

Topological Properties and Destruction Resistance Analysis of Shenzhen Metro Network

Hailong Wang

*Department of Transportation College, Shanghai Maritime University, Shanghai, 201306, China
15221934968@163.com*

ABSTRACT. *In order to analyze the topological properties of complex Metro network, this paper takes Shenzhen as an example, choosing 8 metro lines in Shenzhen as of August 2019 and 168 station nodes as sample data, and constructing a complex network model based on adjacent stations using complex network theory. This method takes metro traffic stations as nodes and metro traffic lines between adjacent stations as edges, which makes the network have the topological properties of complex networks. The characteristics of node degree, average degree, agglomeration coefficient, average shortest path and centrality in the network are analyzed. In addition, the importance of these sites is explored by some important nodes deliberately destroying. By calculating, it is found that the nodes of Shenzhen Metro network occupied a large proportion between 14 and 30, which is 84%. The average node degree of Shenzhen metro network is 29.6, and the average node degree is between 14 and 30. It shows that Shenzhen Metro has serious traffic connectivity. After deleting the important seven nodes, the average node degree decreases by 4, the average shortest path increases by 0.3, and the network diameter or clustering coefficient changes relatively small, showing that these seven nodes play an important role in the transfer trip and can effectively reduce the average transfer times. These laws provide a new reference for optimizing Shenzhen metro traffic network and traffic planning development.*

KEYWORDS: *complex network, network connectivity, network optimization, topology characteristics, metro network*

1. Introduction

In recent years, the development of Metro in various places has shown a rapid momentum. Every place has reported to the central government for the application of establishing rail transit. With the successful closing of the 19th National Congress, the general secretary's proposal to promote the development of transportation has

made the development of Metro glow with a second life. Compared with other means of transportation, subway has obvious advantages. Firstly, it has a large volume of traffic, and its transportation capacity is more than tenfold that of ordinary public transportation. With a population of more than 10 million city, Shenzhen, a developed economy, has completed such a large volume of transportation only by eight rail transit lines. Secondly, the speed of the subway can reach tens of kilometers per hour or even more than 100 kilometers. Considering the traffic jam and speed, ordinary buses can't meet the task of rapid and on-time transportation of such a large traffic flow in city. In addition, the construction of Metro saves land, noise, energy and provides pollution. Now advocating the concept of environment-friendly, resource-saving and sustainable transportation, vigorously developing the subway can make the city's traffic development meet the requirements of the times. Therefore, since 2004, Shenzhen had begun to open its first line and become the fifth Metro City in mainland. At present, only 13 years of development, Shenzhen Metro has made tremendous progress. At present, Shenzhen has eight Metro lines named Line 1, Line 2, Line 3, Line 4, Line 5, Line 7, Line 9 and Line 11, with 168 station nodes. The whole Metro Line in Shenzhen reaches 285 kilometers. The total mileage has developed to the fourth place in China, forming a metro network covering the whole city. According to the survey, the average daily passenger traffic volume of Shenzhen Metro reached 2.37 million in 2014, with an annual passenger traffic volume of 860 million. In 2015, the daily passenger traffic volume reached 2.54 million, and the annual passenger traffic volume reached 930 million. In 2016, the daily passenger traffic volume reached 2.99 million, and the annual passenger traffic volume reached 1.1 billion. The huge passenger volume and increasing passenger demand require Shenzhen Metro to speed up its development and form a wider metro transport network, which all pose new challenges to Shenzhen Metro traffic planning. Therefore, the analysis and evaluation of metro network and network nodes are particularly important. On the one hand, with the rapid development of complex network discipline, the average shortest path, network diameter, agglomeration coefficient, small world network, scale-free network and other reference values can be formed to analyze and evaluate the topological properties of Shenzhen Metro network. On the other hand, the failure of some nodes in the metro network influences the whole Metro network. By calculating the importance of some nodes to the whole network through mathematical methods, we can attach importance to some important nodes and increase the number of important nodes construction, so that the failure rate of important nodes is lower than that of other nodes.

