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An Analysis of a Law That Achieves 60 Fifth-order Kaplerig Numbers—On the Second-order Kaplera Number

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Abstract: *The Kaprekar Number, a class of natural numbers with unique structural properties in number theory, is characterized by its power results remaining equal to the original number when divided by equal-length fractions and summed. Starting with second-order Kaprekar numbers, this study extracts core patterns through number theory derivations and empirical validation. By extending these patterns to higher orders, we establish a cross-order verification framework. Focusing on fifth-order Kaprekar numbers, we conduct comprehensive analysis of both primitive and derived solutions, along with structural deconstruction. Key findings reveal: second-order Kaprekar numbers possess three core features—cyclic number origin, n-fold recursive patterns, and modulo-9 divisibility; their patterns maintain two essential attributes across orders: upgraded modulo operations and equal-length fraction properties; fifth-order Kaprekar numbers exhibit finite primitive solutions and infinite derived solutions, all decomposable into combinations of cyclic number bases and modified terms. The universality of these patterns is constrained by three factors: number base type, number field range, and order threshold. This paper details a rapid method for calculating repeated numbers of any cyclic number using second-order Kaprekar numbers (abbreviated as Kaprekar numbers), demonstrating how one pattern generates 60 fifth-order Kaprekar numbers through a single fifth-order Kaprekar number.*

Keywords: *Invariance principle of Kapurig numbers; Periodicity; Kapurig number formula; Fractional expression*

1. Introduction

The second-order Kapler numbers form the foundation of this study, with their regularity extraction serving as the critical prerequisite for investigating higher-order Kapler numbers. Cross-order extension validation reveals intrinsic connections between Kapler numbers of different orders. As a representative of intermediate and higher-order Kapler numbers, the fifth-order Kapler numbers can fill research gaps through their counting and deconstruction. Exploring the universality of these patterns clarifies the applicable boundaries of Kapler number properties, providing direction for future studies. Therefore, this research holds significant academic value in refining the theoretical framework of Kapler numbers and advancing structural studies of special natural numbers in number theory.

2. Background and Significance of the Study

During a journey, mathematician Kaplerga stumbled upon the Kaplerga number. While encountering a violent storm, he noticed a railway mile marker split in half—one side marked 30, the other 25. He then realized that 30+25=55, and 55^2=3025 (T). Since then, it has been named after its discoverer.the "Kapreka number" (also known as "thunderbolt number") was born.

For convenience, the base 55 in formula (T) is termed the (2-digit) Kapurigah number, the power 3025 is called the Kapurigah square number, and formula (T) is referred to as the general Kapurigah expression. Formula (T) can also be expressed as three fractions: $[5/9\ (2\ \text{cycles})]^2 = [5^2/9^2]$

[2] and $[20/9^2(2) + 1]$ (where the first half and second half of the power are separated by commas, the same applies below) (G). Here, 5/9 (2 cycles) denotes the 2nd cycle number of 5/9, while $5^2/9^2(2)$ represents the first two digits of the 2nd cycle number of $5^2/9^2$ (the same applies below). Equation (G) is called the Kapurigra fraction expression. Equations (T) and (G) are mutually convertible. Of course, all Kapurigra numbers (except those containing 9) can be expressed using both general expressions and

fraction expressions. For Kapurigra numbers with many digits, expressing them in fraction form is not only concise but also allows for quick conversion to general expressions.

3. Research Content and Methods

The validity of the second-order rule and the fidelity of the extension across orders are verified by selecting the classical Kaplerian number examples. The high-order Kaplerian numbers are inductively deduced from the properties of the second-order and fifth-order Kaplerian numbers.[1]

4. Verification of the Kapurigas Number Law

4.1. The capuleg number within one cycle

The story of discovering Kapurigana by chance sparked profound reflection in me: mathematics still holds such fascinating aspects.

Is Kapurajagat a random occurrence or an inevitable phenomenon? Research reveals that among all three-digit cyclic numbers, only four Kapurajagats exist: 001,999,297, and 703. For instance, 703^2 = 494,209, and its cyclic sum...

