

# Research on the Design of High School Mathematics Unit Teaching under PBL Teaching Mode—Taking "Equal Difference Series" as an Example

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**Abstract:** With the deepening of the new curriculum reform and the emphasis on core skills, the large-unit teaching mode has been widely used. This model not only meets the new curriculum standards, but also adapts to the current social needs, and helps to adapt to the trend of curriculum change, thus bringing new possibilities for the innovation of education and teaching methods. PBL teaching mode is a problem-oriented teaching mode, which inspires students' thinking and promotes students' in-depth learning activities using the design of problematic situations. Based on the current situation of high school mathematics unit teaching, this paper discusses the teaching process and strategy under PBL mode by taking the unit design of equidistant series as an example.

**Keywords:** PBL teaching mode; high school mathematics; unit teaching design; equal difference series

## 1. Introduction

The new curriculum requires teachers to activate students' ability to think about problems, organize students to participate in learning activities, and improve the core literacy of mathematics. The Outline of the National Medium- and Long-Term Educational Reform and Development Plan (2010-2020) puts forward the integration of learning and thinking and advocates the use of inspirational, exploratory, discussion, and participatory teaching to help students learn to learn<sup>[1]</sup>. For this reason, mathematics teachers have begun to change their teaching methods and teaching programs, put forward new teaching designs according to students' actual learning and development, and lead students to improve their self-constructive ability. This paper designs unit teaching methods and organises unit teaching activities from the perspective of PBL teaching model. This can broaden the new perspective of teaching design, lead students to build a reasonable knowledge structure in the teaching context, and promote the development of students' core mathematical literacy.

## 2. Problem Formulation

In the mathematics classroom, students' cognitive level and comprehension ability are generally low, which often affects students' subsequent understanding and memorization of mathematical concepts and methods. At the same time, the current mathematics classroom in high school emphasizes too much knowledge learning and neglects the cultivation of students' thinking ability and the enhancement of their thinking consciousness. This means many math teachers in the training of students, can not start from the specific problem to combine the actual situation and specific characteristics, so it can not achieve the expected teaching effect. High school mathematics classrooms should be student-oriented, and teacher-oriented, giving full play to student initiative. However, the traditional didactic teaching method of student participation is weak, the promotion of student independence, and exploratory, innovative consciousness is poor, and can not meet the needs of the overall development of students. Therefore, the PBL teaching mode came into being. The emergence of the PBL teaching mode is in line with the concept of teaching design put forward by the new curriculum reform. Designing the unit teaching method and organizing the unit teaching activities from the perspective of the PBL teaching mode can further develop the new perspective of teaching design, lead the students to establish the connection between the knowledge points in the context, build a reasonable knowledge structure, and promote the students' core mathematical literacy. Formation.

In this paper, the PBL teaching mode is integrated into high school mathematics teaching, and series is an important part of high school mathematics teaching and a widely used mathematical model in life, which is the teaching content of selective compulsory high school mathematics courses. Therefore, this paper takes the unit teaching of "Equal Difference Series" as an example to elaborate on the use of PBL teaching mode in the unit teaching design, which requires that the teaching content should be developed based on the problem to promote the generation of the problem. At the same time, the math classroom should focus on the penetration of the relevant mathematical core literacy and ideological methods, cultivate students' emotional attitudes towards mathematics, and enhance the educational value of mathematics.

### 3. Connotation of PBL Teaching Mode

PBL teaching mode, problem-oriented teaching mode, is a teaching mode that takes students as the main body and guides students to think independently and discuss cooperatively to acquire knowledge through teaching problem situations, with the characteristics of autonomy, inquiry, cooperation, etc.<sup>[2]</sup>. The PBL teaching mode needs three basic factors: problem situations, teachers, and students, the interrelationship is shown in Figure 1. Under the guidance of the teacher, the students explore the questions posed by the teacher. Students experience the process of discovering problems, analysing problems, solving problems and finally getting conclusions. In this process, students' problem-solving ability, independent learning ability and co-operation and communication ability can be improved, and students' learning interest and intrinsic learning motivation can be stimulated.