2. Research status at home and abroad

In recent years, the study of complex networks has gradually become a hot issue for some scholars at home and abroad. Liu Zhixiang constructed urban rail transit as a weighted rail transit network based on complex network theory and graph theory in his research on robustness of weighted network of urban rail transit. This paper focuses on the changes of network traffic after the network is attacked. By studying four aspects of rail transit, namely, the measurement of link weight, network

collapse, cascade failure and identification of traffic bottlenecks, some models and mathematical solutions are established [1]. Qu Yingchun drew the conclusion in the vulnerability analysis of urban rail transit network that the improvement of urban public transport network system can effectively reduce the vulnerability of rail transit [2]. Lai Liping put forward the index to measure the complexity of the urban rail transit network in "Research on the Complexity Characteristics of the Urban Rail Transit Network", and verified the index by comparing and analyzing the different stages of the construction of Beijing rail transit [3]. Li Qian classified the comprehensive importance of the nodes in "The Importance Evaluation of Urban Rail Transit Network Nodes and the Study of Cascade Failure Resistance", and pointed out some suggestions to improve the network survivability optimization [4]. Jin Guang identified the important nodes of the network through the network invulnerability in Analyzing the Network Vulnerability of Rail Transit Based on Complex Theory, and worked out effective methods and strategies to prevent emergencies [5]. Casey P. Shannon from abroad studies the data expression of the whole gene through gene network reasoning in " SABRE: a method for assessing the stability of gene modules in complex tissues and subject populations [6]". Laurie A. Schintler used these to judge the interdependence of the network and the endurance ability of the network to be destroyed in the Using Raster-Based GIS and Graph Theory to Analyze Complex Networks [7]. Kentaro Katahira simulated the complex neural network model of high-order Markov process through the nervous system of birds in " A neural network model for generating complex birdsong syntax [8]". J. Reichardt and D. R. White put forward the concept of structure and modularity in " Role models for complex networks " and applied it to the world trade network [9]. S. Shabbih. U. H. Jafri, P. Johnson explores how to display the behavior of real world networks in random networks in the Modeling complex network systems architecture and growth [10]. Glenn Lawyer's "Measuring the potential of individual airports for pandemic spread over the world airline network " explains some of the network characteristics of pandemic transmission at the macro level [11].

3. Construction and calculation of metro network model

3.1 Introduction to the Current Situation of Shenzhen Metro Network

According to Shenzhen Metro official network information, the length of the whole Shenzhen Metro operation line reaches 285 kilometers, the total mileage of the subway operation ranks 5th in China. There are 8 lines, including Line 1, Line 2, Line 3, Line 4, Line 5, Line 7, Line 9 and Line 11. There are 199 stations (168 nodes in the subway network). Thirty-one nodes are connected to each other. There are 4988 pairs of pairs between nodes. As shown in Table 1.

Table 1 Statistics of Shenzhen Metro Network

City	Metro Line	Metro Node	Relation Pairs
Shenzhen	11	168	4988

3.2 Introduction of Concepts Related to Complex Networks

In this paper, the main content of Shenzhen Metro complex network is the topological index of complex network. Network topology is the basis of studying the characteristics and functions of network structure. The basic concepts of related complex networks are as follows:

3.2.1 Node Degree and Average Degree

The degree k_i of a node v_i is defined as the number of edges connected by that node. Intuitively, the greater the degree of a node, the more important it is in a sense. The average degree of all nodes in the network is called the average degree of the network [12], which is marked as $\langle k \rangle$:

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i \quad (1)$$

3.2.2 Agglomeration coefficient

Generally, assuming that a node in a network has k_i edges and connects it to other nodes, this node is called the neighbor of node a . Obviously, there may be at most $C_{k_i}^2$ edges between these nodes k_i . The ratio of the actual number of edges E_i and the total possible number of edges $k_i(k_i - 1)/2$ between the k_i neighboring nodes of node v_i is defined as the aggregation coefficient C_i [1][12]:

$$C_i = \frac{2E_i}{k_i(k_i-1)} \quad (2)$$

The aggregation coefficient C of the whole network is the average value of the aggregation coefficient C_i of all nodes v_i [12]:

$$C = \frac{1}{N} \sum_{i=1}^N C_i \quad (3)$$

3.2.3 Average Shortest Path

The average path length is the average of the shortest path length between all pairs of nodes in the network, that is, the average shortest path. Average path length can be used to determine the average degree of separation of node pairs in the network, reflecting the links between sites [12]. The formula for calculating the average path length L is as follows:

$$L = \frac{2}{N(N-1)} \sum_{i \geq j}^N d_{ij} \quad (4)$$

N represents the number of network nodes and is the distance between node i and node j .

