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Research on the autonomous cooperative positioning scheme of UAV formation

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Abstract: *In recent years, drones have been widely used. The method of autonomous cooperative positioning of multiple drones to complete tasks is also widely used in various fields, such as search and rescue, surveillance, military missions, and so on. Based on this, this paper constructs a two-dimensional planar cone formation composed of a group of UAVs from the point of view of a circular geometric positioning model. This paper breaks this formation into a combination of regular hexagon drone formation and analyzes the regular hexagon UAV formation. Firstly, the signals emitted by FY00, FY01, and a known numbered drone are used to predict the position of the remaining drones. Then, the model is extended and expanded to predict the position of the remaining drones by constructing the electromagnetic signals emitted by FY00, FY01, and three unknown numbered drones. On this basis, considering the problem of poor electromagnetic signal reception caused by attitude deviation caused by body motion of UAV in real life, the PID algorithm is used in this paper to correct relative position and relative speed, and further improve the geometric autonomous cooperative positioning model of UAV in daily life. The results show that the study of UAV formation based on a geometric model has the advantages of intuitiveness, mathematical controllability, and simplification, which are helpful in understanding, analyzing, and solving formation-related problems.*

Keywords: *UAV, geometric autonomous cooperative positioning model, Regular hexagon formation, PID control algorithm*

1. Introduction

With their low cost, flight time, and low error rate, drones are widely used in scientific and technological urban construction, agriculture, military, and industry. However, due to the constraints of large-scale transportation, mapping, complex paths, and complex environments, it is difficult for a single drone to complete the task by itself, and multi-UAV formation has emerged. When studying the problem of multi-UAV formation, it is necessary to consider the problem of location, signal transmission, obstacle avoidance, and deviation correction. In the complex dynamic field, the autonomous cooperative positioning problem of multi-UAV formation is still a complex problem that varies greatly with time and space.

Huang, Wenhui Ma, and Jiacheng Li [1] combined the depth deterministic strategy gradient and Greedy choice to construct the greedy DDPG algorithm and designed the obstacle avoidance and collision avoidance control strategy of the wingman group. It can be seen from the simulation that the obstacle avoidance ability of UAVs has been effectively improved, which is of great significance for drone formation flying in the location environment. Haisi Li [2] solved the scenario of multiple drones tracking a single moving radiation source. Combining the Kalman filter and extended Kalman filter tracking algorithms and path planning algorithms, he proposed a multi-UAV tracking and positioning path planning model with the current positioning error as a cost function, thus improving the positioning accuracy of moving targets. Aiming at the coverage control problem of river inspection, Dejin Ma, Yang Chen, and Zhenhua Zhu [3] planned the route of multi-UAV river inspection according to the changing dynamic characteristics of the river. Firstly, a dynamically adjusted virtual space is constructed, and two virtual nodes are arranged at both ends to divide different sub-regions of UAV monitoring and evaluate the river. They effectively solve the problem of river inspection in complex spaces. Similarly, Xiumin Zhu [4] studied the high energy efficiency deployment of drones in response to the problem of multi-UAV flight paths. She proposed the problem of three-dimensional deployment and energy consumption of drones and developed a multi-objective optimization model covering utility and energy consumption. To improve the diversity of solutions, she proposed a hybrid initial resolution scheme to obtain a multi-UAV deployment strategy.

The task assignment of multiple drones is also a difficult problem in real life. In this regard, Tao Xie, Jiansheng Guo, Xiaofeng Zhang, et al. [5] proposed a distributed multi-UAV task assignment method based on CNP. Among them, alliance formation is a basic step of the allocation method, which adjusts the resource differences among drones based on the resource consumption allocation algorithm of the Gini coefficient. UAV also play a very important role in the military field. Yuan Niu et al. [6] built a multi-target optimization model based on the suppression task of enemy air defense on a static ground multi-target. At the same time, for the task assignment problem, they improved the NSGA-II algorithm and introduced it into the coding to avoid the time delay. They also make use of target and task hybrid variation operations to improve the uniform distribution of knowledge. In complex environments, UAV cooperative work also has the problem of insufficient obstacle avoidance ability. Junmin Zhao, Haozhe He, Shaoqi Wang [7] et al used the Lyapunov guidance vector field to obtain the target tracking speed command of UAV. The obstacle avoidance speed command of UAV is obtained by using the artificial potential field method. They combined the two methods based on zero space to obtain the integrated UAV speed command so that the UAV can have good coordination. Jie Cheng et al. [8] designed UAV cooperative track planning based on the altitude layer architecture of airspace by using the existing trajectory algorithm and solved the two sub-problems based on the improved artificial potential field method of particle swarm optimization. They provide a feasible multi-aircraft flight path planning method for ultra-low altitude urban logistics scenarios.

