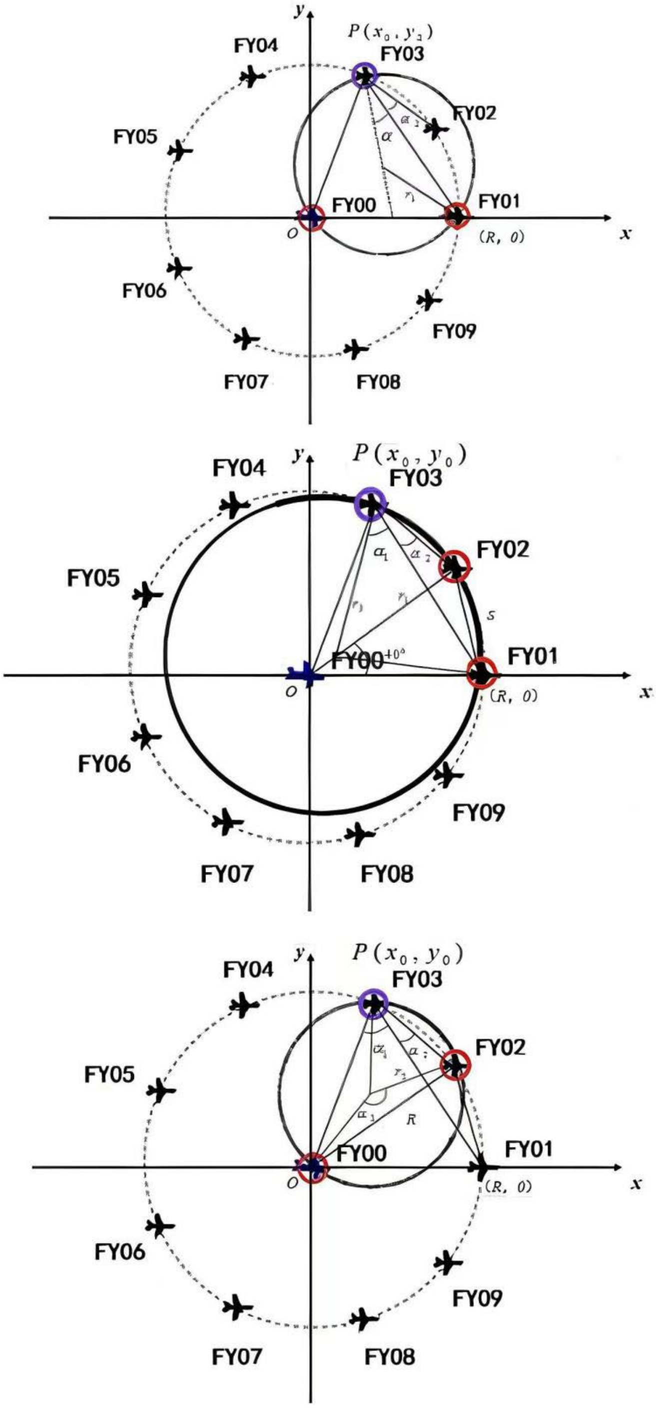

*Figure 1: Schematic diagram of three-point positioning method model*

Two circles intersect A and B, connecting AB, PQ, AQ and AP. AB and PQ intersect at point C, and the coordinates of points A and B can be obtained by simultaneous equations, and then the lengths of PA and AQ can be obtained according to the Euclidean distance[8]. According to the Pythagorean theorem, we can obtain:

$$\left\{\begin{array}{l} \mathrm{PQ} = \mathrm{PC} + \mathrm{QC} \\ \mathrm{QA}^2 = \mathrm{QC}^2 + \mathrm{AC}^2 \\ \mathrm{PA}^2 = \mathrm{PC}^2 + \mathrm{AC}^2 \end{array}\right\} \tag{1}$$

