ISSN 2522-3488 Vol. 7, Issue 3: 47-54, DOI: 10.25236/IJNDES.2023.070308

Newton-Raphson Method Based Power Flow Analysis and Dynamic Security Assessment

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Abstract: *China will accelerate the construction of a new power system under the goal of "carbon neutrality and emission peak". With the change of energy structure and system characteristics, the uncertainty on both the supply and demand sides of the power system has increased significantly, and the problem of small disturbance stability has gradually emerged. In this paper, by performing power flow calculation for the power system, the nodes requiring voltage regulation and the lines with the highest active power losses are identified, and adjustment measures are provided according to their location. In that way, the terminal voltage of each power-using device can be maintained within the specified fluctuation range, reducing the active power losses in the network while satisfying customer demand, which lowers the cost of electrical energy and reduces the waste of energy resources. This paper first introduces the basic mathematical model of the power system and the principle of the Newton-Raphson method for solving power flow. Then simulates the IEEE standard node system and proposes grid operation optimization recommendations such as adjusting the output of generating units or reactive power compensation in conjunction with the simulation results of the calculation example. It enables efficient and targeted measures to be taken to improve system safety and economy in real-time monitoring of power systems, ensuring safe and economic operation.*

Keywords: *new power systems; power flow calculation; Newton-Raphson method; security assessment; economic operation*

1. Introduction

In the context of energy decarbonization, China has proposed a "peak-carbon-neutral" target [1]. As new energy sources such as photovoltaic and wind power are connected to the power grid, and the construction of regional interconnected grids continues, the dynamic characteristics of modern power grids are changing. Therefore, system risks such as the mismatch between supply and demand will become more prominent [2]. In this context, it is urgent to propose a dynamic safety assessment method under the new grid structure to provide a quantitative decision basis for system planning and operation [3]. However, with the formation of large interconnected grids across provinces and regions and large-scale energy grid integration, the power system structure has become more complex. That means it requires the traditional Newton-Raphson method to be improved to meet the new requirements.

At present, the optimization algorithms for power flow calculation are mainly divided into traditional optimization methods and artificial intelligence algorithms. The traditional optimization methods are mainly improvements to the iteration format of the Newton-Raphson algorithm. Literature [4] proposed a new Newton iteration format that approximates the curve using the tangent line at the point of $(x^k, f(x^k))$. Literature [5] simplified the calculation of the Jacobi matrix elements according to the operating characteristics of the power system, which can solve the unbalanced variable separately. Literature [6] used the non-honest Newton method to update the Jacobi matrix and used the Jacobi iterative method to solve the linear algebraic equations. Literature [7] proposed a decomposition solution method combining the positive cut method and the dichotomous method for the shortage of needing to recalculate each element of the Jacobi matrix in each iteration. All of the above papers have improved the iteration format of the Newton-Raphson algorithm, reducing the computational complexity and improving the computational efficiency.

With the rapid development of computer technology, artificial intelligence algorithms have provided new ways and directions for power flow calculation, making up for the shortcomings of traditional optimization methods in many aspects. For example, some methods would be useful like genetic algorithms, quantum evolutionary algorithms and the quantum evolutionary membrane algorithm, which

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combines quantum evolutionary algorithms with membrane computing [8]. Among them, quantum computing can solve power flow problems with exponential acceleration, and problems such as computation, optimization and security assessment in power systems can benefit from quantum computing. Existing methods for introducing quantum computing into traditional power system analysis calculations generally use the quantum linear system algorithm (HHL algorithm) and the quantum iterative algorithm to solve the nodal voltage matrix equations, then obtain the nodal conductance matrix and nodal impedance matrix [9-10]. However, the HHL algorithm is based on an ideal quantum computer whose computational power is not yet available today.

Based on the above analysis, this paper is a comprehensive analysis of the Newton-Raphson method for optimization and safety assessment of power systems to reduce energy losses while meeting the needs of users. Firstly, the basic mathematical model of the power system is analyzed and the principles of the Newton-Raphson method for calculating power flow are introduced. Next, a program is written in MATLAB based on the principles to calculate power flow for the IEEE standard node system, and the results are compared with those run using the Matpower tool to verify the correctness of the program. Then the solution results are analyzed to find out the lowest and highest voltage nodes of each system. Meanwhile, suitable voltage regulation measures are given. Finally, to reduce the power losses in the grid, recommendations are provided for the lines with the highest active power losses in each system.

2. Principle analysis of computer algorithms for power flow

2.1 Power flow calculation

Power flow distribution is a technical term describing the operating state of the power system. It indicates the magnitude and direction of the voltage and current and the distribution of power in the system. Before the distribution can be calculated, the mathematical model of the power system must be introduced.