In this paper, the PID control algorithm is used to correct the flight path, and the geometric autonomous cooperative positioning model of UAV formation is built based on the circular model under certain conditions. It can effectively solve the problem of autonomous cooperative positioning systems of UAV formation.

2. Research on autonomous cooperative positioning of UAV formation based on geometric model

2.1 Build the basic geometric positioning model

First of all, a regular hexagonal drone formation model frame is constructed: as shown in Figure 1, 6 drones are evenly distributed, and 1 is located in the center of the hexagon. Among them, all drones have fixed numbers, several drones in the formation are responsible for transmitting signals, and other drones passively receive signals and extract directional information from them for positioning, that is, the receiving signal drone extracts the Angle information. However, to maintain electromagnetic silence, the geometric autonomous cooperative positioning model needs to select as few drones as possible to transmit signals, to achieve the path correction of the drone formation.

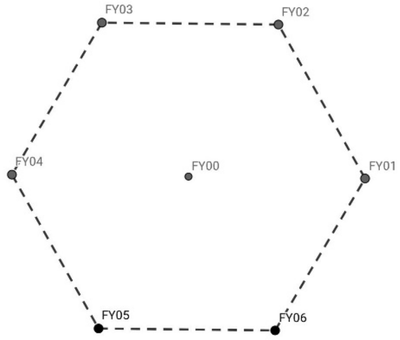

Figure 1: Geometric model of a regular hexagonal drone formation

In this paper, we first consider the case of FY00, FY01, and a UAV with a known number (FY0I) emitting electromagnetic signals, and any UAV in the formation (FY0J) receiving signals.

2.1.1 Scenario 1: FY0J receives signals from FY00 and FY01

First of all, this paper establishes a rectangular coordinate system with FY00 as the origin, FY00 as the positive direction of the X axis along the direction of FY01, and the direction of 90° counterclockwise rotation in the positive direction of the X axis as the positive direction of the Y axis. The following geometric relationship can be obtained.

Let (x,y) be the position of FY0J in the above rectangular coordinate system; R is the distance between FY00 and FY01; $R_1$is the radius of the circle formed by FY00, FY01, and a receiving UAV; α_{oj1} is the angle value of $\angle oj1$ formed by the connection of FY00, FY0J, and FY01, as shown in Figure 2.

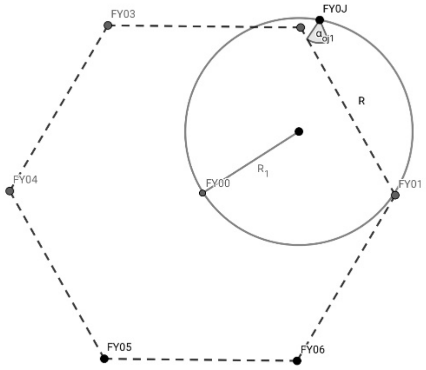

Figure 2: Schematic diagram of FY0J (taking FY02 as an example) receiving signals of FY00 and FY01

Construct the following equation:

$$(x - R_1 * sin\alpha_{oj1})^2 + \left(y - R_1 * \cos\alpha_{oj1}\right)^2 = {R_1}^2 \tag{1}$$

$$R_1 = \frac{R}{2\sin\alpha_{oj1}} \tag{2}$$

2.1.2 Scenario 2: FY0J receives signals from FY01 and FY0I

In this paper, the following images are obtained based on the signal information of FY01 and FY0I received by FY0J, as shown in Figure 3.

