

# A Comparative Study of Mathematical Modeling Teaching in Primary Schools in the United States and Singapore

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**Abstract:** The Chinese Ministry of Education's Compulsory Education Mathematics Curriculum Standards advocate model thinking as a major component of core literacy, in which teachers are to guide students to understand and apply mathematical models based on life cognition, and to develop the ability to use mathematical models to solve problems. The use of mathematical modeling thinking in primary school mathematics teaching is not only a requirement of the new curriculum standards but also improves the problem-solving ability of primary school students. There have been many international studies on mathematical modeling. This paper describes, analyzes, and summarizes the research on teaching mathematical modeling in the United States and Singapore and the implications for China.

**Keywords:** Mathematical Modeling, Primary Mathematics, Model Thinking, Model Awareness

## 1. Introduction

Education has always been an area of global focus. Educational reforms in various countries have been focused on basic education. China's Mathematics Curriculum Standards for Compulsory Education (2022 Edition) emphasizes that the mathematics curriculum should be based on core literacy and requires that "students should be able to express the real world in the language of mathematics [1]." The Mathematics Curriculum Standards (2022 edition) refine modeling ideas into model awareness and model conception. Developing students' modeling thinking is a fundamental goal and an important element of the mathematics curriculum. Model thinking is one of the disciplinary core literacy in primary mathematics, which refers to the requirement for children to be able to have a preliminary sense of mathematical models, to be able to consciously use mathematical models to solve mathematical problems, and to enhance the student's awareness of the application of mathematics and the formation of mathematical model conception [2].

In recent years, with the Chinese curriculum reform of basic education, more and more primary school mathematics teachers recognize the importance of guiding students to construct an awareness of permeable models and actively implement them in their teaching practice. In the process of implementation, there are still some problems. Therefore, it is an objective requirement of the new curriculum reform to study how teachers can cultivate students' awareness of models, as well as infiltrate mathematical modeling ideas into mathematics teaching. In the international context, there have been many studies on integrating mathematical modeling into primary school mathematics teaching. This paper compares the research on teaching mathematical modeling in primary and secondary schools in Singapore and the United States, as well as the implications of teaching mathematical modeling in the two countries for China.

## 2. Background of Mathematical Modeling Teaching in Primary and Secondary Schools in the United States and Singapore

### 2.1 Mathematics Curriculum Standards in the United States and Singapore

Countries around the world have developed curriculum standards for basic education based on the educational context of each country, such as China, which has compiled the Compulsory Education Curriculum Standards, which serve as a guide for the development of teaching materials and the teaching

of teachers. The curriculum guide outline for primary education in the United States is Common Core State Standards for Mathematics. One of the Common Core Standards for Mathematical Practice (Common Core State Standards Initiative, 2010) is Modeling Mathematics. One of the Common Core Standards for Mathematical Practice (Common Core State Standards Initiative, 2010) is Modeling Mathematics, which is required for students to master at every grade level [3]. Obviously, mathematical modeling is a link between mathematics and the actual world, a significant type of mathematical application, and a fundamental method for applying mathematics to solve practical issues. The curricular norm expects students to be able to apply their mathematical knowledge to address problems in their daily lives, and society.

Furthermore, in Singapore's Mathematics education, "Application and Modeling" was introduced in the Mathematics Syllabus for Primary and Secondary Schools, which was implemented in 2007, to help primary and secondary school Mathematics teachers effectively implement Mathematics education in primary school. In the 2013 edition of the Mathematics Syllabus, it is clearly stated that applications and modeling enable students to connect mathematics to real life, promote the understanding of important mathematical concepts and methods, and develop mathematical competence [4]. The newly revised Mathematics Syllabus for Primary Schools 2021 places greater emphasis on the development of problem-solving skills, including mathematical modeling skills. The newly revised Mathematics Syllabus for Primary Schools in 2021 attaches greater importance to the development of problem-solving skills, including mathematical modeling skills [5]. Applying mathematics to real-world problems usually involves modeling. The 2021 version of the syllabus continues to refine the "21st Century Key Competencies" which emphasize the need for students to be able to reason mathematically. Students can develop creative thinking by using different strategies to solve problems and by building mathematical models to represent real-world problems. Therefore, both the United States and Singapore believe that students should have opportunities to use their mathematical problem-solving and reasoning skills to help them solve a variety of problems, including open-ended, real-world problems. In summary, the mathematics syllabi of the two countries have a great emphasis on the penetration of mathematical modeling.