2.1.1 Basic mathematical model of the power system

1) Mathematical models of power lines

For flow calculation, the transmission line is the main carrier of power transmission. Its mathematical model is a π type equivalent circuit, as shown in Figure 1.

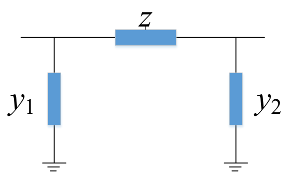

Figure 1: Standard π type equivalent circuit of transmission line

Each parameter is calculated as

$$\begin{cases} y_1 = y_2 = \dfrac{y \times l}{2} \\ z = z_1 \times l \end{cases} \tag{1}$$

2) Mathematical model for generators

The generator is the source of the power supply and its equivalent circuit is shown in figure 2. The generator reactance *XG* is

$$X_G = \frac{X_G\%}{100}\frac{U_{GN}^2}{S_N} \tag{2}$$

Where *XG* % is the generator reactance percentage, *UGN* is the rated generator voltage and *SN* is the rated generator power.

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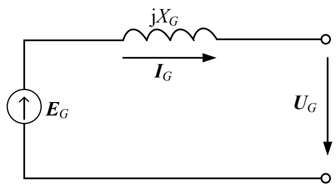

Figure 2: Generator Equivalent Circuit

3) Mathematical model of the transformer

The Γ type equivalent circuit of a double-winding transformer is shown in Figure 3.

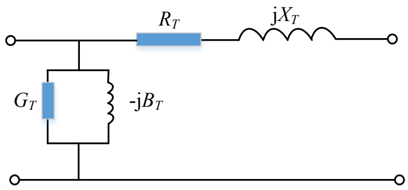

Figure 3: Equivalent circuit of a double-winding transformer

The specific parameters are calculated as follows.

$$\begin{cases} R_T = \dfrac{P_k U_N^2}{1000 S_{\mathrm{N}}^2} \\ X_T = \dfrac{U_k \% U_N^2}{100 S_N} \\ G_T = \dfrac{P_0}{1000 U_N^2} \\ B_T = \dfrac{I_0 \%}{100} \dfrac{S_N}{U_N^2} \end{cases} \tag{3}$$

Where *RT* is the total resistance of the transformer's high and low voltage windings, *Pk* is the three-phase load loss of the transformer, *XT* is the total reactance of the transformer's first and second windings, *Uk* % is the percentage impedance voltage of the transformer, *GT* is the conductance of the transformer, *P*0 is the three-phase no-load loss of the transformer, *BT* is the transformer's electromagnetism, *I*0 % is the percentage no-load current of the transformer.

2.1.2 Nodal voltage equations

In the calculation of currents in complex power systems, it is common to refer to the nodal voltage equation. The nodal voltage equation expresses the branch currents in terms of nodal voltages and lists the system of equations according to Kirchhoff's current law. The nodal voltage equation is then expanded as

$$\begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \vdots \\ \dot{I}_n \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & \cdots & Y_{3n} \\ & & \vdots & \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{pmatrix} \begin{pmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_3 \\ \vdots \\ \dot{U}_n \end{pmatrix} \tag{4}$$

However, in practical power systems, it is generally known neither the nodal current nor the full nodal voltage, but the nodal injected power **SB** , whereupon the nodal voltage equation becomes

$$\mathbf{Y_B U_B} = \frac{\hat{\mathbf{S}}_{\mathbf{B}}}{\hat{\mathbf{U}}_{\mathbf{B}}} \tag{5}$$

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2.2 Principle of the Newton-Raphson method of power flow calculation

The Newton-Raphson method is currently one of the best methods for solving non-linear equations. The point is that the process of solving a non-linear equation would be transformed into the solution of the corresponding linear equation. When using it for power flow calculations, the correction equation is

$$\mathbf{F}\left[\mathbf{X}^{(\mathrm{k})}\right]=\mathbf{J}^{(k)}\mathbf{\Delta X}^{(k)} \tag{6}$$

where **J** is called the Jacobi matrix. After k iterations, the correction is

$$X^{(k+1)}=X^{(k)}-\Delta X^{(k)} \tag{7}$$

By repeatedly solving equations (6) and (7), we can make $X^{(k+1)}$ gradually approach the true solution until it satisfies $|\triangle X^{(k)}|\leq\varepsilon$, i.e. the corresponding $X^{(k+1)}$ is the true solution sought.