The solution is:

$$\mathrm{PC}=\frac{\mathrm{PQ}}{2}+\frac{\mathrm{PA}^2-\mathrm{QA}^2}{2\mathrm{PQ}} \tag{2}$$

Coordinates (x,y) are obtained from the proportion relation, and the three circles are calculated in pairs according to this method, that is, three points are obtained[9], and the mean value of the coordinates of the three points is taken as the coordinates of the unknown point P. Let FY02 coordinate be (X0,Y0), then:

$$\begin{Bmatrix} (\mathrm{x}_1-\mathrm{x}_0)^2+(\mathrm{y}_1-\mathrm{y}_0)^2={r_1}^2 \\ (x_2-x_0)^2+(y_2+y_0)^2={r_2}^2 \\ (x_3-x_0)^2+(y_3-y_0)^2=r^2 \end{Bmatrix} \tag{3}$$

By solving the simultaneous equations, we can get that the coordinates of FY02 satisfy the intersection point of three circles[10].

$$\begin{Bmatrix} (\mathrm{x}_1-\mathrm{x}_0)^2+(y_1-0)^2={r_1}^2 \\ (x_1-R)^2+(y_1-0)^2={r_1}^2 \end{Bmatrix} \tag{4}$$

$$\left(\frac{1}{2}\mathrm{R}\right)^2+\left(\frac{\frac{1}{2}\mathrm{R}}{\tan\frac{\alpha 1}{2}}-\mathrm{r}_1\right)^2={\mathrm{r}_2}^2 \tag{5}$$

The solution is:

$$\begin{cases} x_1^2=\frac{1}{4}R^2 \\ {y_1}^2=R^2\tan^2\frac{\alpha_1}{2}+\frac{R^2}{\tan^2\frac{\alpha_1}{2}}+\frac{7}{4}R^2 \end{cases} \tag{6}$$

$${\mathrm{r}_2}^2=R^2\frac{1}{2(1-\cos 2\alpha_2)} \tag{7}$$

For circle 2:

$$\begin{Bmatrix} (\mathrm{x}_1-\mathrm{x}_0)^2+(\mathrm{y}_1-\mathrm{y}_0)^2={r_1}^2 \\ (x_2-x_0)^2+(y_2-y_0)^2={r_2}^2 \\ (x_3-x_0)^2+(y_3-y_0)^2=r^2 \end{Bmatrix} \tag{8}$$

The solution is:

$$\begin{cases} x_2=\frac{R}{2\cos\theta}-y_2\tan\theta \\ y_2=\sqrt{r_2^2+R^2\left(\frac{1}{2}-\frac{\sin^2\theta}{4}\right)}-\frac{R}{2}\sin\theta \end{cases} \tag{9}$$

$${\mathrm{r}_2}^2=R^2\frac{1}{2(1-\cos 2\alpha_2)} \tag{10}$$

For circle 3:

$$x_3 = \frac{R}{2} + \sqrt{\frac{\sqrt{1-\sin^2\alpha_1}}{\sin^2\beta_1} - \frac{1}{4}R^2 + \frac{R^2(1-\cos\beta_1)}{4\cos^2\alpha_1}}$$

$$y_3 = \frac{2R^2(1-\cos\beta_1)}{4\cos^2\alpha_1} - R^2 + 2x_2R - {x_3}^2 \tag{11}$$

$$r_2^2 = R\frac{\sqrt{2(1-\cos\beta_1)}}{2\cos 2\alpha_1} \tag{12}$$

Therefore, this mathematical model can be used to solve the position of other passively receiving signals UAV.