494+209=703. Among all 9-digit cyclic numbers, there are only 8 Kapurigas; among all 10-digit cyclic numbers, there are merely 32 Kapurigas. Even when expanding to 22-digit cyclic numbers, the number remains at just 128 Kapurigas. To find all 128 Kapurigas in 22-digit cyclic numbers through elementary methods would undoubtedly be "extremely challenging." Therefore, large-scale identification of Kapurigas necessitates alternative approaches. Interestingly, Kapurigas can be independently determined, followed by calculating their Kapuriga-squared numbers. Research reveals that when examining vertically (expanding the scope), within a single cyclic period: when a is a prime number with n=1,2,...,a; and b=1,2,...,a-1, each b/a (n-cyclic) corresponds to one Kapuriga. This demonstrates remarkable fairness and rationality. For instance, when a=11, since every proper fraction b/11 (n-cyclic) corresponds to one Kapuriga,

If there are $b/11$ $b/11$ 10 Kapurigas coexisting with a true fraction (n-cycle), how can we quickly determine these 10 Kapurigas?

We can first determine one Kaplerig number, then derive the remaining nine by applying the invariance principle of Kaplerig numbers. This c/a c/a principle states: "If ≡(k cyclic≡) is a Kaplerig number and ckbn (moda), then

b/a b/a (n-cycle) is also a Kapurig number, with its Kapurig number expression given by:

$[b/a(n$ $[b/a(n$ $]^2$ b^2/a^2 The $]^2$ b^2/a^2 $(ab-b^2)/a^2$ formula for $(ab-b^2)/a^2$ the Kapurigasum is: (a) = [(m*n) + d], (b) = [(m*n) + 1]

where m is the period length of the reduced $ab+b^2$ $ab+b^2$ a^2 true a^2 a^2 $ab+b^2$ fraction $ab+b^2$ a^2 a^2 b/a. When 0<<, d=0; when <<2 a^2

where d= -1 -1 "". Now, we $b/11$ will $b/11$ calculate all Kapurigas in one cycle of (n cycles).

The Kapurajya b/a b/a number ≡(n-cycle≡) is calculated by multiplying the obtained b/a cycle b/a number by the minimum positive integer n in n_1 (moda). Since the cycle number ≡of 1/11 is 09, it ≡is abbreviated as 1/11 (1-cycle) =09 (similar notation applies here). Solving the congruence 09n_1 (mod11) yields ≡the minimum ≡≡positive integer ≡n=5, resulting in the first Kapurajya number 1/11 (5-cycle). Applying the invariance principle of Kapurajya numbers, with c=1, k=5, and b=2, substituting into the congruence c_k b_n (moda) gives $1*5^2n$ (mod11). Solving this yields the minimum positive integer n=8 (which can also be determined using the first Kapurajya number method), yielding the second Kapurajya number 2/11 (8-cycle). By the same logic, the third to tenth Kapurajya numbers can be derived:

3/11 (9 cycles), 4/11 (4 cycles), 5/11 (1 cycle), 6/11 (10 cycles), 7/11 (7 cycles), 8/11 (2 cycles),

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9/11 (3 cycles), 10/11 (6 cycles). According to the Kapurigata formula, derive the fractional expressions of the aforementioned 10 Kapurigata numbers and convert them into general expressions:

(1) $[1/11\ (5\ \text{cycles})]^2 = [1^2/11^2\ (2\times5)]$ and $[10/11^2\ (2\times5) + 1]$

[09 (5 times)]^2 = 00826 44628,08264 46281;

(2) $[2/11\ (8\ \text{cycles})]^2 = [2^2/11^2\ (2\times8)]$ and $[18/11^2\ (2\times8) + 1]$

[18(8)]^2=03305785 12396694,14876033 05785124;

(3) $[3/11\ (9\ \text{cycles})]^2 = [3^2/11^2\ (2\times9)]$ and $[24/11^2\ (2\times9) + 1]$

[27 (9 times)]^2 = 074380165 289256198,198347107 438016529;

(4) $[4/11\ (4\ \text{cycles})]^2 = [4^2/11^2\ (2\times4)]$ and $[28/11^2\ (2\times4) + 1]$

[36 (4 times)]^2 = 1322 3140,2314 0496;

(5) $[5/11\ (1\ \text{cycle})]^2 = [5^2/11^2\ (2\times1)]$ and $[30/11^2\ (2\times1) + 1]$

[45 (1)]^2 = 45^2 = 20,25;

