ISSN 2616-7433 Vol. 1, Issue 07: 128-140, DOI: 10.25236/FSST.2019.010716

Compilation of Different Approaches for Determining Illuminant Direction

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ABSTRACT. *In the areas of computer vision and computer graphics, analyzing information about three-dimensional objects from two-dimensional images is essential. Shape from shading is a very fundamental problem in computer vision, which uses features and patterns in images to infer the shape of the object in our viewing direction. The influence of solving this problem is significant. For example, a typical application of shape from shading is in astronomy, in which we can obtain landscape features of a planet by analyzing the information from the image we get.*

KEYWORDS*: regression model; orientation of surface; Lambertian model*

1. Introduction

Shape from shading is a very fundamental problem in computer vision, which uses features and patterns in images to infer the shape of the object in our viewing direction. The influence of solving this problem is significant. For example, a typical application of shape from shading is in astronomy, in which we can obtain landscape features of a planet by analyzing the information from the image we get[1]. However, letting the computer to achieve what human can easily do is not that simple. the properties of light source include emission direction, the color of emitted light, position and shape. The illuminant direction is one of the essential element of shape from shading. How can we find the location and direction of the light source with limited information? In the following parts of this paper, we study and compare three previous work on determining the light direction and location.

2. Determine the Illumination Direction By the Derivation of Image Intensity Function

To determine the illumination direction, Alex P. Pentland devised a computational

ISSN 2616-7433 Vol. 1, Issue 07: 128-140, DOI: 10.25236/FSST.2019.010716

theory to simulates our biological visual system[2]. In the paper Finding the Illumination Direction [1], Pentland used the first directional derivative of image intensity as input information of the computational theory because humans are more sensitive to changes of image intensity rather than absolute image intensity.

2.1 Background Information

Tilt is the angle between the projection of the surface normal on the image plane and the horizontal axis, and slant is the angle between the surface normal and the optical axis (viewing direction).The average value of a quantity is denoted by a line above the symbol of the quantity (e.g., the average value of A is represented as $\overline{A}$). All vectors are denoted by an arrow above it (e.g., $\vec{B}$).

2.2 Process of Image Generation

For this computational theory, Pentland proposed three assumptions:

1) All surfaces reflectance is the Lambertian Reflectance.

2) All lights are originated from point light sources.

3) The normal vectors of different faces of an object are isotropically distributed.

Then he developed a mathematical model (Fig 1) of image formation:

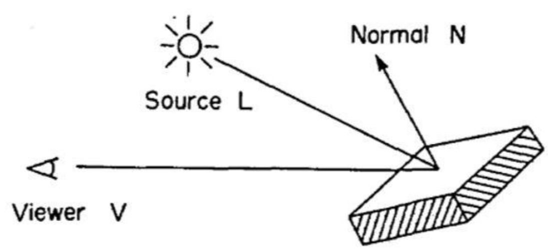

Figure.1 a model of image generation. $\vec{N}$ is surface normal. $\vec{L}$ is illumination direction. $\vec{V}$ is viewing direction.

Three components of image intensity are based on the assumptions mentioned above:

1) Illumination Component:

$$f_N = f(\vec{N} \cdot d\vec{L})$$

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where f_N is the amount of light per meter squared incident upon a patch of a surface with $\vec{N}$ and f is the flux(the amount of light) per unit area got in the adjacent area of the surface of an object from a distant point-source.

2) Surface Reflectance Component:

$$f_R = \rho f_N R(\vec{N},\vec{L},\vec{V}) = \rho f(\vec{N}\cdot\vec{L})$$

In the surface reflectance component, ρ, albedo, is the portion of the amount of light that is reflected, $R(\vec{N},\vec{L},\vec{V}) = R(\vec{N}\cdot\vec{L})$ is the reflectance function which is directly proportional to the cosinebetween the $\vec{N}$ and $\vec{L}$.

3) Projection Component:

$$dA_s = (N\cdot V)^{-1} dA_I$$

In the projection component, dA_s is the projection of the infinitesimal area dA_I which has a foreshortening effect related to the cosine between the $\vec{N}$ and $\vec{L}$.