## 3. Establishment of effective positioning model

The model is established based on the RSSI ranging multilateral positioning method, given d,n,do and other influence factor coefficients. According to the multilateral positioning method, due to practical factors, ranging always has a certain error, when using trilateral positioning, it is often unable to obtain more accurate location information of unknown nodes. According to the corresponding ranging results of multiple known nodes (the number of known nodes is greater than 3), the multilateral positioning rule looks for a point that can minimize the impact of ranging error on positioning accuracy, and takes this point as the position of unknown point. The value of the influence factor in the process of signal propagation is given according to the RSSI formula, so as to determine the distance between nodes.

$$PL(d) = PL(d_0) - 10n\lg\frac{d}{d_0} - N_0 \tag{13}$$

Suppose that the number of nodes z= (x, y) is known, where i= 1,2,3,... n, whose coordinate distribution z, i= 1,2,3... n and the distance between these known points and the unknown points z= (x, y) is r, i= 1,2,3,... n, the following equation can be obtained:

$$\begin{cases} (x-x_1)^2 + (y-y_1)^2 = r_1^2 \\ (x-x_2)^2 + (y-y_2)^2 = r_2^2 \\ \vdots \\ (x-x_n)^2 + (y-y_n)^2 = r_n^2 \end{cases} \tag{14}$$

By eliminating the quadratic term of the unknown node coordinates of the system of equations, an n-1 dimensional linear system of equations is obtained:

$$\begin{cases} 2(x_n-x_1)x + 2(y_n-y_1)y = r_1^2 - r_n^2 - x_1^2 - y_1^2 + x_n^2 + y_n^2 \\ 2(x_n-x_2)x + 2(y_n-y_2)y = r_2^2 - r_n^2 - x_2^2 - y_2^2 + x_n^2 : x_n^2 \\ \vdots \\ 2(x_n-x_{n-1})x + 2(y_n-y_{n-1})y = r_{n-1}^2 - r_n^2 - x_{n-1}^2 - y_{n-1}^2 + x_n^2 + y_n \end{cases} \tag{15}$$

The matrix representation of this equation is AX=B, where the coefficient matrix A and the right-hand value vector B are:

$$A = \begin{bmatrix} 2(x_n-x_1) & 2(y_n-y_1) \\ 2(x_n-x_2) & 2(y_n-y_2) \\ \vdots & \\ 2(x_n-x_{n-1}) & 2(y_n-y_{n-1}) \end{bmatrix} B = \begin{bmatrix} r_1^2 - r_n^2 - x_n^2 - y_1^2 + x_n^2 + y_n^2 \\ r_2^2 - r_n^2 - x_2^2 - y_2^2 + x_n^2 + y_n^2 \\ \vdots \\ r_{n-1}^2 - r_n^2 - x_{n-1}^2 - y_{n-1}^2 + x_n^2 + y_n^2 \end{bmatrix} \tag{16}$$

Due to the existence of errors in measurement, the actual system of linear equations should be expressed as: AX+N=B,N is the n−1 dimensional random error vector. For this system of linear equations, the principle of least squares can be used to minimize the square of the modulus of the random random error vector N=B-AX, that is, $|B-AX|^2=M|^2$, so as to ensure the minimum influence of ranging error on the positioning result.

Now it is assumed that each UAV transmitting signals is on a circle with TY00 as the center and a radius of 100, and the degree of the center Angle formed by the two adjacent Uavs on the circle is 40 degrees.

According to FY0X(X=0,1,2... 9) The solution formula is:

$$\begin{cases} x_X = r\cos\left(\frac{2\pi}{9}x\right) \\ y_X = r\sin\left(\frac{2\pi}{9}x\right) \end{cases} \tag{17}$$

Using the calculator, we can get the Cartesian coordinates of these 10 positions, as shown in Table 1:

*Table 1: Coordinate representation table of model results*

| FY0X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 0 | 100 | 76.604 | 17.365 | -50 | -93.969 | -93.969 | -50 | 17.365 | 76.604 |
| y | 0 | 0 | 64.279 | 98.481 | 86.603 | 34.202 | -34.202 | -86.603 | -98.481 | -64.279 |