3.2.4 Average Shortest Path

The average path length is the average of the shortest path length between all pairs of nodes in the network, that is, the average shortest path. Average path length can be used to determine the average degree of separation of node pairs in the

network, reflecting the links between sites [12]. The formula for calculating the average path length L is as follows:

$$L = \frac{2}{N(N-1)} \sum_{i \geq j}^N d_{ij} \quad (5)$$

3.2.5 Centrality

Median Centrality: The Median Centrality of Node v_i is the normalized Median of Node v_i . What we need to determine here is the maximum possible number of intermediaries. For undirected networks, this value is equal to the maximum possible number of nodes X except node v_i . If the median of node v_i is B_i , its median centrality $C_B(v_i)$ can be defined as[2]:

$$C_B(v_i) = 2B_i / [(N - 1)(N - 2)] \quad (6)$$

Close centrality: proximity is one of the basic concepts in topological space. For two subsets in Euclid space, the closer the two subsets are, if the average Euclid distance between all elements pairs of two subsets is smaller. This concept can be extended to graph theory. Although there is no concept of Euclid distance, it can be represented by the shortest path. The degree of proximity of nodes reflects the degree of centrality of nodes in the network, and it is one of the indicators to measure the centrality of nodes. Define d_{ij} as the distance from point v_i to node v_j . For undirected connected graphs, the most natural definition of compact centrality $C_c(v_i)$ of nodes can be expressed as[4]:

$$C_c(v_i) = \frac{(N-1)}{\sum_{\substack{j=1 \\ j \neq i}}^N d_{ij}} \quad (7)$$

That is, the degree of proximity represents the reciprocal of the sum of the shortest distances from a node to all other nodes multiplied by the number of other nodes. The closer the nodes are, the more important they are in the network.

Eigenvector centrality: It assigns a relative score to each node in the network. In the contribution to a node's score, the connection to the high-score node is larger than that to the low-value node. Google and PageRank are variations of the centrality of eigenvectors. The centrality of eigenvectors is defined by adjacency matrix A . For node v_i , if its centrality score x_i is proportional to the sum of centrality scores of all nodes connected to it [12]:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^N a_{ij} x_j \quad (8)$$

In the formula: N is the total number of nodes: λ is a constant.

3.3 Construction and Solution of Model

In this paper, we construct an undirected network model without considering the distance between sites, only considering whether the sites are connected or not.

GEPHI and MATLAB software are used to calculate the related indicators of the topological properties of Shenzhen Metro network. By discussing the importance of some nodes to the network, the important nodes of Shenzhen metro network are evaluated.

In the model construction, each subway station is regarded as a node of the network, and each node is numbered. For example, the East Airport Station and Hourui Station in Shenzhen Metro Network are regarded as two nodes of the network, and the number of the East Airport Station is No1, while the Hourui Station is No2 (part of the 168 stations are numbered as shown in Table 2). After numbering, the adjacent matrix A (i, j) can be obtained.

$$A(i, j) = \begin{bmatrix} 0 & 1 & 1 & \dots & \dots & \dots & 0 & 0 & 0 \\ 1 & 0 & 1 & \dots & \dots & \dots & 0 & 0 & 0 \\ 1 & 1 & 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \dots & 0 & 1 & 1 \\ 0 & 0 & 0 & \dots & \dots & \dots & 1 & 0 & 1 \\ 0 & 0 & 0 & \dots & \dots & \dots & 1 & 1 & 0 \end{bmatrix}$$

Table 2 Network Node Number of Shenzhen Metro Station (Some Stations)