## ***2.2 Classroom Teaching of Mathematical Modeling in the United States and Singapore***

While the majority of mathematical modeling in traditional American education takes place in high school and college classrooms, some American educational researchers have argued that it is just as important for primary school students to participate in the process. There has always been a strong emphasis on science, technology, engineering, and mathematics (STEM) education in the United States, and mathematical modeling competency is a key component of STEM competency [6]. Thus, teachers integrate mathematical modeling into the STEM curriculum for difficult real-world problems, encouraging students to build models and apply them [7]. When students experienced the modeling process, it was able to strengthen their mathematical problem-solving and application skills. It is important to use mathematical modeling, a core literacy, in instruction, as well as to allow students to use their prior experiences to construct mathematical problems. Mathematical modeling also promotes students' attitudes toward mathematics as well as the integration of mathematical knowledge and practice. The study of mathematical modeling in the primary classroom is a relatively new area of research.

Singapore's mathematics textbooks have developed unconventional problems that allow students to apply their mathematical knowledge in new contexts, and these challenging unconventional problems can be considered as application problems. For example, in the lower primary grades, "Tom has 18 dollars and Tony has twice as much money. How much money do they have altogether?" ; In higher grades, "Amy, Mary, and Daisy collect stamps, Amy has 5 more stamps than Mary, Daisy has 60% of Mary's stamps, and it is also known that Daisy has half of Amy's stamps, so how many stamps do they have in total?" And so on with math problems. Furthermore, Classrooms in Singapore use a modeling approach to solving mathematical problems early on. Singapore has developed the Primary Spiral Curriculum, which uses a modeling approach to solve increasingly difficult problems. The teacher effectively uses modeling methods in math instruction based on curriculum content at different grade levels. Teachers cultivate the use of modeling methods to solve fraction problems and application problems. The program encourages teachers to use the concrete-graphic-abstract teaching sequence. For example, teachers guide students to transform word problems into modeling diagrams or letter symbols to solve problems. Singapore's CPA course promotes the concrete-graphic-abstract teaching mode, through the modeling of abstract problems into concrete, comprehensively enhancing the students' calculation, application, logical thinking, induction, and other mathematical skills.

In summary, the United States places great emphasis on students' problem-solving skills and their

ability to use math tools. Singapore promotes flexibility in the development of mathematical talents at different levels and of different types. Different teaching contents are set according to different age levels to meet the mathematical development needs of different students. Moreover, China emphasizes the penetration of modeling ideas at the basic education stage, while the United States and Singapore attach importance to the cultivation of mathematical modeling competence from an early age.

### **3. Commonalities in the Teaching of Mathematical Modeling in the United States and Singapore**

#### ***3.1 Mathematical modeling as a new approach to mathematical problem solving***

Mathematical modeling has been advocated internationally as a new direction in the study of mathematical problem-solving and is considered to be the most important goal of mathematics education. Learning to solve problems is an important direction for students in their current math learning process [8]. One of the goals of mathematics education in Singapore is to "acquire and apply mathematical concepts and skills in a variety of contexts, including unconventional, open-ended and real-world problems". Traditionally, primary school students' problem-solving skills have been developed through teachers' teaching of knowledge and students' practice of mathematical problems, and these mathematical problems have been primarily normative. Therefore, it is necessary to study how to improve students' problem-solving skills. The United States and Singapore also often conduct activities such as investigations and modeling activities as open-ended tasks to provide students with opportunities to explore, experiment, and discuss mathematical ideas. This not only enhances students' problem-solving skills but also develops their mathematical thinking.

#### ***3.2 Extremely high demands are placed on the competence of primary school math teachers***

Primary teacher preparation programs in the United States and Singapore have one or two math courses. The content of the math courses focuses primarily on geometry and measurement as well as algebra. So both countries want future math teachers to have several competencies dedicated to teaching modeling, that is, a) Theoretical competence, the mastery of modeling-related knowledge. b) Task-related competencies, which require teachers to be able to have the ability to solve, analyze, and create modeling tasks. c) Pedagogical competence, which requires teachers to be able to plan and execute modeling sessions and make appropriate interventions during student modeling. d) Evaluative abilities, where the teacher can be able to identify the stages in the modeling process of the students as well as the ability to evaluate the students [9]. These are several competencies that are extremely helpful for teachers to teach mathematical modeling, so teaching mathematical modeling is extremely demanding for mathematics teachers [10]. Moreover, This kind of modeling instruction in the primary classroom, where teachers need to encourage students to model tasks with multiple mathematical representations of real-world problems, poses a significant challenge for teachers.