When we assume that the existing FY00, FY01, FYOK, FYOJ (K, J=2,3,4... 9 and K! =J) Four Uavs to estimate the position, and display the distance from the target position and the target position is not equal to the position of the four Uavs. When we cycled 10 times with four UAV models, 10 distance values between the true value and the estimated value were obtained. Similarly, 10 distance values are obtained with 5 aircraft:

*Table 2: Table of location prediction values*

| Number of times | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 unmanned aerial vehicles | 9.78 | 7.99 | 24.38 | 12.16 | 22.86 | 40.32 | 8.75 | 8.28 | 10.27 | 23.77 |
| 5 unmanned aerial vehicles | 14.53 | 10.88 | 8.07 | 10.69 | 15.17 | 15.17 | 48.06 | 0.67 | 6.46 | 0.76 |

Table 2 the 20 data are presented in line chart, from which it can be seen intuitively that there is not much difference between the two groups of data, so it can be considered that 4 Uavs are the optimal solution, as shown in Figure 2.

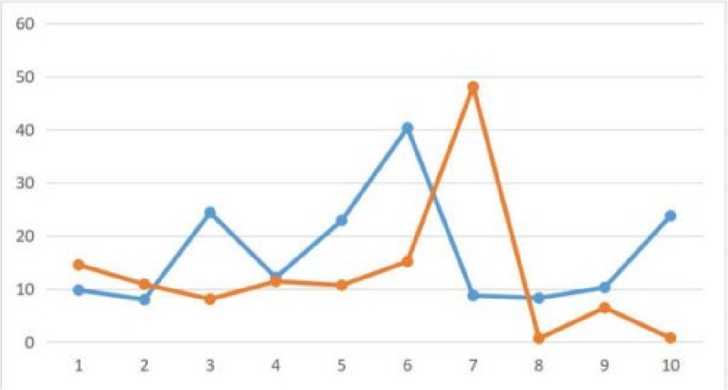

*Figure 2: Numerical line diagram of position prediction*

By analyzing the polar coordinate data of the initial position of each UAV in Table 1 given by the topic, it can be observed that the polar diameter of No.2, No.5 and No.8 UAV is the same as 98m; The polar diameter of No.3, No.6 and No.9 UAV is 112m; The polar diameter of No.4 and No.7 UAV is 105m.According to the information given, it is found that the formation requires 9 Uavs to be evenly distributed on a circle with a radius of 100m, which is the best. For this, a circle with a radius of 98m should be established as the target circle. By default, No. 2, 5 and 8 are the other three unmanned signal transmitters in addition to FY00, which is the center of the circle. Firstly, the modeling starts from the position prediction, because each UAV will have deviation, we default to ignore the height error of each aircraft, and four aircraft No. 0, 2, 5 and 8 have formed a circle with a radius of 98m.At the same time we set up a bias system K,$K = \frac{x}{\delta_1} \times 100\%$. Is the ratio of UAV movement deviation.The following figure shows the initial position information of ten UUAS, in which the red triangle represents the signal transmitter, and the red dot is the target position of UAV No. 9 after adjustment. The circle is a plane figure with a radius of 98m. As shown in Figure 3.

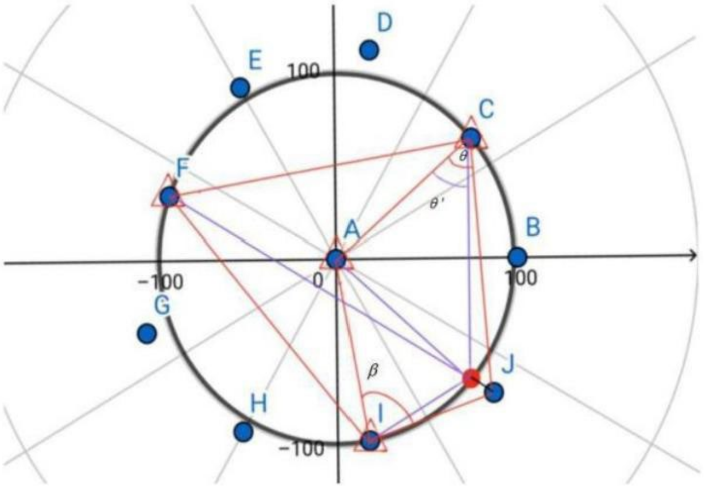

*Figure 3: UAV dynamic adjustment planning diagram*

Through multiple data tests, the following results are obtained:

