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Tips for solving plane geometry problems in middle school math

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Abstract: *Plane geometry is a crucial element in the teaching of junior high school mathematics, and it is also a compulsory element in the examination. Plane geometry topics are diverse and flexible, and in the process of solving the problems, students are required to master the basic knowledge but also require students to be familiar with and be able to flexibly utilize the problem-solving skills to quickly answer the questions and improve the efficiency of solving the problems. This paper analyzes the problem-solving skills of plane geometry students in junior high school mathematics.*

Keywords: *Middle school familiarization; Plane geometry; Problem solving skills*

1. Introduction

In middle school, plane geometry involves lines and angles, quadrilaterals, triangles, circles and so on. Therefore, these questions mainly test students' familiarity with geometry, ability to comprehensively apply their knowledge of geometry, and mastery of the close connection between various knowledge points. Familiarity with and mastery of plane geometry problem-solving skills can help improve students' comprehensive ability to use geometric knowledge, which helps solve this kind of math problem. In this article, we will explain standard problem-solving techniques with practical examples to deepen students' understanding and mastery of the techniques.

2. Solving problems using categorical discussion methods

The idea of "classification and discussion" refers to distinguishing the mathematical objects under study into different kinds according to specific essential characteristics and then examining them separately. In plane geometry problem-solving in junior high school, we can combine the meaning of the problem with the way of classification and discussion and then solve the problem by considering it comprehensively. The reform and development of quality education so that the junior high school mathematics teaching system gradually transitions to quality education, classified discussion of ideas to help improve the rigour of students' mathematical thinking and mathematical knowledge of the practical, to help students to generalize and summarize the knowledge they have learned.[1]

Example 1 An isosceles triangle has always had two sides of length m , n , if m , n satisfy $|m-3|+(n-5)^2=0$, then his area is.

Ans. Based on the known conditions in the question, you can determine $m=3$ and $n=5$, but you cannot determine the waist length of the isosceles triangle, so you need to classify and discuss while solving the problem.

(1) When 3 is the waist length, then 5 is the bottom edge length.

$\therefore$ The height of an isosceles triangle: $\sqrt{3^2-2.5^2}=\dfrac{\sqrt{11}}{2}$.

$\therefore$ The area is $\dfrac{1}{2}\times\dfrac{\sqrt{11}}{2}\times 5=\dfrac{5\sqrt{11}}{4}$.

(2) When 5 is the length of the waist, then 3 is the length of the bottom edge.

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$\therefore$ The height of an isosceles triangle: $\sqrt{5^2 - 1.5^2} = \frac{\sqrt{91}}{2}$.

$\therefore$ The area is $\frac{1}{2} \times \frac{\sqrt{91}}{2} \times 3 = \frac{3\sqrt{91}}{4}$.

Therefore, the area of this isosceles triangle is $\frac{5\sqrt{11}}{4}$ or $\frac{3\sqrt{91}}{4}$.

Students can classify the shapes based on their characteristics when it comes to different types of geometric shapes. For example, triangle problems can be classified according to factors such as side lengths, angle sizes, shapes, etc. This problem requires to organize the side lengths of isosceles triangles. Such categorization can help students better understand the problem and find an appropriate solution.

In addition to this, the categorical discussion idea can also be used when solving problems of calculating line segments and angles, as shown in the following example:

Example 2 It is known that the points A , B , C are on the same line, the line segments $AB = 6cm$, $BC = 4cm$,the point E is the midpoint of the line segment AB , the point F is the midpoint of BC , then the length of EF is.

The idea of categorical discussion can still be used in this question. Since we are not sure where the point C is located, i.e., whether the point C is to the left or to the right of the point B , we can categorize it.

If the point C is located to the left of the point B , the image is as shown in figure 1:

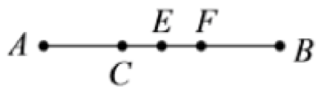

Figure 1: Axis plot for the first case

Then the length of EF can be expressed as $EF = BE - BF = \frac{1}{2}AB - \frac{1}{2}BC = \frac{1}{2} \times 6cm - \frac{1}{2} \times 4cm = 1cm$.