(6) [6/11 (10 cycles)]^2= [6^2/11^2 (2×10)] and [30/11^2 (2×10) + 1]

[54 (10 times)]^2 = 29752 06611 57024 79338,24793 38842 97520 66116;

(7) $[7/11\ (7\ \text{cycles})]^2 = [7^2/11^2\ (2\times7) -1]$, $[28/11^2\ (2\times7) + 1]$

[63 (7 times)]^2 = 4049586 7768594,2314049 5867769;

(8) $[8/11\ (2\ \text{cycles})]^2 = [8^2/11^2\ (2\times2) -1]$, $[24/11^2\ (2\times2) + 1]$

[72 (2 times)]^2 = 72; 72^2 = 5288.1984;

(9) $[9/11\ (3\ \text{cycles})]^2 = [9^2/11^2\ (2\times3) -1]$, $[18/11^2\ (2\times3) + 1]$

[81(3)]^2=669420,148761;

(10) [10/11 (6 cycles)]^2= [10^2/11^2 (2×6) -1], [10/11^2 (2×6) + 1]

[90 (6 times)]^2 = 826446 280990,082644 628100.

We have calculated 10 Kaplerigas within one cycle. By applying the periodic variation rule, we can derive all Kaplerigas (where k is a natural number) for the rational fraction b/11(n cycles).

1/11 [(11k+5) cycle], 2/11 [(11k+8) cycle], 3/11 [(11k+9) cycle], 4/11 [(11k+4) cycle], 5/11 [(11k+1) cycle], 6/11 [(11k+10) cycle], 7/11 [(11k+7) cycle], 8/11 [(11k+2) cycle], 9/11 [(11k+3) cycle], 10/11 [(11k+6) cycle]

Similarly, we can derive the fractional and general expressions for these Kapurigas (omitted). Of course, any Kapurigas can also be quickly calculated. For example, to find the Kapurigas of the repeating number 2024 in a cycle: Since 2024/9999 = 184/909, i.e. 2024/9999 = 184/909,

The number 184 is a cyclic number with a period of 909, where ≡≡the period length m=4. To solve the congruence ≡≡equation 2024n1 (mod909), we simplify the left side to 206n1 (mod909), yielding the minimum positive integer n=278. This shows that writing 2024 as 278 times results in a Kapurigah number. Using the Kapurigah number formula, we can derive its fractional expression:

$[184/909\ (278\ \text{cycles})]^2=[184^2/909^2\ (\text{the first } 4*278)]$, $[133400/909^2\ (\text{the first } 4*278) + 1]$. Similarly, due to 2025

The cycle number is 225/1111, with a period length of m≡=4. ≡By solving the congruence equation 2025n1 (mod1111), we obtain the smallest positive integer n=485.

Thus, the Kapurigana number is obtained as 225/1111 (485 cycles). Similarly, its Kapurigana number in fractional form can be derived as follows:

$[225/1111\ (485\ \text{cycles})]^2 = [225^2/1111^2\ (485 \times 4)]$ and $[199350/1111^2\ (485 \times 4) + 1]$.

Furthermore, since ≡≡184*278248=248*1 (mod909≡≡) and 225*485247=247*1 (mod1111), the invariance principle of Kapurigasum yields that 248/909 (1 cycle) equals 2728, and 247/1111 (1 cycle) equals 2223 are also Kapurigasums. Their general expressions are: 2728^2=0744,1984; 2223^2=0494,1729.

Clearly, calculating the second-order Kapurigasum (abbreviated as Kapurigasum) is a breeze and can be done quickly. We've already covered the straightforward method for Kapurigasum in detail. As for the third-and fourth-order Kapurigasum, we'll cover them later[2].

4.2. The Fifth Kapuriga Number

The fifth-order Kapurigasum requires calculating all its values in the b/a(n-cycle) formula, as the study of its variation patterns demands a broader scope, making the task progressively more challenging. Below is the fractional expression for the fifth-order Kapurigasum:

[5/7 (136 cycles)]^5=[5^5/7^5 (6×136) -1], [3488/7^5 (6×136) + 2], [1264/7^5 (6×136) -1]

[2291/7^5 (pre-6*136) + 1], [1837/7^5 (pre-6*136)] (J)

For simplicity, the denominator on the right side of the formula (J) and the first 6×136 are omitted, and any additions or subtractions are also omitted.