When using the Newton-Raphson method to calculate the flow distribution, the basic equation is equation (5), which is expanded into a power equation as

$$\left(P_i+\mathrm{j}Q_i\right)-\hat{U}_i\sum_{j=1}^{\mathrm{n}}\hat{Y}_{ij}\hat{U}_j=0 \tag{8}$$

The difference between the two is the nodal power inequality. The problem that needs to be solved is what is the value of the voltage at each node when the nodal power is close to zero. Using equation (8) as a non-linear function equation F(X)=0, the variable X is the node voltage. Once the correspondence has been established, the correction equation can be listed and solved iteratively. When the node voltages are expressed in right-angle coordinates, the correction equation of a system with n nodes expressed in matrix form is as follows.

$$\begin{pmatrix}\Delta P_1\\ \Delta Q_1\\ \Delta P_2\\ \Delta Q_2\\ \vdots\\ \Delta P_p\\ \Delta U_p^2\\ \vdots\\ \Delta P_n\\ \Delta U_n^2\end{pmatrix}=\begin{pmatrix}H_{11} & N_{11} & H_{12} & N_{12} & \cdots & H_{1p} & N_{1p} & \cdots & H_{1n} & N_{1n}\\ J_{11} & L_{11} & J_{12} & L_{12} & \cdots & J_{1p} & L_{1p} & \cdots & J_{1n} & L_{1n}\\ H_{21} & N_{21} & H_{22} & N_{22} & \cdots & H_{2p} & N_{2p} & \cdots & H_{2n} & N_{2n}\\ J_{21} & L_{21} & J_{22} & L_{22} & \cdots & J_{2p} & L_{2p} & \cdots & J_{2n} & L_{2n}\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots\\ H_{p1} & N_{p1} & H_{p2} & N_{p2} & \cdots & H_{pp} & N_{pp} & \cdots & H_{pn} & N_{pn}\\ R_{p1} & S_{p1} & R_{p2} & S_{p2} & \cdots & R_{pp} & S_{pp} & \cdots & R_{pn} & S_{pn}\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots\\ H_{n1} & N_{n1} & H_{n2} & N_{n2} & \cdots & H_{np} & N_{np} & \cdots & H_{nn} & N_{nn}\\ R_{n1} & S_{n1} & R_{n2} & S_{n2} & \cdots & R_{np} & S_{np} & \cdots & R_{nn} & S_{nn}\end{pmatrix}\begin{pmatrix}\Delta f_1\\ \Delta e_1\\ \Delta f_2\\ \Delta e_2\\ \vdots\\ \Delta f_p\\ \Delta e_p\\ \vdots\\ \Delta f_n\\ \Delta e_n\end{pmatrix} \tag{9}$$

where the elements of the Jacobi matrix are

$$\begin{cases}H_{ij}=\dfrac{\partial\Delta P_i}{\partial f_j} & N_{ij}=\dfrac{\partial\Delta P_i}{\partial e_j}\\ J_{ij}=\dfrac{\partial\Delta Q_i}{\partial f_j} & L_{ij}=\dfrac{\partial\Delta Q_i}{\partial e_j}\\ R_{ij}=\dfrac{\partial\Delta U_i^2}{\partial f_j} & S_{ij}=\dfrac{\partial\Delta U_i^2}{\partial e_j}\end{cases} \tag{10}$$

The above is a specific method for calculating power flow by the Newton-Raphson method. The block diagram of the procedure is shown in Figure 4.

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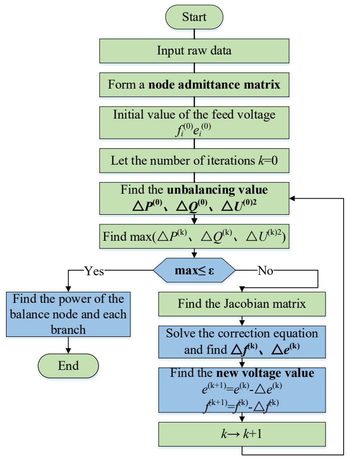
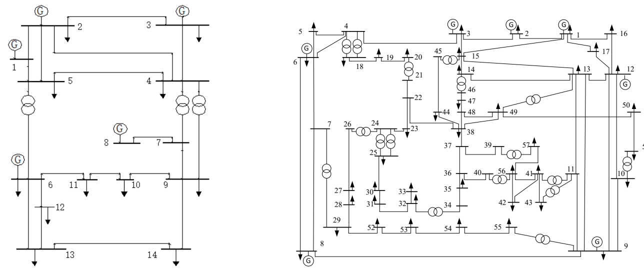


Figure 4: Block diagram of the Newton-Raphson method of power flow calculation

3. Simulation model and analysis of results

3.1 Analysis of examples

The Newton-Raphson method was used to perform power flow calculations for the IEEE14, IEEE57 and IEEE118 node systems. The equivalent circuits of the IEEE14 and 57 node systems are shown in Figure 5. Based on the above principles, the program was written using MATLAB. Then the final code run results obtained were in general agreement with the Matpower calculation results, illustrating the correctness of the program. The final calculation results are shown in Tables 1-2. The line graphs of voltage and power for each node are shown in Figure 6 and that of power loss for each branch are shown in Figure 7.