#### ***3.3 Advocates the creation of realistic situations for modeling tasks***

American and Singaporean teachers will establish real situations for teaching tasks. They will design a real situation for students, throw out realistic problems, let students think independently, and go through a whole process of modeling by discovering problems, building models, solving models, hypothesizing, and verifying [11]. This type of modeling task necessitates students' ability to comprehend the problem, know how to simplify it, form the model by establishing mathematical relationships, examine the mathematical results, and modify the model to solve the problem. Students need to be competent enough to successfully solve modeling problems. This kind of modeling task appears in the teaching of primary school mathematics in the United States and Singapore, and its operation is very difficult. Not only is it demanding for teachers, but it also requires a long-term process for training students.

### **4. Implications for Mathematical Modeling Teaching in Primary Schools in China**

#### ***4.1 Teachers should emphasize the cultivation of modeling ideas in primary school students***

The United States and Singapore are extremely committed to teaching mathematical modeling at the primary school level, which also requires students to apply the mathematical knowledge they have acquired to solve problems in their daily lives. First, simplify complex problems, then use mathematical knowledge to interpret mathematical situations to arrive at results, reflect on whether the results make

sense, and finally improve the model, among other things. In Chinese mathematics teaching, although mathematical modeling is viewed as a way to develop students' ability to apply mathematics, mathematical modeling in the curriculum standard proposes that it is based on the application of modeling thinking/modeling awareness and is a penetration of thought <sup>[12]</sup>. Although many primary school teachers have begun to pay attention to mathematics, they are still not perfect enough and do not leave enough opportunities for students to participate in mathematical modeling activities, which are treated only as mathematical knowledge. In China's primary school mathematics education, the psychological characteristics of Chinese primary school students should be combined to establish methods and strategies for teaching mathematical modeling to primary school students that are suitable for China's national conditions as much as possible. The state and schools are appealing to more primary mathematics teachers to incorporate mathematical modeling and modeling thinking/modeling awareness into the instruction of primary mathematics.

#### ***4.2 Teachers should guide students to investigate math problems***

American and Singaporean teaching of mathematical modeling, their teacher-designed modeling tasks, and problem-guided approaches are well worth learning from Chinese teachers. In the process of mathematical modeling, Chinese primary school mathematics teachers pay attention to choosing appropriate problem situations to provide students with an entry point and asking appropriate questions for students to try to answer. When modeling needs to be used to explain certain mathematical concepts, the teacher should lead a class discussion to meet the course objectives. In the classroom, the teacher's role should be to be a suggestion, guide, or facilitator. Teaching modeling involves problem-solving, and problem-solving is a challenging process. If a task is not at all difficult for the student, it is not a problem, but merely an exercise. Therefore, when teachers conduct modeling instruction, they should pay attention to the following: provide students with appropriate modeling tasks, pay attention to the trade-off between difficulty and ease, as well as be able to prompt students to think independently with minimal guidance from the teacher. Teachers should encourage students to propose diverse solutions, be able to intervene in student modeling activities at the right time and guide students' problem-solving.

#### ***4.3 Teachers need to help students face the challenges of mathematical modeling***

The study of teaching mathematical modeling in primary schools in both America and Singapore shows that teaching mathematical modeling requires great theoretical support and is extremely demanding for teachers. Teachers' understanding of knowledge, design of modeling tasks, and teaching strategies have a direct impact on the effectiveness of modeling instruction. According to the developmental characteristics of Chinese primary school students, it is very important to connect math problems with the reality of life. Mathematical modeling is challenging for primary school students, and it is somewhat difficult for students to learn to deal with some complex problems at the primary school stage. Therefore, teachers should encourage students to discover examples from their lives, and when students tackle problems that make sense, they will become true problem solvers. Teachers can create real model situations and gradually increase the difficulty of tasks to promote students' thinking change. Students should recognize in the modeling process that the teacher is no longer the answer provider and that students should learn to solve difficult problems independently. Additionally, when students can formulate their problems and execute the process of mathematical modeling, not only are their problem-solving skills improved, but their logical thinking is also greatly developed.

### **5. Conclusions**

A study of the teaching of mathematical modeling in the United States and Singapore found that China is more about the penetration of model thinking/model awareness. Therefore, China should also try to guide primary school students to build real mathematical models, which is very helpful in improving students' mathematical thinking. In conclusion, the teaching of mathematical modeling is of great significance to the teaching of current primary schools, where teachers help students to build mathematical models based on students' real lives and guide them to form the ability to build models.

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