The PBL referred to in this study is a kind of teaching mode, and this paper is the application of the whole set of teaching procedures of PBL in the teaching of the unit of equivariant series in high school, which has an independent guiding ideology and complete implementation steps, and it is a coherent link from the initial posing of the problem to the initial solving of the problem, and the final evaluation of the results. First, students independently collect information around the teacher's questions. When they encounter obstacles, they carry out group co-operative investigation and finally solve the questions, so as to improve students' independent learning ability and creativity through this teaching mode.

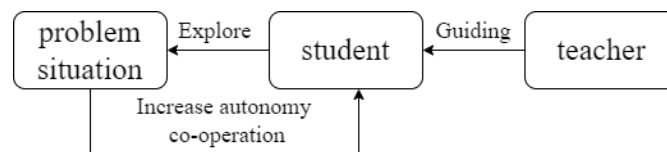


Figure 1: The three basic elements of the relationship

### 4. The value of unit teaching based on PBL mode

Unit teaching is based on the learning characteristics and psychology of high school students, closely integrating the relevant knowledge in the mathematics textbooks and the relationship between them, under the guidance of teachers. This teaching method aims to coordinate and optimize the content of mathematics teaching materials, integrate the optimized and screened mathematics teaching content, and construct independent unit teaching content to highlight the main line of teaching mathematical knowledge, i.e., to strengthen the correlation between mathematical knowledge. Based on this, a series of mathematical teaching activities can be designed. For the unit teaching mode, the focus is on highlighting the connection and wholeness between mathematical knowledge, emphasizing the coherence of students' thinking logic, and cooperating with systematic mathematical thinking to deepen the understanding of the learned mathematical unit knowledge. With the support of the PBL model, students' independent learning can be combined with the teacher's lectures to promote students' independent exploration and cooperative exchanges, as shown in figure 2. The PBL teaching model is centred on problem solving in the process of implementation, and is not confined to a single point of knowledge, which is just right to combine with the concept of unit teaching. Teachers systematically integrate the teaching content to make the teaching design more comprehensive, further optimise the teaching effect, and mobilise students' creative and logical thinking. While allowing students to deepen their understanding of the knowledge of the mathematical units they have learnt, it promotes the successful development of students' core mathematical literacy<sup>[3]</sup>.

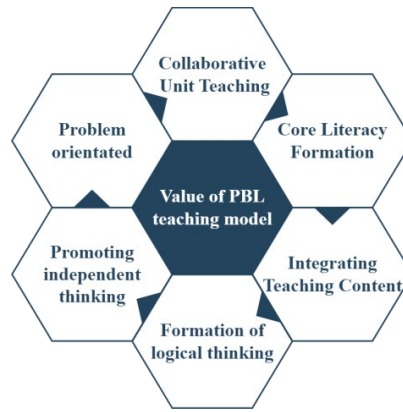


Figure 2: The value of teaching units in the PBL model

**5. Based on the PBL teaching model of high school mathematics "equal difference series" unit teaching design**

"Equal difference series" is the teaching part of the "series" chapter, before the study of "equal difference series", students have already understood the concept of series, on this basis, further extension, into the "equal difference series" study. The study of "Equal Difference Series". In the unit of "Equivalent Difference Series", students need to learn the concept of equal difference series, the general formula, the formula for the sum of the first n terms, the basic properties of the equal difference series and its application, and so on. The teaching goal of this unit is to make students realize that the equivarient series is a special kind of series, to understand the equivarient series in life in a wide range of applications. In order to prompt students to establish mathematical models in solving practical problems, understand the implied quantitative relationships, and improve the modelling ability of mathematics as well as problem solving ability<sup>[4]</sup>. To better implement the unit teaching objectives, in the teaching design, teachers should further split the unit teaching objectives, according to the content of the unit teaching will be divided into three classroom objectives, to provide a basis for unit teaching design, specifically:

**5.1 Planning lesson learning objectives**

Learning objectives of the first lesson: (1) students understand the concept of the equivarient series and master the method of determining the equivarient series; (2) students think independently, summarise the basic properties of the equivarient series and the general formula, and master the relevant calculation methods of the equivarient series; (3) students solve the practical problems of the equivarient series in the group cooperation and investigation;

Learning objectives of the second lesson: (1) students master the characteristics of the equidistant series in group work, complete the exploration of mathematical laws, learn to prove the conjecture and summarise the application of the properties of the equidistant series, and realise a deeper understanding; (2) students master the method of argumentation in logical reasoning through experiencing the learning steps of computation and validation, and improve the core mathematical literacy.