NO	Station name	No.	Station name	No.	Station name
1	Airport East	16	Window of the Century	31	Red Bay
2	Hou Rui	17	Overseas Chinese City	32	Shekou port
3	Fixed Garrison	18	Qiaochengdong	33	Sea World
4	Xixiang	19	Bamboo forest	34	Water Bay
5	Peng Chau	20	Chegong Temple	35	Dongjiaotou
6	Treasure body	21	Xiangmi Lake	36	Bay Mansion
7	Baoan Center	22	Shopping Parks	37	Sea moon
8	Xin'an	23	Exhibition Center	38	Deng Liang
9	Front Bay	24	Gangxia	39	Houhai
10	Cyprinus carpio	25	Huaqiang Road	40	Keyuan
11	Daxin	26	Science Museum	41	Mangrove Bay
12	peach orchard	27	Grand Theatre	42	Overseas Chinese
13	Shenzhen University	28	Old Street	43	Town North
14	High-tech Park	29	China World Trade Center	44	Shen Kang
15	Baishizhou	30	Luohu	45	Mount Anto

In the adjacency matrix, if the numbered i station and the numbered j station are on the same metro line and connected with each other, the assignment value is 1,

and if the numbered i station and the numbered j station are not on the same metro line or cannot connect with each other, the assignment value is 0. By establishing the adjacency matrix and the list of adjacent edges, the model of Shenzhen Metro network is drawn by using Gephi software as follows:

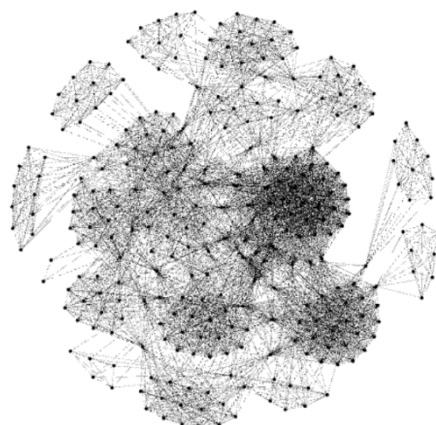


Figure. 1 Model of Shenzhen Metro Network

3.3.1 Calculation results of node degree of Shenzhen Metro

The degree of 168 nodes can be calculated. The number of nodes on line 1 is 30, the average node degree is 35.5, and the largest node degree is Chegongmiao Station, the value is 91; the number of nodes on line 2 is 29, the average node degree is 35, the node degree of node degree is the largest is Futian Station, the value is 73; the number of nodes on line 3 is 30, the average node degree is 36.9, and the node degree of node degree is the largest. Futian Station, the number of nodes is 73; the number of nodes on Line 4 is 15, the average node degree is 24.7, the largest node degree is Junior Palace Station and Convention and Exhibition Center Station, the number of nodes on Line 5 is 43; the number of nodes on Line 5 is 27, the average node degree is 33.2, and the largest node degree is Buji Station, the value is 55; the number of nodes on Line 7 is 28, the average node degree is 23. 37.1, the largest node degree is Chegongmiao Station, the value is 91; the number of nodes on Line 9 is 22, the average node degree is 29.3, the largest node degree is Chegongmiao Station, the value is 91; the number of nodes on Line 11 is 18, the average node degree is 29.8, and the largest node degree is Chegongmiao Station, the value is 91. In summary, the lines with the largest number of nodes are Line 1 and Line 3, with 30 nodes. The line with the largest average node degree is Line 7, and the line with the smallest average node degree is Line 4. On the whole, the average node degree of Shenzhen Metro network is 29.6, which means that there are nearly 30 edges connected to each node in the network. Among all the nodes, Chegongmiao Station is the node with the largest node degree, and its node degree value is 91, which indicates that Chegongmiao Station is of great importance in this metro network node. Node degrees between 14 and 30 account for 84% of the total number of

nodes. Most of the values of node degrees are concentrated between 14 and 30. The overall distribution of node degrees in Shenzhen Metro network is balanced and does not have the characteristics of scale-free network. It shows that the overall connectivity of Shenzhen Metro network is good, and the station is convenient to change.