If the point C is located to the left of the point B , the image is as shown in figure 2:

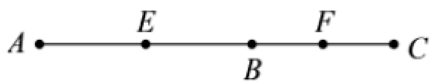

Figure 2: Axis plot for the second case

Then the length of EF can be expressed as $EF = BE + BF = \frac{1}{2}AB + \frac{1}{2}BC = \frac{1}{2} \times 6cm + \frac{1}{2} \times 4cm = 5cm$.

So the answer is 1cm or 5cm.

3. Using symbolic-graphic combination to answer questions

Symbolic-graphic combination is to combine abstract mathematical language and quantitative relations with intuitive geometric figures and positional relations. Through the combination of abstract thinking and image thinking, complex problems can be simplified, and conceptual issues can be concretized to optimize the way to solve problems. At the same time, the idea of combining numbers and shapes is a bridge to use algebraic methods to solve geometric problems in junior high school mathematics[2], which can exercise students' abstract logical thinking, enable students to solve practical

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problems efficiently, and form good problem-solving habits and thinking skills.

In general, symbolic-graphic combination is mainly used to solve number problems, but the idea can sometimes be used in geometric issues; for example, when calculating area and volume, you can combine algebra and geometry to derive formulas for the area and volume of various shapes.

Example 3 As shown in the figure, the three sides of the right triangle ABC are known to be 6, 8, 10, respectively, and three semicircles are made upwards with its three sides as diameters, find the area of the shaded part of the figure 3.

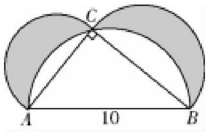

Figure 3: Pictures for example three

Ans The key point of this question is how to express the area of the shaded portion as the area of a shape that is familiar to the student, as we find by looking at the graph:

First you can find the semicircular surface made with two right-angled edges as diameters

Product: $S_1 = \frac{1}{2}\pi \times 3^2 + \frac{1}{2}\pi \times 4^2 = \frac{25}{2}\pi$.

Then find the area of the unshaded part of the two semicircles, i.e., the area of the semicircle with the hypotenuse as its diameter minus the area of the triangle: $S_2 = \frac{1}{2}\pi \times 5^2 - \frac{1}{2} \times 6 \times 8 = \frac{25}{2}\pi - 24$.

Then the area of the shaded part of the figure $S = S_1 - S_2 = 24$.

In addition, the idea of combining numbers and shapes can also be used in English to solve similar triangles, where students can use proportionality and algebraic expressions to prove the similarity of shapes in order to achieve twice the result with half the effort.

Example 4 As shown in the figure 4, PB is the tangent to⊙O the tangent line of B , the point is the tangent point, the line PO to⊙O at the point E , F ,through the point B for PO the vertical line BA , the foot of the vertical line is the point, intersection with the point, extend with the point, the vertical line is the tangent line to the point. D ⊙O with the point A , extend AO and⊙O Intersect BC with the point C , connect, AF .

(1) Prove that the line PA is⊙O tangent to the line;

(2) Try to investigate the equivalence between the line segments EF , OD , OP and prove it;

(3) If $BC = 6$, $\tan \angle F = \frac{1}{2}$, find the value of $\cos \angle ACB$ and the length of the line PE .

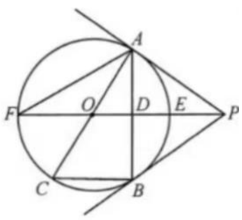

Figure 4: Pictures for example four

(1) This question mainly utilizes the properties of congruent triangles in combination with the determination theorem of tangent lines to get the conclusion.

PROOF: As shown in figure 5, connect OB .

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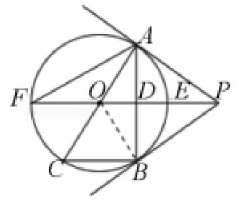

Figure 5: Make an auxiliary line to the picture in example four

$\because$ PB is the⊙O the tangent of $\therefore$ $\angle PBO = 90^\circ$.

$\because$ $OA = OB$, $BA \perp PO$ at point D.

$\therefore$ $AD = BD$, $\angle POA = \angle POB$.

Also $\because$ $PO = PO$, $\therefore$ $\Delta PAO \cong \Delta PBO$.

$\therefore$ $\angle PAO = \angle PBO = 90^\circ$. $\therefore$ $OA \perp PA$.

$\therefore$ The line PA is⊙O tangent to the line.

