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Conceptual Lesson Teaching Example Based on Mathematical Core Literacy: Concept of Function in Middle School

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Abstract: *In this teaching example, the concept of function in Middle school mathematics is used to systematically illustrate how to integrate the core literacy of mathematics and effectively implement the requirements of the<Chinese Mathematics Curriculum Standards (Version 2022)>. Besides, students are taught in the way of concept formation, through life situations, imaginary situations, and pure mathematical situations, as well as positive and counter example, knowledge transferring and classroom exercises, differentiated homework, etc., so that students can learn to observe the world with mathematical eyes, think about the world with mathematical thinking, and express the world with mathematical language, thus cultivating their reasoning ability, model concept, and application awareness. Based on the design of this lesson, three suggestions are given: (1) the teaching of multiple scenarios can start from project-based teaching; (2) the comparison of the concept of the equation and the concept of function is added to further help students clarify the concept; (3) Not only the homework but also the teaching objectives and teaching activities can be arranged in a tiered way.*

Keywords: *Core literacy, Function concept, Concept formation*

1. Introduction

Mathematical concepts, the basic units of mathematical theory in the process of teaching mathematics, are the cornerstones for establishing formulas, laws, and theorems, as well as the basis for mathematical reasoning and induction. Therefore, teaching mathematical concepts is necessary for the formation and improvement of basic mathematical knowledge and skills, and is an important vehicle for developing core literacy.

In 2022, a new version of the Chinese Compulsory Education Mathematics Curriculum Standards was promulgated, defining the core literacy-oriented curriculum objectives, and reflecting the curriculum philosophy of the nurturing value of the subject of mathematics[1].

The function is an important point of knowledge that transforms from a constant quantity to a variable. Students can understand that mathematical knowledge comes from life and is applied to life through the study of functions and strengthen their grasp of the essence of mathematics[2]. It is not only an abstract concept encountered throughout middle school mathematics, but also one of the first abstract concepts encountered in high school mathematics and is one of the most difficult concepts to learn and teach in middle school mathematics[3]. Shi Ningzhong[4] has talked about the need to highlight functions in the teaching of mathematics at the secondary school level, which is a long-standing practical study by mathematicians.

This case is intended to use the Beijing Normal University version of the textbook <Compulsory Education Textbook: Mathematics> in the volume one of grade 8[5]. Chapter 4 Primary Functions: Section 1-Functions as the object of study, the student's prior knowledge paved with, Integers and their addition and subtraction, Multiplication and division of integers, Real numbers, Position, and coordinates. The student's prior knowledge includes multiplication and division of integers, relationships between variables, real numbers, and positions and coordinates. In addition, in terms of content classification, function definition belongs to the content of concept teaching, and there are two common teaching modes of concept acquisition: concept formation and concept assimilation[6]. Concept formation, i.e., students

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discover key elements from many cases to obtain concepts, concept assimilation, i.e., students use their existing knowledge to deduce new concepts. This case uses the concept formation approach to teach the definition of a function.

2. Value judgment

2.1. From cognitive awareness

The International Research Project on Trends in Mathematics and Science Education[7] in the assessment of mathematical cognition of junior high school students, mathematical cognition is divided into three dimensions, knowing, using, and reasoning. Among them, the application dimension includes identification, representation, and implementation while the reasoning dimension includes analysis, evaluation, conclusion, integration, generalization, and argumentation. These two dimensions accounted for 60% of the overall cognitive assessment. From the research project, it can be found that middle school mathematics should focus more on students' higher-order thinking, using mathematical knowledge, and solving real problems originating from real life. At the same time, mathematical reasoning models are built in real-life scenarios to enhance students' thinking development and cognitive level.

2.2. From the curriculum standards

The Standards (2022 Edition)[8] state in the connotation of core literacy that (1)students need to can observe the real world with a mathematical eye: through the observation of basic quantitative relationships in the real world, students can intuitively understand the mathematical knowledge and knowledge context they have learned. (2)Students can think about the real world with a mathematical mind through experiencing an independent mathematical thinking process, students can understand the generation and development of basic mathematical concepts, and the connections between basic mathematical concepts and between mathematics and the real world. (3)Students will express the real world in the language of mathematics: they will be able to construct universal mathematical models to express and solve problems in real life.

The general goal of the Standards (2022 Edition) states that students will acquire the basic knowledge, skills, ideas, and activity experiences in mathematics necessary for further development in their future lives. At the same time, students need to feel the connections between mathematics and life, identify problems, ask questions, and solve problems while exploring the relationships embedded in real situations.

The Standards (2022 Edition) state in the teaching suggestions that each specific learning content has the role of developing related core literacy, focusing on establishing the relevance of the related content and core literacy.