Table 3 Network node degree of Shenzhen Metro (part of nodes)

Node Number	Node Degree
1	29
2	29
3	29
4	29
5	29
...	...
164	17
165	17
166	17
167	17
168	17

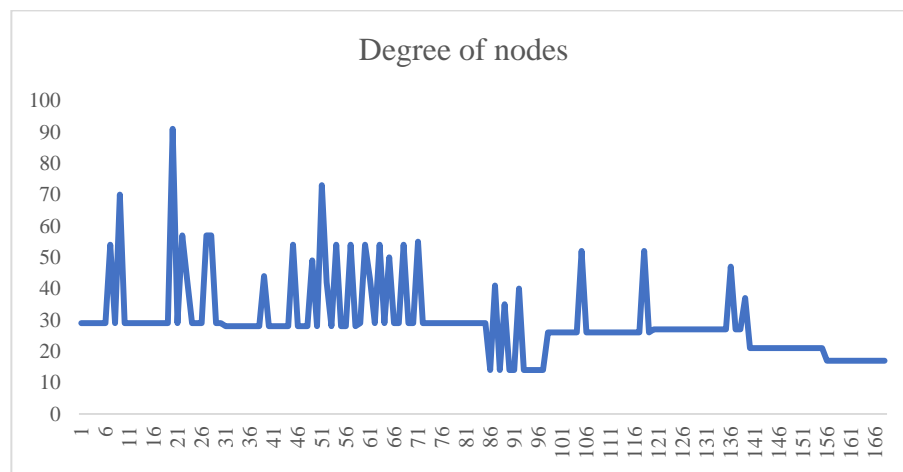


Figure. 2 Distribution of Node Degree of Shenzhen Metro Network

3.3.2 Average Shortest Path and Network Diameter Calculation

The shortest path represents a path that has the least number of edges in the process of starting from one node to another. The average shortest path of the network is the average of the shortest path between two nodes in the network. In this paper, the average number of transfers between any two stations in the subway network is reflected, which is an important index to evaluate the convenience of

Shenzhen Metro. The average shortest path of the network is calculated to be 1.9, which means that the average commuter needs to transfer once. The network diameter represents the maximum of all the shortest paths in a complex network. This paper reflects the maximum number of possible transfers of passengers during a trip, which is a limiting case in the network. The diameter of Shenzhen metro network is calculated to be 3. This shows that the destination can be reached by two transfers at most between any two stations in Shenzhen Metro Traffic Network. The calculation results also show that the average number of transfers needed to arrive between any two stations in Shenzhen Metro transportation network is 2.

3.3.3 Agglomeration coefficient

The aggregation coefficient of complex networks is an important index to describe the degree of network tightness. The clustering coefficient of the whole network is the average value of the clustering coefficient of all points in the network. Through the calculation of MATALAB software, the clustering coefficient of the whole network of Shenzhen Metro is 0.5, which shows that Shenzhen Rail Transit Network is a closely connected and well-planned transportation network. And because Shenzhen Metro network also has a relatively small shortest path, it shows that it is a rail transit network with small world network characteristics.

3.3.4 Centrality

The centrality and compactness of eigenvectors are calculated as shown in Table 4.

Table 4 Centrality and Close Centrality of Node Eigenvector

Node Number	Eigenvector Centrality	Close Centrality
1	0.0795191546056408	0.0032786885245902
2	0.0795191546056408	0.0032786885245902
3	0.0795191546056408	0.0032786885245902
...
166	0.0281103712896574	0.0030674846625767
167	0.0281103712896574	0.0030674846625767
168	0.0281103712896574	0.0030674846625767