(1) This question mainly uses the properties of similar triangles to establish the relationship between the lengths of different side lengths, using the idea of combining numbers and shapes to investigate the equivalence between the line segments EF, OD, OP.

$EF^2 = 4OD \cdot OP$. Proof: $\because$ $\angle PAO = \angle PDA = 90^\circ$,.

$\therefore$ $\angle OAD + \angle AOD = 90^\circ$, $\angle OPA + \angle AOP = 90^\circ$.

$\therefore$ $\angle OAD = \angle OPA$. $\therefore$ $\Delta OAD \sim \Delta OPA$. $\therefore$ $\dfrac{OD}{OA} = \dfrac{OA}{OP}$, i.e. $OA^2 = OD \cdot OP$.

Also $\because$ $EF = 2OA$, $\therefore$ $EF^2 = 4OD \cdot OP$.

(2) The answer to this question can be obtained by combining the idea of combining numbers and shapes with knowledge of the median, trigonometric functions, and the Pythagorean theorem.

$\because$ $OA = OC$, $AD = BD$, $BC = 6$, $\therefore$ $OD = \dfrac{1}{2}BC = 3$ (triangle median theorem).

Set $AD = x$, $\because$ $\tan \angle F = \dfrac{1}{2}$, $\therefore$ $FD = 2x$, $OA = OF = 2x - 3$.

In $Rt\Delta AOD$, from the Pythagorean Theorem, we have $(2x-3)^2 = x^2 + 3^2$, which is solved by $x_1 = 4$, $x_2 = 0$ (not relevant, discarded). $\therefore$ $AD = 4$, $OA = 2x - 3 = 5$. $\because$ AC is the diameter of⊙O the diameter of. $\therefore$ $\angle ABC = 90^\circ$

Also $\because$ $AC = 2OA = 10$, $BC = 6$, $\therefore$ $\cos \angle ACB = \dfrac{6}{10} = \dfrac{3}{5}$.

$\because$ $OA^2 = OD \cdot OP$, $\therefore$ $3(PE = 5) = 25$. $\therefore$ $PE = \dfrac{10}{3}$.

4. Answering questions using the idea of reduction

Regression is a way of thinking about problem-solving that transforms a more complex problem into a simpler or more familiar one. In solving geometry problems, reduction usually refers to changing a problematic geometry problem into a known or relatively easy geometry problem so that the answer can be found more easily. The idea of reduction helps students make connections between prior experience and abstract knowledge to apply appropriate mathematical methods and mathematical

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thinking to solve problems. This method of thinking requires students to master the principles of simplicity, concreteness, harmony, familiarity and formal standardization[3].

Example 5 As shown in the figure 6, it is known that the length of the side of the rhombus $ABCD$ is 4, $\angle ABC = 60°$, the point N is the midpoint of BC, and the point M is a point on the diagonal AC, then the minimum value of $MB + MN$ is.

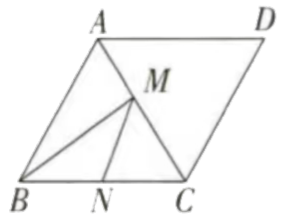

Figure 6: Pictures for example five

In this question, we have to use the idea of reduction, the problem will be transformed into a convenient form for students to observe, in this question to combine the knowledge of the "shortest straight line distance between two points", first according to the meaning of the transformation of the location of the point N, and then find out the shortest straight line distance between the point B and the point N' after the transformation.

Make a point of symmetry of point N about line AC on Figure 7.

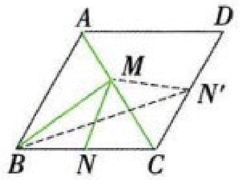

Figure 7: Make auxiliary lines to the picture in example five

$\because$ The quadrilateral $ABCD$ is a rhombus, $\angle ABC = 60°$, the

$\therefore AB = BC$,

$\therefore \angle ACB = \angle ACD = 60°$ (math.) genus

$\therefore$ Tap N' on DC $N'C = NC = \frac{1}{2}BC = 2$.

Connections MN', BN'.

$\therefore MB + MN = MB + MN' \geq BN'$ (math.) genus

When, B M N' three points of the common line, $MB + MN$ to obtain the minimum value, the minimum value is the length of BN', through the point N' for $N'E \perp BC$ to BC the extension of the line at the point E.