2.3. From the content of the materials

As they enter Key Stage 4, students describe quantitative relationships and patterns of change in real-world problems using algebraic equations, equations, inequalities, and functions and develop appropriate operational ideas to solve problems. In addition, students need to develop abstraction skills, model concepts, further develop arithmetic skills, explore the identification and formulation of problems from a mathematical perspective in different contexts, and apply mathematical knowledge to seek ways to analyze and solve problems.

2.4. From the evaluation recommendations

The Standards (2022 edition) state that teaching evaluation should be diversified, including but not limited to written quizzes, activity reports, classroom presentations, post-class assignments, and formative records. This requires teachers, not only to focus on the mastery of students' knowledge and skills but also on the development of students ability to analyze and solve problems. In addition, in the test paper proposition, it is clear that the mathematical knowledge to be examined in each test question is presented accordingly with the core literacy.

Based on this, the objectives and key points of this lesson:

Teaching Objectives:(1) To master the concept and representation of functions based on the

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understanding of real-life scenarios.(2) Combining the independent variables of a function with real-world elements to determine the range of values of the independent variables.(3) Model real-world problems as functions and learn to solve real-world problems.

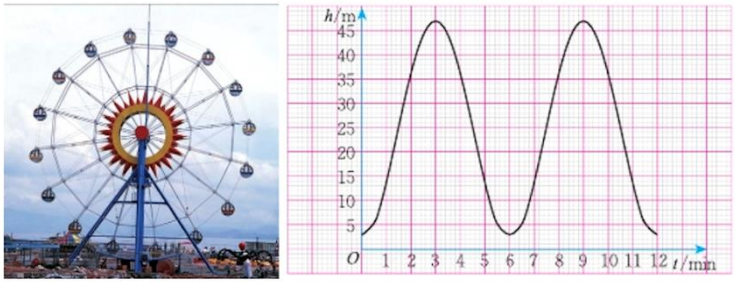
Teaching focus: The concept of function is deconstructed through the understanding of real scenarios.

Teaching Difficulties: Determine the range of values of the function's independent variable in both purely mathematical scenarios and real-world scenarios.

3. Course Case Design

3.1. Set the context and observe the commonality

Question 1: When riding the Ferris wheel in the Figure 1, how does the height of the wheel on the ground change with time?



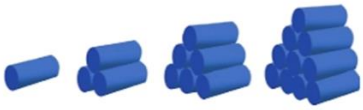
t/min	0	1	2	3	4	5	…
h/m							…

Figure 1: The relationship between time and height in sky wheel

Teacher-Student Interaction: work in small groups to develop a discussion from two perspectives.(1) Complete the table according to the icons.(2) For a given time t, is the corresponding height h determined?

Design intent: Using real-life situations to help students learn abstract shapes based on the image perception of shapes. Another Outside, through group discussions, experience looking at the real world through the eyes of mathematics. While completing the task, you can realize the rationality of representing function problems by the image and the table method.

Question 2: Looking at the Figure 2 below, how does the total number of cylindrical objects change as the number of layers increases?



Filling the blank.

Layer n	1	2	3	4	5	.....
Number of cylindrical objects y						

Figure 2: The relationship between number of plies and number of cylinder

Teacher-Student Interaction: after students have filled in the data in the table, follow up by asking them:(1) When $n = 10, y =$?; when $n = 2022, y =$? Leading students to think about the necessity and importance of finding patterns. (2) Elaborate on what patterns you have found. For example, discovering in terms of layers and strata, or simply thinking in terms of changes in numbers. (3) Write the equation for y versus n. (4) Differences from question (1).

Design intent: the fictitious situations allow students to exercise the mathematical knowledge they find embedded in them. Students are allowed to elaborate on the laws they discover, learn to express the world in the language of mathematics, develop the ability to express and communicate mathematics, and develop a sense of application. They learn to express the world in mathematical language, develop the

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ability to express and communicate mathematics, and develop a sense of application. In addition, students are guided to write the relationship between y and n, from which they can appreciate the rationality of using. In addition, students are guided to write the relationship between y and n, from which they learn the rationality of using analytical equations to represent function problems. Finally, the comparison of problems 1 and 2 enables students to have a better understanding of the different ways of presenting function problems. Finally, the comparison of problems 1 and 2 enables students to have a deeper understanding of the different ways of presenting function problems and to exercise their reasoning skills and model view.

Question 3: What is the lengths y of its neighboring sides when the lengths x of one side of the rectangle are 3 m, 3.5 m, 4 m, and 4.5 m? Does the value of y vary with the value of x?

Teacher-Student Interaction: when x takes different values, what is the corresponding y? Also, write the equation for the relationship between x and y? Immediately after, answer how the value of y changes with the value of x?