The formula (J) is abbreviated as:

[5/7 (136 cycles)]^5=[5^5,3488,1264,2291,1837] (J1)

The expression (J1) is referred to as the fractional simplified expression of (J) (the same applies hereafter). Given the following values: A=13718557838, B=73094049058223, C=22965496021662, D=246148460455877, and M=1001995802786540, we derive the fractional simplified expression of a fifth-order Kaplera number along with its associated x-value condition.

[225/1111 (n cyclic)]^5=[225^5, A, B, C, D] (where n = N*1111^2 + 136027, N = 1116430, x = M) (K);

Since the left side of equation (K) is constant, to simplify the equation, this paper omits the n values in the parentheses on both sides of equation (K), thereby condensing it into a simplified fraction expression with the seven elements of the x value condition:

[225^5, A, B, C, D] (where N=1116430, x=M) (K1).

Interestingly, a seven-element form of a fifth-order Kapler number (K) or (K1) can be transformed into the following 60 seven-element forms of fifth-order Kapler numbers [including (K1)] (N and x values to be simplified) (where E=668×1111³, F=-893×1111³, G=-218×1111³, P=443×1111³, R=116629, S=539712×1111³):

(1) [225^5, A, B, C, D] (where N=1116430, x=M) (K1); [225^5, A, B, C, D] (where N=1116430, x=M) (K1);

(2) [225^5, A+1*E, B+1*F, C+1*G, D+1*P] (where N=1116430+1*R, x=M +1*S) (K2);

(3) [225^5, A+2*E, B+2*F, C+2*G, D+2*P] (where N=1116430+2*R, x=M +2*S) (K3);

(4) [225^5, A+3*E, B+3*F, C+3*G, D+3*P] (where N=1116430+2*R, x=M +3*S) (K4);

(5) [225^5, A+4*E, B+4*F, C+4*G, D+4*P] (where N=1116430+4*R, x=M +4*S) (K5);

……………………;

(58) [225^5, A+57*E, B+57*F, C+57*G, D+57*P] (where N=1116430+57*R, x=M +57*S) (K58);

(59)[225^5, A+58*E, B+58*F, C+58*G, D+58*P] (where N=1116430+58*R, x=M +58*S) (K59);

(60)[225^5, A+59*E, B+59*F, C+59*G, D+59*P] (where N=1116430+59*R, x=M +59*S) (K60);

In equation (K1), the value 225^5 in parentheses remains constant, while the other four numbers are sequentially multiplied by 668*1111^3, -893*1111^3, and-218*

1111^3, 3,443*1111^3. The value of N in the parentheses plus 116629, x plus 539712*1111^3, yields Equation (K2). By extension: Equation (K2) leads to Equation (K3); Equation (K3) to Equation (K4); Equation (K4) to Equation (K5);...; Equation (K58) to Equation (K59); Equation (K59) to Equation (K60). Here, the values of N and x require simplification. To facilitate reading and research, we present a partial reduction of the simplified expression of the 60 fifth-order Kaplerian numbers and the seven-element form of the additional condition x value back to simplified expression and x value form, as follows (where N is simplified under modulo 1111^2, and the simplified N value is substituted into n=N*1111^2+136027; x is simplified under modulo 1111^5):

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(1) [225/1111(n cycle)]^5=[225^5,13718557838,73094049058223,22965496021662, 246148460455877] (where n=1116430 × 1111² + 136027, x = 1001995802786540) (K1);

(2) [225/1111(n cycle)]^5=[225^5,929767419346,71869450804740,22666545944104, 246755959925410] (where n=1233059 × 1111² + 136027, x = 49457204518261) (K2);

(3) [225/1111(n cycle)]^5=[225^5,1845816280854,70644852551257,22367595866546, 247363459394943] (where n=115367 × 1111² + 136027, x = 789580802036533) (K3);

(4) [225/1111(n cycle)]^5=[225^5,2761865142362,69420254297774,22068645788988, 247970958864476] (where n=231996 × 1111² + 136027, x = 1529704399554805) (K4);

(5) [225/1111(n cycle)]^5=[225^5,3677914003870,68195656044291,21769695711430, 248578458334009] (where n=348625 × 1111² + 136027, x = 577165801286526) (K5);