$\because \angle N'CE = 180° - \angle BCD = 60°$,

$\therefore CE = \frac{1}{2}CN' = 1$,, $N'E = \frac{\sqrt{3}}{2}CN' = \sqrt{3}$

$\therefore BE = BC + CE = 5$ (math.) genus

$\therefore BN' = \sqrt{BE^2 + N'E^2} = \sqrt{5^2 + \left(\sqrt{3}\right)^2} = 2\sqrt{7}$,

$\therefore MB + MN$ The minimum value is $2\sqrt{7}$.

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The above question combines the idea of reduction with the two-point-one-line model, using the symmetry property of geometric shapes to make geometric deformations and transform the problem into an equivalent but easier-to-solve form. In addition, the idea of reduction can be linked to other knowledge to solve the problem, such as the use of similar triangles: when encountering geometric problems involving triangles, especially when it comes to proportionality, you can consider using the properties of similar triangles to simplify the problem, by proving that the two triangles are identical to each other, you can get the proportionality of the length of the corresponding side, to solve the problem.

The auxiliary construction of angles is also a common point in geometry questions on the midterm. Sometimes, problems can be made easier to handle by introducing additional line segments, points or angles, known as an auxiliary construction. By skillfully introducing new geometric elements, the structure of a problem can be changed to make it more solvable.

Example 6 In ΔABC , $\angle ABC = 90°$, $\frac{AB}{BC} = n$, M are points on BC , connect AM .

(1)As in figure 8(1), if $n = 1$, N is a point on the extension line of AB , and CN is perpendicular to AM , show that $BM = BN$.

(2)Through the point B make $BP \perp AM$, P is the pendant, connect CP and extend to intersect AB with the point Q .

(i) As in figure 8(2), if $n = 1$, find: $\frac{CP}{PQ} = \frac{BM}{BQ}$.

(ii) As in figure 8(3), if M is the midpoint of BC , write the value of $\tan \angle BPQ$ directly.

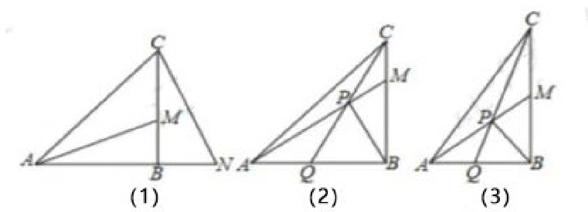

Figure 8: Pictures for example six

Proof:(1) Extend AM to intersect CN with the point H .

$\because AM$ Perpendicular to CN , $\angle ABC = 90°$.

$\therefore \angle BAM + \angle N = 90°$,, $\angle BCM + \angle N = 90°$

$\therefore \angle BAM = \angle BCN$.

$\because n = 1$, $\angle ABC = 90°$.

$\therefore AB = BC$, $\angle ABC = \angle CBN$.

$\therefore \Delta ABM \cong \Delta CBN$,

$\therefore BM = BN$.

(2)① Prove that, as shown in the figure 9, the point C is crossed by $CD // BP$,and the extension line of AB is crossed at the point D , then AM is perpendicular to CD .

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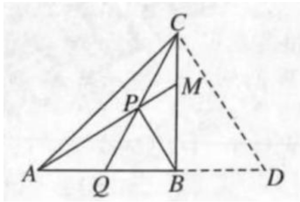

Figure 9: Make an auxiliary line to the picture in the first part of the second question of example 6

Rooted in question (1) yields $BM = BD$.

$\because CD \parallel BP$, $\therefore \dfrac{CP}{PQ} = \dfrac{DB}{BQ}$, i.e. $\dfrac{CP}{PQ} = \dfrac{BM}{BQ}$.

② $\dfrac{1}{n}$. As shown in the figure 10, cross the point C as $CN \parallel BP$, and intersect the extension line of AM at the point N.

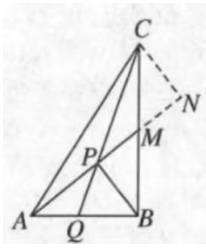

Figure 10: Make an auxiliary line to the picture in the second part of the second question of example 6

Then, $\angle BPQ = \angle NCP$ $\angle CNM = \angle BPM = 90°$.

Also, $\because CM = BM$ $\angle BMP = \angle CMN$.