Design intent: the original problem 3 in the textbook - about the quantitative relationship between thermodynamic temperature $T(K)$and Celsius temperature t(°C) $T = t + 273$ was abandoned because for Chinese students, past life experiences and learning experiences, there were no scenarios involving thermodynamic problems that could well meet the original design intent: a purely mathematical situation. Therefore, the case of the function in-19.1 under the eighth grade of the Renminbi version is used here[9], thus continuing to follow a pure mathematical context, and developing students higher-order thinking. The above design achieves the goal of getting students to think mathematically about the real world and use mathematical methods such as symbolic operations and formulaic reasoning to analyze and solve mathematical problems. From this, the analytic method of representing functions is also truly felt.

3.2. Extracting the essence and forming concept

Question 4: What is the common denominator in the above questions?

Teacher-student interaction:

(1) In small groups, have students come to the table to see which group finds the common denominator, fast and often.

(2) Teacher's summary: 1) the number of variables 2) the relationship between two variables 3) the method of representation.

(3) Developing the concept of a function.

(4) Give counterexamples and give notes: a function is not a number; it refers to the relationship between two variables in a given change.

Design intent: to apply to learn from life scenes to life scenes.

3.3. Positive examples and negative examples reinforce the concept

Question 5: In the following two graphs in Figure 3, are both functions? Why?

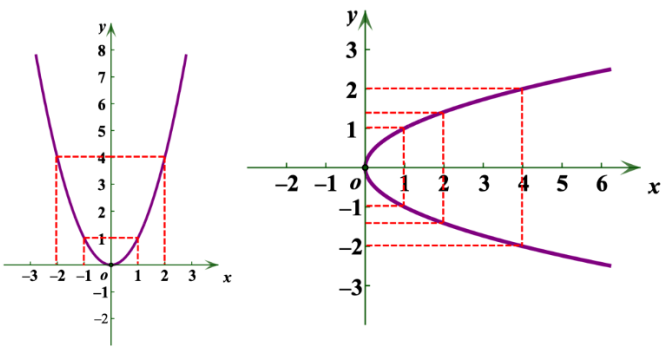

Figure 3: Curves in different positions

Design intent: Through the demonstration of positive and negative examples, using discussion,

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communication, and reflection, etc. Strengthen students' understanding of the concept of function and refine the correspondence of function: one One-to-one or many-to-one, but not one-to-many.

3.4. Step-by-step knowledge progression

Question 6: What is the range of values of the independent variables in questions 1-3?

Teacher-student interaction: (1) Question 1: The independent variable t takes on a range of values $t \geq 0$, can t be taken as a decimal? (2) Question 2: Is the range of values of the independent variable n a positive integer for n and can n be a negative or decimal number? (3) Question 3: What is the range of values of the independent variable x? How was it considered? (4) Question 4: What is the range of values of the independent variable to be considered?

Design intent: when examining the range of values of the independent variables, it is important to consider not only the need to make sense of the analytic formula of the function, but also the practical meaning represented by each variable. Thus, students develop the concept of mathematical models while understanding the importance of satisfying realistic meanings.

3.5. Knowledge transfer Practical application

Question 7: The car has 60 L of gasoline in the tank, and if no more gasoline is added, then the amount of gasoline in the tank y (unit: L) decreases with the increase in mileage x (unit: km), and the average fuel consumption is 0.1 L/km.

Teacher-Student Interaction: (1) Write the equation that represents y as a function of x.(2) Indicate the range of values of the independent variable x.(3) Can the car travel up to 250 km?

Design intent: From real scenes to real scenes, to achieve the goal of using functions to model real-world problems and solve real-world problems.

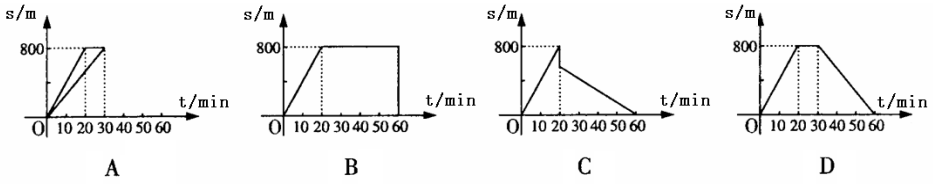
3.6. Classroom exercises to reinforce understanding

1) Let the distance be s, the time be t, and the speed is v. When v =60, the relationship between distance and time is _____. In this relationship, _____ is a constant, _____ is a variable, and _____ is a function of _____.

2) Each of following expressions is not a representation of y as a function of x ( )

$$A.y = 3x^2 \qquad B.y = \frac{1}{x}. \qquad C.y = \pm\sqrt{x}(x \geq 0). \qquad D.y = 2022x.$$

3) Xiao Ming's grandfather goes out for morning exercise and walks 20 min from home to a park 800 m away from home. Then the graph of the function between the distance s (in m) and the time t (in min) of Xiaoming's grandfather is approximately ( ).