(6) [225/1111(n cycle)]^5=[225^5,4593962865378,66971057790808,21470745633872, 249185957803542] (where n=465254 × 1111² + 136027, x = 1317289398804798) (K6);

(7) [225/1111(n cycle)]^5=[225^5,5510011726886,65746459537325,21171795556314, 249793457273075] (where n=581883 × 1111² + 136027, x = 364750800536519) (K7);

(8) [225/1111(n cycle)]^5=[225^5,6426060588394,64521861283842,20872845478756, 250400956742608] (where n=698512 × 1111² + 136027, x = 1104874398054791) (K8);

(9) [225/1111(n cycle)]^5=[225^5,7342109449902,63297263030359,20573895401198, 251008456212141] (where n=815141 × 1111² + 136027, x = 152335799786512) (K9);

(10) [225/1111(n cycle)]^5=[225^5,8258158311410,62072664776876,20274945323640, 251615955681674] (where n=931770 × 1111² + 136027, x = 892459397304784) (K10);

……………………;

(51) [225/1111(n cycle)]^5=[225^5,45816161633238,11864136384073,8017992143762, 276523433932527] (where n=776275 × 1111² + 136027, x = 769607371396018) (K51);

(52) [225/1111(n cycle)]^5=[225^5,46732210494746,10639538130590,7719042066204, 277130933402060] (where n=892904 × 1111² + 136027, x = 1509730968914290) (K52);

(53) [225/1111(n cycle)]^5=[225^5,47648259356254,9414939877107,7420091988646, 277738432871593] (where n=1009533 × 1111² + 136027, x = 557192370646011) (K53);

(54) [225/1111(n cycle)]^5=[225^5,48564308217762,8190341623624,7121141911088, 278345932341126] (where n=1126162 × 1111^2 + 136027, x = 1297315968164283) (K54);

(55) [225/1111(n cycle)]^5=[225^5,49480357079270,6965743370141,6822191833530, 278953431810659] (where n=8470 × 1111² + 136027, x = 344777369896004) (K55);

(56) [225/1111(n cycle)]^5=[225^5,50396405940778,5741145116658,6523241755972, 279560931280192] (where n=125099 × 1111² + 136027, x = 1084900967414276) (K56);

(57)[225/1111(n cycle)]^5=[225^5,51312454802286,4516546863175,6224291678414, 280168430749725] (where n=241728 × 1111² + 136027, x = 132362369145997) (K57);

(58)[225/1111(n cycle)]^5=[225^5,52228503663794,3291948609692,5925341600856, 280775930219258] (where n=358357 × 1111² + 136027, x = 872485966664269) (K58);

(59)[225/1111(n cycle)]^5=[225^5,53144552525302,2067350356209,5626391523298, 281383429688791] (where n=474986 * 1111^2 + 136027, x = 1612609564182541) (K59)

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(60)[225/1111(n cycle)]^5=[225^5,54060601386810,842752102726,5327441445740,

281990929158324] (where n=591615 × 1111^2 + 136027, x = 660070965914262) (K60).

Through the four basic arithmetic operations and remainder calculation, we effortlessly derived 60 quintic Kapurigas using a single quintic Kapurigas. This pattern not only governs the progression from second to fifth order but also directly determines the quantity and composition of quintic Kapurigas. The generation of these 60 solutions is precisely the inevitable outcome of this pattern under specific numerical constraints, embodying the core principle of "pattern-dominant existence" in number theory. You might find it hard to believe mathematics could exhibit such remarkable phenomena. Whether you believe it or not, we can verify this using the following "fractional expression formula for quintic power operations of cyclic numbers":

When $10^{m*n} \equiv 10^{m*n} \equiv b^5\ b^5 \equiv y_1$ x, $y_1\ b^5\ b^5$ ($\equiv y_2$ x y_2 - b^5 5 $\equiv y_1 \equiv y_3$) $y_3\ y_1\ b^5 \equiv y_4$, (b^5 x $\equiv y_2$ +10 y_2), b^5 ($\equiv y_3$ x y_3 -10 b^5) $\equiv y_4$, (x+5) b^5

where $a^5\ a^5$ m is the period length of the rational number b/a (with a being a positive integer not divisible by 2 or 5)

a(n cycle) $]^5 =$ $b^5\ b^5\ a^5$ [a^5 m* d_1 n] y_1 + a^5 [m d_1 * y_1 d_2 n a^5] $]^5$ d_2