$\therefore \Delta BPM \cong \Delta CNM$ (math.) genus

$\therefore PM = NM$.

Available from, $\Delta ABM \sim \Delta CNM$

$\therefore \dfrac{BM}{MN} = \dfrac{AB}{CN}$,

$\therefore \dfrac{2BM}{AB} = \dfrac{2MN}{CN}$,

i.e.,, $\dfrac{BC}{AB} = \dfrac{PN}{CN}$

$\therefore \tan\angle BAC = \tan\angle NCP$,

$\therefore \tan\angle BPQ = \tan\angle NCP = \tan\angle BAC = \dfrac{BC}{AB} = \dfrac{1}{n}$.

5. Solving Problems Using Equation Thinking

In middle school math, geometry problems can also be solved using equations. For the middle school geometry content involved in some line segments, angles and area problems, equations can be more easily and quickly solved. The critical step in solving these problems is to find the equivalence relationship and then make equations, that is, according to the meaning of the problem and the

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relationship between the graphs, to find out the solution to the problem and the known conditions implied by the equivalence relationship between the equations and the establishment of equations or systems of equations. [4]By constructing equations to solve problems, students can transform geometric problems into equation-solving problems, which can help students turn abstract problems into simple computational problems but also help students master new ideas.

Example 7 As shown in the figure 11, in the isosceles ΔABC, $\angle B = 90°$, $AB = BC = 8cm$, the moving point P starts from the point A and moves along AB toward the point B as, $PQ \parallel AC$ $PR \parallel BC$,when the area of the quadrilateral $PQCR$ is half of the area of ΔABC, the distance moved by the point P is.

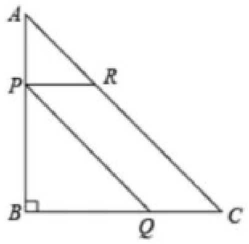

Figure 11: Pictures for example seven

Since the area of the quadrilateral $PQCR$ and the area of ΔABC change with the position of the point P, we can choose a reference point A to express the change of the position of the point P, set $AP = xcm$, and use the length of AP to express the area of the quadrilateral $PQCR$ and the area of ΔABC. Finally, combine the meaning of the question "the area of the quadrilateral $PQCR$ is half of the area of ΔABC " to establish the expression of the equation.

Let be $AP = xcm$ $PB = (8 - x)cm$, then

$\because$ $\angle B = 90°$,, $AB = BC = 8cm$

$\therefore$ $\angle A = 45°$,

$\because$ $PR \parallel BC$ (math.) genus

$\therefore$ $\angle APR = 90°$ (math.) genus

$\therefore$ $PR = PA = xcm$ (math.) genus

$\because$ The area of the quadrilateral $PQCR$ is half the area of ΔABC.

$\therefore$ $x \cdot (8 - x) = \frac{1}{2} \times \frac{1}{2} \times 8 \times 8$ (math.) genus

Solve for: $x_1 = x_2 = 4$, and

$\therefore$ The point P traveled the distance $4cm$.

If the skill is summarized, the main steps to solve the problem by using equations are: firstly, you need to read the problem carefully, figure out the given information and the unknowns required by the problem, and at the same time, understand the nature of the geometric shapes and relationships; secondly, choose the appropriate variables to represent the unknowns in the problem, and usually you can use the letters, such as x, y, etc., to describe the length, angle, etc.; after determining the unknowns, according to the problem, we will use the algebraic equations to find the values of the unknowns. After deciding the unknowns, based on the geometric relationships given in the problem, algebraic equations are established. In junior high school knowledge, the theorem of hook and strand, the property of similar triangles, the property of parallel lines, and other knowledge points are often used. Finally, the equations are solved to find the values of the unknowns, mainly by using algebraic methods, which may use the techniques of combining terms, shifting terms, factorization, and so on.

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Equational thinking helps translate geometric problems into algebraic problems, making problem-solving more systematic and precise.

6. Problem-solving using reverse thinking

Reverse thinking is a way of thinking that starts from the result or goal, analyzes the problem in reverse and looks for solutions. It is the opposite of traditional forward-thinking, which usually begins from known conditions and deduces conclusions step by step. Conversely, reverse thinking starts from a desired outcome or goal and traces the possible ways to reach that outcome in reverse.