(a)IEEE14 node system network topology *(b)IEEE57 node system network topology*

Figure 5: IEEE standard node system network topology

Table 1: Voltage and power at each node of the IEEE14 node system

Bus	Voltage Magnitude	Voltage Angle	Generation		Load	
			P	Q	P	Q
1	1.060	0	232.39	-16.55	-	-
2	1.045	-4.983	40.00	43.56	21.70	12.70
3	1.010	-12.725	0.00	25.08	94.20	19.00
4	1.018	-10.313	-	-	47.80	-3.90
5	1.020	-8.774	-	-	7.60	1.60
6	1.070	-14.221	0.00	12.73	11.20	7.50
7	1.062	-13.360	-	-	-	-
8	1.090	-13.360	0.00	17.62	-	-
9	1.056	-14.939	-	-	29.50	16.60
10	1.051	-15.097	-	-	9.00	5.80
11	1.057	-14.791	-	-	3.50	1.80
12	1.055	-15.076	-	-	6.10	1.60
13	1.050	-15.156	-	-	13.50	5.80
14	1.036	-16.034	-	-	14.90	5.00

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Table 2: IEEE14 node system power losses by the branch circuit

Branch	From	To	P Loss(MW)	Q Loss(MVar)
1	1	2	4.298	13.12
2	1	5	2.763	11.41
3	2	3	2.323	9.79
4	2	4	1.677	5.09
5	2	5	0.904	2.76
6	3	4	0.373	0.95
7	4	5	0.514	1.62
8	4	7	0.000	1.70
9	4	9	0.000	1.30
10	5	6	0.000	4.42
11	6	11	0.055	0.12
12	6	12	0.072	0.15
13	6	13	0.212	0.42
14	7	8	0.000	0.46
15	7	9	0.000	0.80
16	9	10	0.013	0.03
17	9	14	0.116	0.25
18	10	11	0.013	0.03
19	12	13	0.006	0.01
20	13	14	0.054	0.11

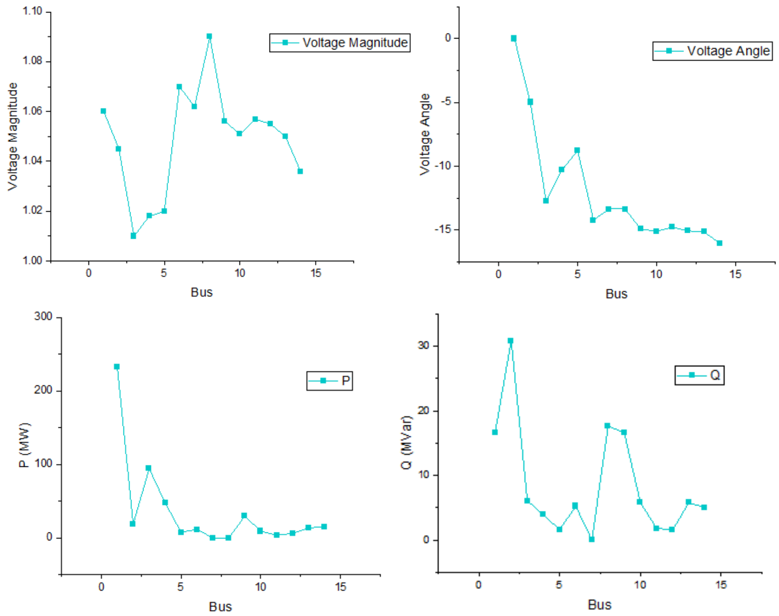

Figure 6: Voltage and power at each node of the IEEE14 node system

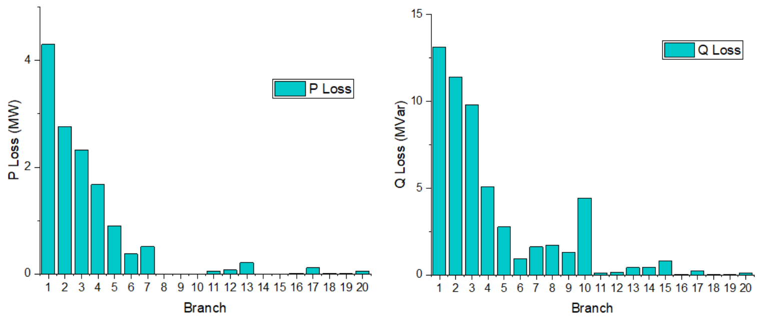

Figure 7: Power loss in each branch of the IEEE14 node system

The program written was then used to perform flow calculations for the other node systems. The

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voltage and line power losses at each node are shown in Tables 3-5.