Based on components mentioned above, the paper obtains the image intensity:

$$I = \rho f(\vec{N}\cdot\vec{L})$$

Although there are many variables, the paper assumes $\vec{L}$, ρ, and $\vec{f}$ are constant, focusing on a relatively small patch of area, so changes in image intensity are only related to $\vec{N}$ and the first derivative of image intensity is:

$$dI = \rho f(d\vec{N}\cdot\vec{L}+\vec{N}+d\vec{L}) = \rho f(d\vec{N}\cdot\vec{L})$$

2.3 Estimation of Illumination Direction from Its Effect

Based on Pentland's previous assumptions, the changes in $\vec{N}$ are also isotropically distributed. In this case, the sum of changesin $\vec{N}$ is zero. Let $\vec{L} = (x_L, y_L, z_L)$, $d\vec{N} = (dx_N, dy_N, dz_N)$. Thus, the function of the expected value of the derivative of image intensity is:

$$E(dI) = k(d\bar{x}_N x_L + d\bar{y}_N y_L)$$

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Where k is a constant generated by ρ, illumination strength, and the variance of $d\vec{N}$ within that region. we can construct a linear regression between the average of dI and the x and y component of the illumination direction. Later, k could be evaluated from the average of dI and the variance of the distribution of dI, which could be used to determine the slant of the illumination vector. Pentland then introduces the differential term dr which could be considered as $d\vec{N}$ so that $xdr = d\bar{x}_N$ and $ydr = d\bar{y}_N$.

Letting dI be the mean value of dI over the region along image direction, the regression can be constructed.

$$\begin{bmatrix} d\vec{I}_1 \\ d\vec{I}_2 \\ \vdots \\ d\vec{I}_n \end{bmatrix} = \begin{bmatrix} dx_1 dy_1 \\ dx_2 dy_1 \\ \vdots \quad \vdots \\ dx_n dy_1 \end{bmatrix} \begin{bmatrix} \hat{x}_L \\ \hat{y}_L \end{bmatrix}$$

From this, τ could be determined as

$$\tau = \tan^{-1}\left(\frac{\hat{y}_L}{\hat{x}_L}\right)$$

The calculation of k needs the variance of dI, which can be estimated by least-square regression, so the paper again sets $Var(dx_N) = Var(dy_N) = Var(dz_N) = dr^2$ thereby obtaining the variance:

$$\begin{aligned} E(dI^2) &= \rho^2 f^2[(xx_L + yy_L)^2 dr^2 + (x_L^2 + y_L^2 + z_L^2)dr^2] \\ &= \rho^2 f^2[(xx_L + yy_L)^2 dr^2 + dr^2] \end{aligned}$$

So we can get $E(dI)^2 - E(dI^2) = \rho^2 f^2 dr^2$

Where $k = \rho f dr = [E(dI^2) - E(dI)^2]^{1/2}$.

Since the vector of illumination direction is a unit vector, we can get the z component

$$z_L = [1 - \frac{(\hat{x}_L^2 + \hat{y}_L^2)}{2}]^{1/2}$$

Because the arccosine of the z component of the normal vector is slant,

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$$\sigma_L = \cos^{-1}[(1-\frac{(\hat{x}_L^2+\hat{y}_L^2)}{k^2})]^{1/2}$$

However, this algorithm may not yield an accurate result, particularly when applied on planar and cylindrical surfaces, because of either the derivative of $\vec{N}$ is zero or $d\vec{N}$ are in one direction.

3. Determine the Illumination Direction by Surface Orientation and Tilt-Slant Technique

3.1 Introduction

This method first uses a statistical approach to determine the orientation of the surface (by Witkin [4]), then calculates the tilt and slant of the illumination source (Lee & Rosenfield [5]).

3.2 Estimating Surface Orientation

Projected quantities will be denoted by the same letter as their original counterpart with an asterisk as a superscript. Two planes are introduced: the surface plane and the image plane, denoted as S and I respectively. A curve in S, called a contour generator, will be denoted as a function $C(s)$ The normal vectors of S and I are denoted as N_S and N_I.

In order to determine the orientation of S with respect to I, two angles σ and τ (for slant and tilt) are used, with σ being the angle between I and S, and τ being the angle between the projection of N_S onto I, and the x-axis in I. The angle between $C(s)$'s tangent and a fixed coordinate axis in the surface plane S will be denoted as β, shown in Figure 2.