[($y_2\ y_2\ a^5$ m a^5 *n) + 1] y_3, $a^5\ d_3$ [(y_3 m $d_4\ a^5\ y_4\ a^5$ *n) + 1], d_4 [(y_4 m a^5 *n) + 1], d_3

where

$d_1\ H_1\ d_1\ b^5\ y_1\ a^5\ d_2\ H_2\ b^5\ y_2\ a^5\ d_3\ H_3\ H_1\ b^5\ y_1\ a^5\ d_2\ H_2\ b^5\ y_2\ a^5\ d_3\ H_3\ b^5\ y_3\ a^5 =$ (b^5 -5-) y_3 /, = a^5 (+10-)/, = (-10-)/,

$d_4\ H_4\ d_4\ b^5\ y_4\ a^5\ b^5 \equiv H_1\ y_1 \equiv H_2\ y_2 \equiv H_3\ y_3 \equiv H_4\ b^5\ y_4\ a^5\ b^5 \equiv H_1\ y_1 \equiv H_2\ y_2 \equiv H_3\ y_3 \equiv H_4\ a^5 =$ H_4 (+5- a^5) / x, x, x, x (mod)

Substituting b=225, a=1111, and the additional condition x=660070965914262 (where x represents the simplified remainder x in the formula) into equation (K60), we verify its correctness. Using the data in the right parentheses of $d_1\ d_1\ d_2$ (K60 $d_3\ d_2$) d_4, d_3 the formula d_4 can quickly calculate: =0, =1, =0, =0. The sum of the five numbers in the right parentheses equals 225×1111^4, and the cyclic sum =225×1111^4/1111^5=225/1111 matches the fraction in the left parentheses of (K60). Thus, (K60) serves not only as a simplified fraction expression for the fifth-order Kaplerigah number but can also be reduced to its fractional form (note: the cyclic number of 225/1111 is 2025, with a cycle length m=4).

[225/1111(n-cycle)]^5=[225^5/1111^5(4n+1)], [54060601386810/1111^5(4n+1)]

[842752102726/1111^5 (first 4n)], [5327441445740/1111^5 (first 4n)]

[281990929158324/1111^5 (the first 4n)] (where n=591615 × 1111^2 + 136027, x = 660070965914262).

Certainly, the simplified fraction expressions for the remaining 59 fifth-order Kapurigas can all be reduced to their fraction expressions using the given formulas (details omitted). According to the above formulas, any fifth-power operation of a repeating number (excluding the repeating number of 9) can be expressed in fractional form. If the sum of the five fractions on the right side of this expression (i.e., the cyclic sum) equals b/a, then this expression is a fifth-order Kapurigas fraction expression. Similarly, the nth-power operation of a repeating number can also be expressed using another formula (the fractional expression formula is omitted)[3].

Research has revealed that when n equals 1111^2 + 136027, the number 225/1111 (n cyclic) or 2025 (n repeating) contains 1017 quintic Kapluga numbers. This demonstrates that the 60 quintic Kapluga numbers previously identified merely represent a fraction of the total. Naturally, attempting to compute all quintic Kapluga numbers for 225/1111 (n cyclic) or 2025 (n repeating) within a single 1111^4 cycle would be computationally infeasible and practically insignificant. The 60 quintic Kapluga numbers thus obtained are but the tip of the iceberg among all quintic Kapluga numbers in 2025 repeating sequences. Therefore, we offer this preliminary exploration to broaden readers' perspectives, proving that higher-order Kapluga numbers not only exist but can be calculated when discernible patterns are

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identified.

5. Conclusions

The story of the Kapurigas' birth makes them seem like fleeting figures—arriving and departing in haste—yet as long as Seize the "high and mighty" nature of Kapluga numbers, and by solving a congruence equation, it takes just a few minutes to find a second-order Kapluga number (abbreviated as Kapluga number). This law not only permeates the expansion process from the second to the fifth order but also directly determines the quantity and composition of fifth-order Kaprekar numbers—60 solutions are the inevitable outcome of this law under specific digit constraints, embodying the core idea of "laws governing existence" in number theory.

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