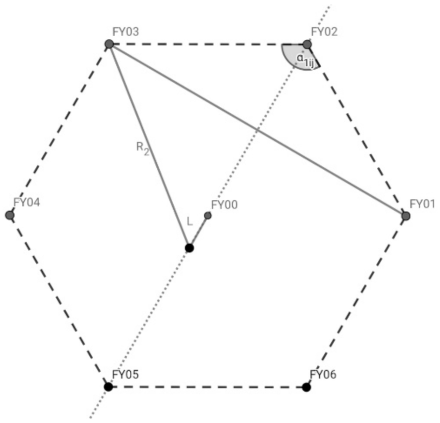

Figure 3: Schematic diagram of FY0J (using FY02 as an example) receiving signals for FY01 and FY0I (using FY03 as an example)

Where R_2 is the radius of the circle formed by FY01, FY0I and FY0J; l is the length of the line segment connecting the center of the circle where R_2 is and FY00, and α_{1ji} is the value of $\angle 1ji$ formed by the connection of FY01, FY0J and FY0I in turn. The equation constructed in analogy case l can be obtained as follows:

$$(x - l * cos\frac{\pi}{6}(i-1))^2 + (y - l * sin\frac{\pi}{6}(i-1))^2 = {R_2}^2 \tag{3}$$

$$R_2 = \frac{Rsin(\frac{\pi}{6}*(i-1))}{\sin\frac{\alpha_{1ij}}{2}} \tag{4}$$

$$l = Rcos\frac{\pi}{6}(i-1) + R_2 cos\alpha_{1ji} \tag{5}$$

The equation constructed in cases 1 and 2 can solve the position factors of x and y concerning FY00.

Therefore, this paper can obtain the position information of any UAV in the regular hexagonal UAV formation through the electromagnetic signals emitted by FY00 and any two known numbered drones, so that the position of the UAV can be corrected through this information.

2.2 Improvement of geometric model

In the basic geometric positioning model, FY00 and two drones with known numbers are crucial, but the drones transmitting signals may have unknown numbers, so this paper needs to consider this case. To this end, this paper deepens the definition of the model: FY00, FY01, and three unknown numbered drones emit signals, which can ensure that all drones on the regular hexagon can be positioned by the signals of these five drones. To test this model, this paper assumes that FY02 is a UAV with position deviation that needs to be corrected and explains it as follows.

2.2.1 FY00 and FY01 transmit signals to FY02

As shown in Figure 2, the following equation can be obtained through signal detection in this paper:

$$(\mathrm{x}-\mathrm{R}'\sin\alpha_1)^2+(y-R'cos\alpha_1)^2=R'^2 \tag{6}$$

$$R'=\frac{\rho}{2\sin(\pi-\theta_1-\alpha_1)} \tag{7}$$

Where R' is the radius of the circle formed by FY00, FY01 and FY02; θ_1 is the Angle formed by the connection of FY02, FY00 and FY01 in turn; α_1 is the angle formed by the connection of FY00, FY02 and FY01; (x,y) is the coordinates of FY02 in this coordinate system; ρ is the distance between FY00 and FY02.

2.2.2 FY0I (FY0J, FY0k) and FY00 transmit signals to FY02

At the same time, this paper adds a drone with an unknown number FY0I and makes it emit signals, as shown in Figure 4.

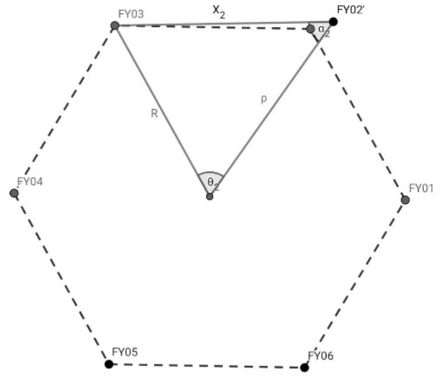

Figure 4: FY0I (FY0J, FY0k) and FY00 transmit signals to FY02

Using the sine theorem, the cosine theorem and geometric relations, the following equation can be obtained:

$$\frac{\sin(\pi-\theta_2-\alpha_2)}{\rho}=\frac{sin\alpha_2}{R}=\frac{sin\theta_2}{X_2} \tag{8}$$

$$\cos\alpha_2=\frac{{X_2}^2+\rho^2-R^2}{2X_2\rho} \tag{9}$$

$$\theta_2=\frac{2\pi(\mathrm{I}-1)}{6}-\theta_1 \tag{10}$$

Among them,$\alpha_2$is the angle formed by the connection of FY0I, FY02 and FY00 in turn. X2 is the distance between FY0I and FY02;$\theta_2$is the Angle formed by connecting FY0I, FY00, and FY02 successively.