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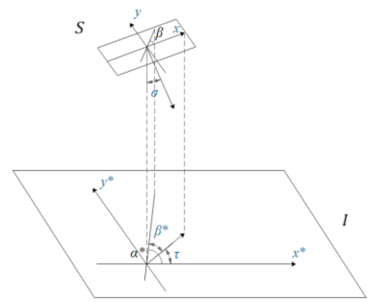

Figure. 2 A diagram showing the angles σ , τ , β , and α .

This model uses a statistical method to determine the most possible orientation of the surface, instead of trying to model the texture or assume that the texture of the surface is simple, for natural textures are too complicated and diversed to be modelled accurately.

Here, we use the probability density function (pdf) to estimate the most possible solution for σ and τ .

3.2.1Deduction of the Theorem

Based on the definition of tilt and slant, the process of projection can be modelled reversely as a rotation from I by the angles of (σ,τ) to S .

Therefore, the tangent vector $t=[\cos\beta \ \sin\beta]$ for $C(s)$ will be projected to the vector $t^*=[\cos\beta\cos\sigma \ \ \sin\beta]$. Therefore, the projected tangent angle will be

$$\beta^* = \tan^{-1}\frac{\tan\beta}{\cos\sigma}$$

To reintroduce τ , we define a^* as the angle between the projected tangent line and the x-axis of the image plane, as shown in Figure 2. Thus,

$$a^* = \beta^* + \tau = \tan^{-1}\frac{\tan\beta}{\cos\sigma} + \tau$$

According to the projecting rules, N_I can be achieved by rotating N_S by (σ,τ). We can model all possible orientations of N_S by creating a sphere, called the Gaussian Sphere. The set containing the orientation of all normal vectors every patch of surface contains all possibilities of N_S. Moreover, every normal vector can be represented as

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rotating a predefined normal vector by (σ, τ), as shown in Figure 3.

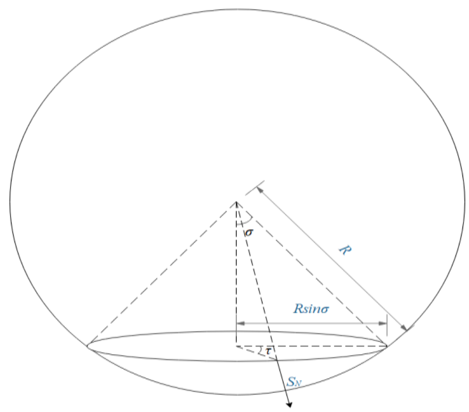

Figure. 3 A diagram showing one possible representation method of one of the normal vectors, along with the Gaussian Sphere.

We assume that the possibility of N_S lying on every possible direction of a normal vector on the Gaussian sphere is equivalent. By considering the relationships between those quantities, we have:

$$\mathrm{pdf}(\beta, \sigma, \tau) = \frac{1}{\pi} \cdot \frac{1}{\pi} \cdot \sin\sigma = \frac{\sin\sigma}{\pi^2}$$

By substituting and integrating, we can deduce that:

$$\mathrm{pdf}(\sigma, \tau | A^*) = \prod_{i=1}^{n} \frac{\pi^{-2} \sin\sigma}{\cos^2(a_i^* - \tau) + \sin^2(a_i^* - \tau)\cos\sigma}$$

Detailed deduction process can be found in [3].

3.3 Estimating the Illuminant Direction

The slant and tilt of the surface are denoted as σ and τ, while the slant and tilt of the illumination direction are denoted as σ_L and τ_L. The expectation of some value is denoted with a function $E\{\}$ (e.g., $E\{A\}$).

In the image process, there are three distinguished directions: the viewing direction, the illumination direction, and the surface normal. We choose to establish a coordinate

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system represented as S whose axes are denoted as x', y', and z', where

$$x' = (\cos\sigma_L \cos\tau_L, \cos\sigma_L \sin\tau_L, -\sin\sigma_L)$$
$$y' = (-\sin\tau_L, \cos\tau_L, 0)$$
$$z' = (\sin\sigma_L \cos\tau_L, \sin\sigma_L \sin\tau_L, \cos\sigma_L)$$

for z' is the illumination direction, which is also denoted as L; x' has the same tilt as z'; $x'y'$ is a plane that is perpendicular to z'. x^{-}, y^{-}, and z^{-} axes, on the other hand, are used to denote the image-based Cartesian coordinate system, where the image plane is the plane $z = 0$. We will call the plane $x'y'$ the S plane and xy the V plane, as shown in Figure 5.