If k>1, it means that the UAV moves too far in the direction of the target, and the transmitter should issue instructions to make the UAV move in the opposite direction. If k=1, it indicates that the UAV is just adjusted to the required target position (that is, just on the target circumference), and the transmitter should remain relatively stationary with the receiver; If 0<k<1, it indicates that the adjusted position of the UAV is still outside the target circle, and the transmitter should continue to send down the instructions to make the UAV continue to act. In this regard, the adjustment and movement of the control UAV can be judged by the change of the deviation coefficient K.

FY00, FY02, FY05 and FY08 are set up as unmanned signal transmitters, and FY09 is adjusted to illustrate the adjustment model. For ease of understanding, we label FY00 as A, and so on, FY09 as J. From the background of the problem, three direction information, namely a, $a_2$ and a3 can be obtained from three unmanned signal transmitters and one unmanned signal receiver. First, we set C as the unmanned signal receiver, A, F, and J as the unmanned signal transmitter, and J 'as the target position of J after adjustment. Thus, it can be concluded that $\angle ACJ=0$, $\angle ACJ'=\theta'$, and from the theorem of circular angles, $\angle ALJ'=\angle ACJ'$. It can be seen from the simulation circle that Uavs 2, 5 and 8 are all on the target circle at this time. According to the triangle theorem and the circle Angle theorem, $FC = FI = CI, \angle FCI = \angle CIF = \angle IFC = 60°$, $\angle IAC = 120°, \angle AC = \angle AIC = 30°$. Uav J starts to adjust, and θ becomes larger or unchanged. Even if K increases, we cannot determine whether FY09 flies to the target position, and B becomes larger or unchanged.

If the UAV flies exactly to the target position, that is, on the circle, and keeps a uniform distance from other Uavs, then $AJ = AJ'$. According to the circular Angle criterion, the chord of $\angle ICJ'$ is $U'$, so $\angle ICJ' = 40° = \angle IAJ' = 40°$. The Angle offset range is $8°$ to $12°$, and the B Angle is also synchronously reduced by the corresponding degree, and the K value is in line with the normal situation. To sum up, UAV J has flown to the target circle.

Explain through this example: Other Uavs 1, 3, 4, 6 and 7 can also be adjusted by using the multiple target analysis method through this mathematical model, and the simulation experiment of this process is carried out through matlab, and a fitting circle with similar target circumference is obtained. Therefore, according to the above method, the adjustment scheme given by us can make 9 Uavs evenly distributed on the circle.

## 4. Conclusion

In this paper, the problem of pure azimuth passive positioning in UAV formation flight is studied, and a solution based on three-point positioning method is proposed. By analyzing the Angle formed by the two Uavs sending signals in the formation and the geometric relationship between the two Uavs receiving signals and these two Uavs, we establish the localization model and discuss the coping strategy under different measurement errors.

The research shows that the azimuth pure passive positioning method can effectively deal with the electromagnetic interference problem in UAV formation flight, and ensure the accuracy and stability of position positioning. In the process of solving the model, we consider the possible impact of measurement errors, and propose the corresponding mathematical calculation and error processing methods to ensure the reliability of the positioning results.

Future research can further explore the application under complex environmental conditions, such as the influence of wind speed changes, weather effects and other factors on the positioning accuracy. In addition, the use of more advanced sensor technologies and data processing algorithms can also be considered to further improve the efficiency and reliability of pure azimuth passive positioning in UAV formation flight.

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