Similarly, FY0J and FY00 transmit signals to FY02, and the following equation can also be obtained:

$$\frac{\sin(\pi-\theta_3-\alpha_3)}{\rho}=\frac{sin\alpha_3}{R}=\frac{sin\theta_3}{X_3} \tag{11}$$

$$\cos\alpha_3 = \frac{{X_3}^2+\rho^2-R^2}{2X_3\rho} \tag{12}$$

$$\theta_3 = \frac{2\pi(\mathrm{J}-1)}{6} - \theta_1 \tag{13}$$

By analyzing these equations, θ_1, ρ, i are represented by X_2,that is:

$$\theta_1 = f(X_2), \rho = g(X_2), i = h(X_2) \tag{14}$$

Combining the above equations, the unknowns are X_2, j and X_3. X_2, j and X_3 are three unknowns, two of which can be represented by the same unknown, and X_2 and j are represented by X_3, that is:

$$X_2 = k(X_3), j = l(X_3) \tag{15}$$

Combined with the formula of FY0k and FY00 transmitting signals to FY02, we can see that there are three unknowns x_4, x_3, and k.

However, after the introduction of FY0k, there must be two Angle inclusion relationships among α_2,$\alpha_3$and α_4, as shown in Figure 5.

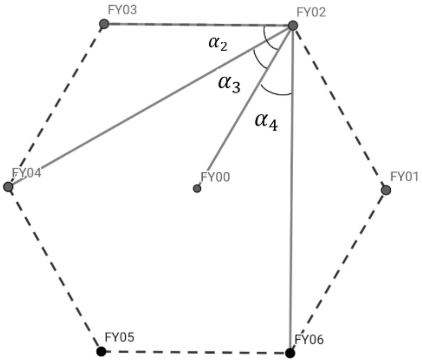

Figure 5: The inclusion relationship among $\alpha_2, \alpha_3, \alpha_4$

The corresponding result of each inclusion relation is the same, so this paper only uses the geometric relation in Figure 5, in Δ (FY02, FY03, FY04), the distance between FY03 and FY04 is known, set as d, we know that:

$$\boldsymbol{d} = \frac{\mathbf{Rsi\,n}\left(\frac{2\pi}{6}|j-i|\right)}{\mathbf{co\,s}\left(\frac{\pi|j-i|}{6}\right)} \tag{16}$$

$$cos\alpha = \frac{x_2^2+x_3^2-d^2}{2x_2x_3} \tag{17}$$

The distance between FY03 and FY02 is x_2. The distance between FY04 and FY02 is x_3. Set $\alpha = \alpha_2 - \alpha_3$ (α_2, α_3 are known angle, reflected by receiving information of unmanned aerial vehicle (UAV)).

A total of four equations with three unknowns, can solve the specific position of FY02. For any UAV, the above method can be used to locate, and only FY02 is taken as an example here.

To sum up, this paper can correct the geometric position of any UAV on the circle through FY00, FY01, and three UAV models with unknown numbers.

2.3 Model optimization

During flight, the speed and direction of the UAV formation are prone to change. In order not to be affected, the PID control algorithm is adopted in this paper, in which the relative position and speed of the UAV depend on the formation coordinate system and the body coordinate system, and the coordinate system is converted through the transformation matrix [9].

The body coordinate system and formation coordinate system are converted as follows:

$$R_{\frac{b}{nv}} = R_\phi R_\theta R_\psi$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \mathrm{sIn}\,\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \mathrm{sIn}\,\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} = \begin{bmatrix} \cos\mu\cos\varphi & \sin\varphi & \sin\mu\cos\varphi \\ -\cos\mu\sin\varphi & \cos\varphi & -\sin\mu\sin\varphi \\ -\sin\mu & 0 & \cos\mu \end{bmatrix} \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} \tag{19}$$

Where the x-axis is the horizontal motion direction, the y-axis is the horizontal vertical direction and the right wing is positive, the z-axis can be up or down, φ and θ are the track azimuth, μ is the track tilt Angle, R_ϕ is the yaw Angle rotation matrix, R_θ is the pitch angle conversion matrix, and R_ψ is the roll Angle rotation matrix.

After the coordinate system is converted, the PID control algorithm needs to be established in this coordinate system to obtain the deviation values of the direction Angle and velocity, to accurately correct the direction Angle and velocity. PID is an algorithm based on the integration of proportion, integral, and differential.