Reverse thinking in mathematics can help students deepen mathematical concepts, improve problem-solving efficiency, enhance their creative ability, and help them solve complex problems. Students can apply this thinking to problem-solving strategies and use it appropriately in simple arithmetic to improve computational efficiency[5].

Example 8 As shown in the figure 12, E, F are two moving points on the hypotenuse BC of the isosceles, $Rt\Delta ABC$ $\angle EAF = 45°$ $CD \perp BC$ and $CD = BE$.

(1) Seeking: $\Delta ABE \cong \Delta ACD$

(2) Ask for proof: $EF^2 = BE^2 + CF^2$

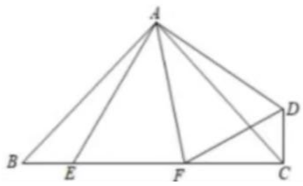

Figure 12: Pictures for example eight

(1) According to the meaning of the question, it is relatively easy to find that ΔABE and ΔACD have two sides that are equal, i.e. $AB = AC$ and $CD = BE$. The key to this question is to use reverse thinking to find the key between $\angle B$ and $\angle ACD$, i.e. from the condition that $CD \perp BC$ and ΔABC are isosceles right triangles, to discover the relationship between $\angle B = \angle ACD$.

$\because$ ΔABC is an isosceles right triangle.

$\therefore$ $AB = AC$, $\angle BAC = 90°$.

$\therefore$ $\angle ABC = \angle ACB = 45°$.

$\because$ $CD \perp BC$,, $\because$ $\ell DCB = 90°$

$\therefore$ $\angle DCA = 90° - 45° = 45 = \angle ABE$.

In ΔABE and ΔACD $\begin{cases} AB = AC, \\ \angle ABE = \angle ACD, \\ BE = CD, \end{cases}$

$\therefore$ $\Delta ABE \cong \Delta ACD(SAS)$.

(2) This question also requires students to use reverse thinking to solve the problem. First of all, according to the conclusion of problem (1) to get the relationship between two sides and two angles: $AE = AD$ and $\angle BAE = \angle CAD$, and then according to the conditions of the topic to get $\angle EAF = \angle DAF$, according to the conditions of the proof of similar triangles can be obtained to conclude $\Delta AEF \cong \Delta ADF$. Finally, the proof can be completed by combining the condition of equality of sides and the Pythagorean theorem.

$\because$ $\Delta ABE \cong \Delta ACD$ (math.) genus

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$\therefore \angle BAE = \angle CAD$, $AE = AD$.

$\because \angle EAF = 45°$,

$\therefore \angle BAE + \angle FAC = 90° - 45° = 45°$,

$\therefore \angle FAD = \angle FAC + \angle CAD = \angle FAC + \angle BAF = 45°$.

In ΔAEF and ΔADF $\begin{cases} AE = AD, \\ \angle EAF = \angle DAF, \\ AF = AF, \end{cases}$

$\therefore \Delta AEF \cong \Delta ADF(SAS)$, $\therefore EF = DF$.

In $Rt\Delta CDF$, according to the Pythagorean theorem, the

$DF^2 = CD^2 + CF^2$

assume (office) $EF^2 = BE^2 + CF^2$

The core steps of reverse thinking to solve a problem are: first, to clarify the goal of the problem and to understand the unknowns or geometric relationships that are required to be found in the problem; in the second step, you can first speculate on the process on your own, and if the student is able to observe the answer to the problem in terms of the question, you can try to think about what steps or conditions lead to this particular result; subsequently, starting from the desired result, you can progressively consider possible geometrical properties or conditions, and using the known information in the problem to try to find geometric relationships related to the objective; after obtaining the solution, they can also use positive thinking to reverse the process to verify that the solution they have made is correct.

7. Summary

In this paper, different mathematical ideas are used in other problem scenarios to show the variety of problem-solving ideas in junior high school geometry, and students should be familiar with these techniques and methods and be able to use them skillfully in solving problems. It is worth mentioning that the moderately complex questions or final questions may need to use a variety of problem-solving techniques, so it is even more critical for students to try to master these methods, the use of learned geometric formulas and similarity and be able to independently make the auxiliary lines conducive to the solution of the problem, and to solve the more difficult questions to find the point of entry faster, to solve the problem successfully.

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