Table 3: IEEE14 node system flow calculation results

	Minimum	Position	Maximum	Position	Total
Voltage Magnitude	1.010	bus 3	1.090	bus 8	-
Voltage Angle	-16.03	bus 14	0.00	bus 1	-
P Losses	-	-	4.30MW	line 1-2	13.39
Q Losses	-	-	13.12MVar	line 1-2	8.94

Table 4: IEEE57 node system flow calculation results

	Minimum	Position	Maximum	Position	Total
Voltage Magnitude	0.936	bus 31	1.060	bus 46	-
Voltage Angle	-19.38	bus 31	0.00	bus 1	-
P Losses	-	-	3.90MW	line 1-15	27.86
Q Losses	-	-	19.96MVar	line 1-15	15.32

Table 5: IEEE118 node system flow calculation results

	Minimum	Position	Maximum	Position	Total
Voltage Magnitude	0.943	bus 76	1.050	bus 10	-
Voltage Angle	7.05	bus 41	39.75	bus 89	-
P Losses	-	-	6.40MW	line 25-27	132.86
Q Losses	-	-	59.22MVar	line 9-10	642.32

3.2 Analysis of simulation results

First of all, according to Figure 7, the analysis of the voltage and power at each node shows that the node voltage depends mainly on the reactive power, which flows from the node with a high voltage to the node with a low voltage; the voltage phase angle depends mainly on the active power, which flows from the node with an over-phase angle to the node with a lagging phase angle.

Tables 3 - 5 give the nodes with the largest and smallest voltage modulus for each system. In the power system, the performance of the power equipment will be affected if its end voltage deviates from the rated voltage. What is worse, when the end voltage of the power equipment rises or falls sharply, it leads to equipment damage, causing the system "voltage collapse". Therefore, during normal operation of the power system, the node voltages should be maintained at the nominal value or within the permissible offset values. For the IEEE14 node system, the lowest voltage is at node 3. Since this node is in the vicinity of the generator, voltage regulation by changing the generator terminal voltage should be considered first for it requires no additional investment. The highest voltage node is node 8, which has a 9% deviation from the nominal voltage and should be regulated. As it is also in the vicinity of the generator, it can be regulated in the same way as node 3. For the IEEE57 node system, the lowest voltage is at node 31, which is far away from the generator and does not work well with the previous regulation method. When the reactive power of the system is abundant, the transformer tap can be changed to regulate the voltage, without additional investment. In conditions where the reactive power is insufficient, reactive power compensation should be carried out, adding reactive power compensation equipment such as shunt regulators, static compensators or capacitors. The highest voltage node is node 46, which is offset by 6% compared to the nominal voltage. As this node is at the end of the transformer, adjusting the transformer tap would be the best way to regulate the voltage.

In addition, the active power loss of the power line directly affects the economic efficiency of the power system, and is an important indicator for the design and operation of the power system. From Table 3 - 5, it can be concluded that the IEEE14 node system active power loss of the largest line for the line between node 1 and node 2. As the injected active power at each node other than the balancing node is already given, the active power loss of the network is only related to the injected reactive power at each node. As nodes 1 and 2 are both directly connected to the end of the generator, the inductive reactive power emitted by the generator can be changed to reduce the active power losses on this line. The line between node 1 and node 15 consumes the most active power in the IEEE57 node system. To reduce the active power losses, not only the generator can be changed, but also by adding static capacitors or static compensators would do.

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4. Conclusion

This paper introduces the mathematical model of the power system and the principle of the Newton-Raphson method for calculating power flow, and further analyses the simulation results of an example. It is also possible to find the line with the highest active power loss in the network and reduce the power loss by changing the generator output or by reactive power compensation, thus making the grid more economical. This reflects the significance and importance of power flow calculation in the power system, which in practice can not only efficiently find the nodes that require voltage regulation, but also could reduce line losses and ensure safer and more economical operation of the power system.

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