Generally, the algorithm [10] is expressed as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt} \tag{20}$$

First, for the directional angle of the UAV, the following symbol definition is made in this paper: R(t) represents the motor output change; k_{pv}, k_{iv}, and k_{dv} represent the proportional coefficient, integral coefficient, and differential coefficient on speed adjustment respectively, and $e(t)$ represents the weighted error on time. $\sum_{n=0}^{t} e(n)$ represents the error integral after discretization; I_x represents the derivative coefficient of the X-axis motion direction error in the body coordinate system; I_v represents the velocity derivative error coefficient; $(x_{li} - x_{li}^l)$ represents the x-direction position error between the body under test and the host; $v_l^l cos\theta_i^l$ represents the estimated value of i relative to the velocity in the host coordinate system; $(v_l^l cos\theta_i^l - v_i)$ represents the relative speed difference between the UAV under test and the main engine FY00, v_i represents the speed of the UAV under test; E_i represents the offset of the body position of the i station, and $e(t) - e(t-1)$ represents the error within each time interval.

Based on the relative speed of UAV, the following formula is obtained in this paper:

$$R(\mathrm{t}) = k_{vp} * e(t) + k_{vi} \int_0^t e(\varepsilon)d\varepsilon + k_{vd} \frac{de(t)}{dt} \tag{21}$$

By discretizing this formula, we can get:

$$R(t) = k_{pv} e(t) + k_{iv} \sum_{n=0}^{t} e(n) + k_{dv}\,[e(t) - e(t-1)] \tag{22}$$

$$e(t) = I_x(x_{li} - x_{li}^l) + I_v(v_l^l cos\theta_i^l - v_i) + E_i \tag{23}$$

The above three formulas mainly control the speed, use the given integral and differential to control the feedback of the formula, calculate the position error, and then adjust, you can get the vector difference on the speed. Where $k_{dv}[e(t) - e(t-1)]$ gets the amount of adjustment, that is the amount of adjustment assigned to the motor to output power.

This paper has the same algorithm for the direction Angle of UAV. This article first makes the following symbol description: $W(t)$ represents the change of steering gear direction, k_{pp} and k_{pd} represent the proportional coefficient and differential coefficient of direction adjustment respectively, I_y represents the derivation coefficient of Y-axis motion direction error in the body coordinate system, I_ρ represents the derivation coefficient of direction error, $(y_{li} - y_{li}^l)$ represents the Y direction position error between the tested body and the main engine. ρ indicates the heading Angle, $\Delta\rho$ indicates the error between the measured direction and the actual direction, and H_i indicates the direction offset of the body under test.

The following equation can be obtained:

$$W(t) = k_{pp} e'(t) + k_{pd}[e'(t) - e'(t-1)] \tag{24}$$

$$\Delta\rho = \rho_l - \rho_i \;, \rho\epsilon[0,2\pi] \tag{25}$$

$$e' = I_y(y_{li} - y_{li}^l) + I_\rho \Delta\rho + H_i \tag{26}$$

Through the above formula, this paper can analyze the direction Angle correction through the measured data, obtain the relative error, and correct the error, to make the geometric autonomous cooperative positioning model of the UAV more accurate. The error of measurement model result caused by deviation of direction Angle and velocity is avoided effectively.

3. Result analysis

For the regular hexagon formation geometric drone formation positioning model constructed, this paper has two geometric positioning methods to determine the specific position of the north UAV, namely FY00, FY01, and a UAV with a known number (FY0I) emitting electromagnetic signals, FY00, FY01, and three unknown unmanned aerial vehicles emitting electromagnetic signals. In this paper, the corresponding equation can be obtained by geometric relation and three-circle intersection positioning method, and the specific position of any measured UAV on the regular hexagon UAV formation can be obtained by solving the equation.

At the same time, to ensure the accuracy of the model, the PID algorithm is used to determine the relative position and relative speed of each UAV in the UAV formation and correct their travel path, to make the geometric autonomous cooperative positioning model more accurate.

4. Conclusion

Aiming at the established regular hexagon UAV formation, this paper obtains a geometric autonomous positioning model, which can obtain the specific relative position and relative speed of the UAV and the scheme of adjusting the UAV formation. This method makes the design of the formation control strategy more intuitive. PID correction of the UAV position improves the stability of the formation and visualizes the path of the UAV, thus enabling the researchers to understand the spatial position in this model, which greatly improves the efficiency of the UAV.

The geometric autonomous cooperative positioning model of UAV formation can be applied in military, civil, traffic management, and environmental monitoring, and has important guiding significance. In general, the study of UAV formation based on a geometric model provides a powerful theoretical basis and engineering method for realizing a safer, more efficient, and multi-functional UAV formation system.

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