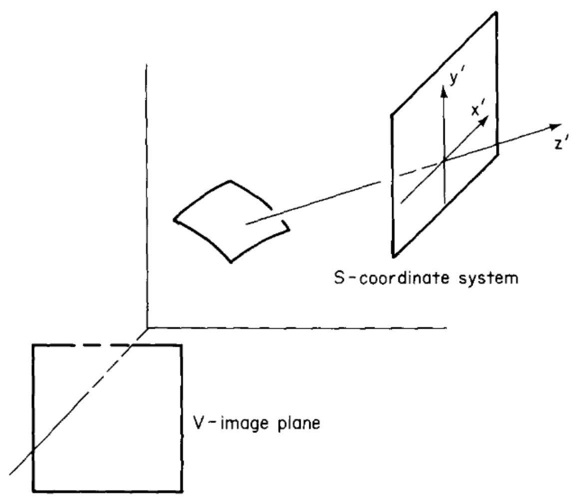

Figure. 4 A diagram showing the V -coordinate system and S -coordinate system, along with axes x', y', and z'.

The rotation from V to S, expressed in the form of matrix multiplication is

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\sigma_L \cos\tau_L & \cos\sigma_L \sin\tau_L & -\sin\sigma_L \\ -\sin\tau_L & \cos\tau_L & 0 \\ \sin\sigma_L \cos\tau_L & \sin\sigma_L \cos\tau_L & \cos\sigma_L \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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3.4 Estimating the Illumination Direction Using τ and σ of the Surface Plane

1) Theorem 1

The tilt τ_L of the illumination direction is $\tan^{-1}\dfrac{E\{I_y\}}{E\{I_x\}}$, where the expectation is taken over the given image region.

2) Theorem

The slant σ_L of the illumination direction satisfies

$$E\{I\} = \frac{\lambda\rho(\pi\cos\sigma_L - \sigma_L\cos\sigma_L + \sin\sigma_L)}{2(1+\cos\sigma_L)}$$

$$E\{I^2\} = \frac{\lambda^2\rho^2}{3}(1+\cos\sigma_L)$$

Where the expectation is taken over all the points with the same tilt as the illumination direction.

3) Theorem 3

The slant of the illumination direction also satisfies:

$$E\{I\} = \frac{4\lambda\rho(\sin\sigma_L - \sigma_L\cos\sigma_L + \pi\cos\sigma_L)}{3(1+\cos\sigma_L)}$$

where the expectation is taken over the whole image.

Theorem 1focuses on estimating the tilt; Theorem 2 points out the first and second moments of image intensity over those surface points having the same tilt as that of the light direction are related to the albedo and the slant of the light source direction. Theorem 3point out a two-dimensional analysis which led to a formula describing the first moment of the intensity over the image region in terms of the slant of the illumination direction. Unlike Theorem 2, Theorem 3 does not rely on points that need to be identified whether the tilt of the point is the same as the illumination direction.

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4. Determine the Illumination Direction from a Single View Without the Light Distance Assumption

4.1 Background Information

Generally, the main idea of determining illumination position is to estimate the surface reflectance parameter. With the light source's position(distance and tilts), the reflectance parameter can be estimated by separating the specular reflection components and Lambertian fitting algorithm. After this, we can determine the light source direction in the vector diagram. More details could be seen from[6].However, if light source conditions are unclear, for example, reflectance model of indoor environments, the fitting algorithm is not executable. Approach 3 uses a smart way to estimate the light source location by applying the Phong model. In this approach, assumptions are given that:

1) The camera projection matrix is given.

2) In the scene, there is only one illumination source.

3) This method has to be given with at least one specular peak.