Learning objectives of the third lesson: (1) students explore the calculation method of the sum of the first n terms of the equivarient series under the guidance of the teacher's prompts; (2) students show and organise their personal learning outcomes and state their understanding of the formula of the equivarient series; (3) teachers summarise the different methods of argumentation, which prompts the students to master a variety of methods of proof or argumentation.

The specific breakdown of the learning objectives for the three lessons is shown in Table 1.

Table 1: Breakdown of learning tasks for the three lessons

Tasks	Objective 1	Objective 2	Objective 3
Task 1	Understand the concept of an isotropic series	Summarizing properties	Exploring problems
Task 2	Deeper Understanding of Equal Difference Series	Mastering calculations	Mastering logical argumentation
Task 3	the calculation of the sum of the first n terms series	Enhance understanding of series calculations	Master multiple methods of proof

**5.2 Designing the Problem Context of the Core Unit**

Starting from the problem situation, designing driving problems that can stimulate students' interest in learning is a key part of the PBL teaching mode. In the "equal difference series" unit teaching design, teachers should be linked to the problem situation and teaching content, the design of the core problem situation with a guiding role. With the situation as a carrier to lead to the core problem, each problem corresponds to the core knowledge of the unit, prompting students to actively think about the problem under the drive of the problem, summarize the mathematical knowledge in the exploration of the problem, and lead the students out of the passive acceptance of the state of learning, and to become the explorers of mathematical learning<sup>[5]</sup>. The core problems and their corresponding core knowledge designed by teachers in the unit teaching design are as follows:

Core problem situation: a company has purchased a piece of equipment valued at 2.2 million yuan, and as the equipment ages in use, its value decreases each year. Experience has shown that its value decreases by  $d$  ( $d$  is a normal number) million dollars for every year that passes. The equipment is known to have a useful life of 10 years, beyond which its value will be less than 5% of its purchase value and the equipment will be scrapped. The resulting core question posed by the teacher is: "How can we recognize the equivariant series in terms of the change in value and useful life of this equipment as it ages?" This core question corresponds to the core knowledge points:

- (1) The definition and properties of the equal-difference series;
- (2) the general formula of the equal-difference series:  $a_n = a_1 + (n - 1)d$
- (3) the formula and application of the sum of the first  $n$  terms of the equal-difference series:  $S_n = na_1 + \frac{n(n-1)}{2}d$

The four core problems and the corresponding knowledge points are shown in Table 2.

First, the core question is "How long will this piece of equipment be obsolete?" The design of this problem can lead students to transform life problems into mathematical problems, leading students to think: what kind of a series is this? What is the first term of the series? What is the tolerance of the series? How to prove whether the known series belongs to an equal-difference series? The design of such a problem situation is to lead students from the series transition to the isotropic series, learn the definition and properties of the isotropic series, realize the transfer of core knowledge conversion, to develop students' mathematical transformation ideas.

Second, the core problem is "How to appropriately use the formula to express the annual value of this equipment?" The corresponding core knowledge is to learn to use the representation of the general formula of the equivariant series, through practical problems, so that students can experience the application of the equivariant series.

Third, the core question is "How can you use the formula to express the sum of the decreases in value of this piece of equipment when it is used for 5 years?" The corresponding knowledge point is to express the sum of the first  $n$  terms in terms of an equal-difference series and to clarify the purpose of summing an equal-difference series.

Fourth, the central question is: "How many years after use will this piece of equipment be scrapped?" The corresponding knowledge is to learn the application of the formula for the sum of the first  $n$  terms of the equivariant series and to improve students' mathematical abstraction and mathematical modeling core literacy using the first  $n$  terms of the equivariant series in practical problems.