4) Find the range of values of the independent variable x in the following functions:

(1) $y = 10x - 1$.

(2) $y = \frac{1}{11x-1}$.

(3) $y = \frac{1}{11x-1} + 10x - 1$.

(4) $y = \sqrt{10x - 1}$.

(5) $y = \frac{1}{\sqrt{10x-1}} + \sqrt{10x - 1}$.

5) The daytime cab fare in a city is 8 yuan for a ride of up to 3 kilometers, and 1.8 yuan per kilometer for a ride of more than 3 kilometers. Let the mileage of the cab ride be x (km) (x is an integer), and the

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corresponding charge is y (yuan). (1) Please write separately when $0 < x \leq 3$ and $x > 3$ when the equation expresses y versus x and write the value of y when $x = 2$ and $x = 6$. (2) When $0 < x \leq 3$ and $x > 3$ when y is a function of x? Why?

3.7. Perceive the thoughts and summarize the sublimation

Think: What did you learn from this lesson? What investigations did you experience? What knowledge did you learn? What methods of thought were experienced?

Teacher-student interaction: Find students to share what they have learned and felt today.

Design Intention: Through the review of the inquiry process in this lesson, let students appreciate that the formation of the concept of function has a background and is logical and step-by-step. At the same time, students will develop the ability to sort out knowledge and construct a knowledge system.

3.8. After-class assignments Tiered instruction

Assignment design: students in layer A (basic layer) do basic training. Layer B (general layer) students do basic training and proficiency improvement. Students in layer C (improvement layer) do basic Training, proficiency improvement and integrated application.

Design Intention: The Curriculum Standard (2022 Edition) suggests that the mathematics curriculum should be dedicated to achieving the training goals of compulsory education, be open to all students, and be suitable for student's individual development so that everyone can get a good mathematics education and different people can develop differently in mathematics. At the same time, teachers' teaching should be based on students' cognitive development level and existing experience, with emphasis on heuristic teaching and teaching according to their abilities.

3.9. Sublimation of connotation and conclusion of famous lines

Conclusion: Time is a constant, but for the diligent, it is a variable. A person who counts time in minutes has 59 times more time than a person who counts time in hours.——Soviet Historian Rybakov.

Design intent: to sublimate knowledge and develop students' mathematical literacy and comprehensive quality.

4. Discussion and Outlook

4.1. Discussion

In this lesson, we take the teaching of Definition of Functions in Middle school mathematics as an example to illustrate how to integrate core literacy into the curriculum and teach mathematics based on core literacy.

The entire curriculum is designed to proceed based on the concept formation model. Through life situations cases, help students learn abstract graphics from the image perception basis of graphics, experience observe the world with mathematical eyes; through the imaginary situations, let students elaborate on the laws they found, will use the language of mathematics to express the world, the formation of mathematical expression and communication skills, the development of application awareness; through pure mathematical situations, let students think about the real world with mathematical thinking, can use symbols The students will be able to analyze and solve mathematical problems by using mathematical methods such as symbols, formulas, and reasoning.

After the concept is formed, positive and negative examples, knowledge transferring and classroom exercises are used to further strengthen students' understanding and application of the definition of functions, and to develop students' ability to model mathematical concepts. The student's ability to model mathematical concepts was also developed. Finally, in the post-lesson homework, tiered homework is designed to meet the needs of students at different levels to master the learning tasks and get maximum improvement in the nearest development zone. At the same time, this lesson is in line with the New Curriculum for individualized instruction and teaching according to students' abilities.

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4.2. Outlook

Based on the above analysis, this lesson example also has some informative suggestions, as follows:

Above all, since the teaching of the concept of function is mostly initiated by contextual cases, the design of this lesson is also suitable for project-based learning, such as exploring the relationship between time and flow of people, the relationship between study length and academic performance, and the relationship between age and height, so as to implement the application awareness, learn by doing and use in learning, and further develop the core literacy of observing the world through the eyes of mathematics.

Furthermore, the concept of function is a very important element in algebra, which has been studied throughout middle school and high school. Among them, the comparison between the concept of the equation and the concept of function has always troubled many students. Therefore, if the comparison of the two can be covered in the lesson, it can not only help students sort out the connotation of knowledge but also exercise students' abstraction and reasoning abilities.

Last but not least, the homework session of the lesson example is designed to teach in a tiered way. This is a good illustration of the teaching concept of the New Curriculum. It will be more helpful and meaningful to students if the idea of tiered teaching can be incorporated throughout the teaching sessions, such as teaching objectives and teaching process.

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