4) This model is given explicitly as a linear combination of the Lambertian diffuse model and Torrance sparrow model [7]

5)The 2D image pixel intensity have already been given with specular components(so we understand where we locate the specular peak)[7]

4.2 Light Source Positions

4.2.1 Phong Model

Phong Model is also an idealistic model which is formed under a specular light condition. This model indicates that the incidence angle is equal to the reflection angle[8], and the unit vector of specular light L_p is given as:

$$L_P = 2(N_T^P L_P)N_p - L_p$$

Where N_p means the normal vector of P, reflection point. The equation could be illustrated as the diagram below:

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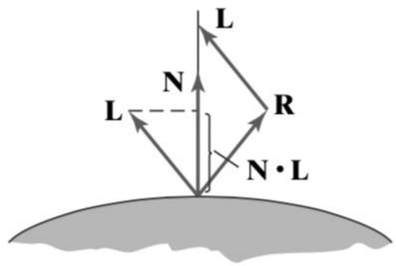

Figure. 5 Phong model [8]

where the projection of neither L or R onto the direction of the normal vector N has a magnitude equal to $N \cdot T$. Finally, we could explain the equation by translating L to R.

4.2.2 The Location Vector of the Light Source

By knowing the unit vector of the light source, the location vector of the light source is given as [8]:

$$L = P + tL_p$$

Where t is the distance between light source and object position and L_p is the unit vector of specular light. Also, P is the vector location of the object ,which could be calculated by the camera projection matrix. In the next section, t value could be estimated by 2 steps.

4.3 Light Distance Estimation

4.3.1 Diffuse Region Extraction

To find the diffuse region, diffuse region and specular region are needed to be separated individually. The diffuse region is given as:

$$\Omega_d = \{(i, j) \in \Omega \mid (i - i_p)^2 + (j - j_p)^2 > T^2\}$$

where (i, j) is the points of 2D image of the object surface and T is a positive integer related to the radius of the specular region. In this method of estimation of illumination directions, it is hard to find the most appropriate value of T because we are not able to find the roughness of a certain texture[6], but this method uses an alternative method to find the light distance(t). As described, we simultaneously estimate the diffuse and

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specular reflection components in estimating the 2D image value by iterative relaxation scheme. The final results do not relate to the T value.

4.3.2 Range of t Estimated by Diffuse-Based Algorithm

By applying the diffuse term of pixel value into the diffuse region, we have the equation shown below:

$$t^* = \arg\min \sum_{(i,j)\in\Omega_d} \left[u(i,j,t) - \frac{1}{N_p} \sum_{(i,j)\in\Omega_d} u(i,j,t) \right]^2$$

The equation is written in non-homogeneous coordinates because 2D image coordinates are converted into 3D image coordinates and u is a set of 3D coordinates which is converted from the surface of 2D coordinates by adding t component. The domain of u is given as below:

$$u(i,j,t) = \frac{I(i,j)r(i,j,t)2}{\cos\theta_i(i,j,t)}$$

Where the $I(i,j)$ is the 2D pixel value, $r(i,j,t)$ is the distance between light source and image surface, and $\cos\theta_i(i,j,t)$ is the surface incident angle.

Analytically speaking, it is hard to estimate the t value, so this paper manually searches the source distance in the limited range. Users could define $t_{\min}$, $t_{\max}$, and Δt The interval t should be chosen comparable to or larger than the sampling interval (spatial resolution) R of the input image (t=R^1). After t value have been estimated, the light reflectance parameter could be calculated according to t^* .Finally, the location vector of the light source is determined.

5. Conclusion

In this paper, we have compiled three different approaches in estimating the illumination directions; the first approach is designed to estimate the degrees of tilts and slants by applying Lambertian reflection model, besides, the essential part in approach 1 is to calculate the image intensity by three different illumination components. The second approach could be regarded as the extensions of approach 1. It could be done from a real image and 3D geometric model of the object by estimating reflectance parameters of the object; this method is based on the use of the iterative separating-and-fitting algorithm. Taking a specular component image as input, the specular reflection parameters and the light source positions are estimated simultaneously by linearizing the Torrance-Sparrow specular reflection. Similarly, the third approach also utilizes the tilt-slant technique. Approach 3 first uses a statistical method to determine the orientation of

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the surface; then, a method of determining the slant and tilt of the illumination source is given.

Scientific reference has approved the rationality of three approaches. However, there are still some limitations of those approaches. For instances, under multiple light sources conditions, the light intensity equation is not available because of numerous specular peaks. In the later parts of Finding the Illuminant Direction. The user-defined t value in approach two sound might be restrictive, but there are no references related to t value specifically until now. For the last approach, the statistical model is best applied on planar surfaces; curved surfaces requires more computational power and special case rejections. Breaking those barriers are left out as challeges of the future.

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