*Table 2: Core Problem Correspondence*

Core question one	How long will it take for this equipment to become obsolete?	Transitioning from a series to an equal and different series.
Core question two	How to appropriately use the formula to express the annual value of this piece of equipment?	Learn to represent the general formula of an equal and different series.
Core question three	How to use a formula to show the total decrease in value of this piece of equipment after 5 years of use?	Represent the first $n$ terms of an equal and different series.
Core question four	How many years will this piece of equipment be obsolete?	Improve students' core skills

**5.3 Cooperative group inquiry problems**

Group cooperative learning is a common learning method in the implementation of PBL teaching mode, which can stimulate students' subjective initiative in learning. Teachers should put forward the

unit problem situation from the perspective of group cooperation in the unit teaching design, lead students to investigate the problem in group learning, summarize the results of the discussion on behalf of the group, and encourage students to show the results of the group's cooperative research in the summary section. Students through group collaborative inquiry discussion concluded that the value of this equipment after  $n$  years of use constitutes a series  $\{a_n\}$ . The meaning of the question is within 10 years (including 10 years), the value of this piece of equipment should not be less than  $(220 \times 5\%) = 110,000$  yuan; and after 10 years, the value of this piece of equipment should be less than 110,000 yuan. You can use the general formula of  $\{a_n\}$  to make an inequality to solve the problem.

Teachers also need to improve students' ability to learn math and research in cooperative learning, break through the zone of nearest development, and gain practice in all learning skills. For example, teachers can put forward group discussion questions in the teaching of the unit of equidistant series: (1) How did we abstract the concept of series in the previous unit of study? Can the equivariant series be obtained in the same way? (2) What are the basic components and concepts in an equal-difference series? How are the generalized and summation formulas derived? Read the case material in small groups so that students can solve the following problems in cooperative learning: (1) What mathematical information do you get from the material? (2) How many years will factory equipment be scrapped? Is it possible to find the total value of loss of value in a particular year? Through the way of assigning cooperative learning tasks, promoting students' cooperative learning, cultivating students' problem-solving ability, unity, and cooperation ability, learning to learn, being able to acquire new knowledge in cooperative learning, improving the knowledge structure, and mastering the method of mathematical learning.

#### **5.4 Implementing Multiple Teaching Evaluation**

Teachers should optimize the design of unit teaching evaluation and improve the teaching evaluation system based on the perspective of the PBL teaching mode.

The effective implementation of teaching evaluation promotes the improvement of teaching quality. Here are some methods that math teachers can use:

(1) Regular classroom assessment of assignments and projects: such as quick quizzes, class discussions, question and answer sessions, whiteboard assignments, and so on. This helps teachers to keep abreast of students' mastery of new knowledge. Teachers can design assignments or projects of different types and levels of difficulty, including written assignments, inquiry-based projects, mathematical modeling, etc. By evaluating students' assignments and projects, we can understand their independent thinking ability and math application ability.

(2) Evaluation of group work, encouraging self-assessment and peer assessment: Students are organized to work on group projects to observe their ability to work together in a team, their leadership skills, and their communication skills. By observing group activities, students' teamwork and communication skills are assessed.

Students are also encouraged to carry out self-evaluation, and they can also be guided to carry out peer evaluation to learn and communicate with each other. This method of mutual evaluation helps students to understand mathematical concepts more comprehensively and to increase their respect for and cooperation with each other.

(3) Comprehensive assessment and feedback: Teachers combine multiple assessment methods and conduct comprehensive assessments to provide students with timely, specific, and personalized feedback. This helps students understand their strengths and areas for improvement and motivates them to learn. In addition to standardized tests, teachers can use other forms of assessment, such as oral quizzes, open-ended questions and answers, and practical application problems.

## **6. Conclusion**

PBL teaching, as an effective way to cultivate students' mathematical thinking, can promote changes in students' learning attitudes and methods. Based on core literacy, teachers should make full use of the advantages of PBL to reform the curriculum and create a good situation for students to experience the fun of mathematical knowledge while enhancing their learning ability. In addition, PBL teaching allows students to learn and explore problems independently in the math classroom, which helps teachers carry out overall unit teaching design. Mathematics teachers should make full use of classroom time to create

a good problem environment and unit planning for students to develop their thinking ability, mathematical logic ability, and practical application ability.

### Acknowledgements

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