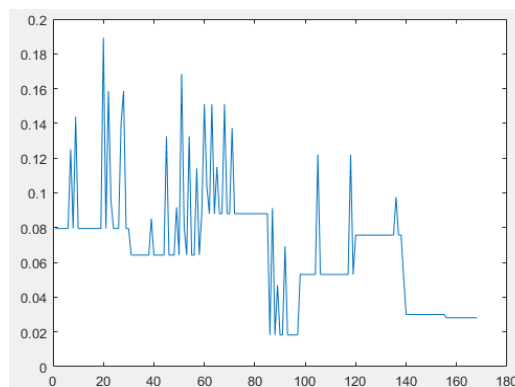


Figure. 3 Centrality of eigenvectors

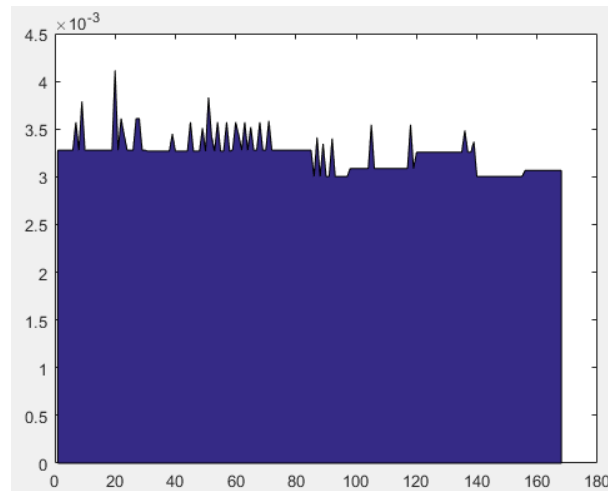


Figure. 4 Close centrality

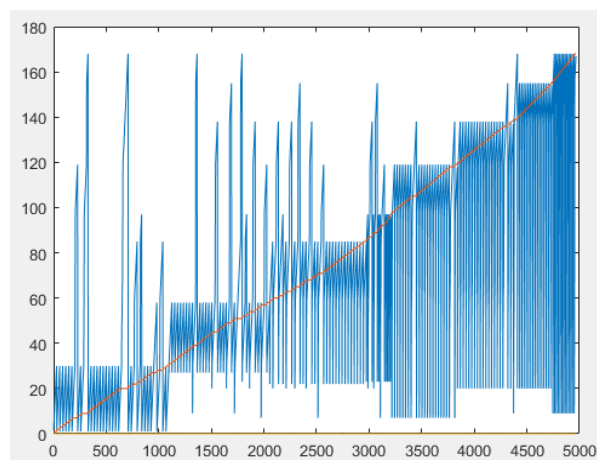


Figure. 5 Median Centrality

3.3.5 Analysis of Important Nodes in Metro Network

After the destruction and deletion of Chegongmiao Station (Node 20), Qianhaiwan Station (Node 9), Futian Station (Node 51), Buji Station (Node 71),

Shenzhen North Station (Node 92), Xili Station (Node 105), Jingtian Station (Node 49), the eigenvalues of the network nodes can be calculated:

The average node degree is 25.7, the network diameter is 3, the average shortest path is 2.2, and the aggregation coefficient is 0.6. By comparison, it is found that after destroying the above seven nodes, the average node degree decreases by 3.9, the average shortest path increases by 0.3, and the network diameter and clustering coefficient change slightly. This shows that these seven nodes play an important role in the transfer trip and can effectively reduce the average transfer times.

4. Conclusion

Taking Shenzhen Metro as an example, this paper establishes a complex network model and draws the following conclusions by using the complex network principle:

1. The average nodal degree of Shenzhen Metro complex network is 29.6. The distribution of the overall nodal degree and the average nodal degree is relatively balanced. The nodal degree of each line and the average nodal degree of each line are not very different, and they do not have the characteristics of scale-free network. These reflect the geographical advantages of Shenzhen Metro network, which shows that the overall connectivity of Shenzhen Metro network is good.

2. The average shortest path of Shenzhen Metro network is 1.9, that is to say, people need to transfer once on average. The diameter of the network is 3, which indicates that any two stations in Shenzhen Metro Traffic Network can reach their destination by two transfers at most, and the station is convenient to change.

3. The clustering coefficient of Shenzhen Metro Network is 0.5, which shows that Shenzhen Metro Network is a closely connected and well-planned transportation network. And because Shenzhen Metro network also has a relatively small shortest path, it shows that it is a rail transit network with small world network characteristics.

4. When Chegongmiao Station, Qianhaiwan Station, Futian Station, Buji Station, Shenzhen North Station, Xili Station and Jingtian Station are destroyed, the average node degree of Shenzhen Metro Network decreases by 13.2%, the average shortest path increases by 13.6%, and the number of transfers increases, which reduces the convenience of transfers. Therefore, similar important sites should be fully protected.

Metro system is a relatively complex system. Taking Shenzhen Metro Network as an example, the network characteristics of Shenzhen Rail Transit can be obtained through the study of Shenzhen traffic network topology and important nodes. Starting from the key nodes, the feasible strategies of protecting key nodes and optimizing the metro network are put forward, which can provide reference for increasing the convenience of Shenzhen Metro travel and the safety and reliability of the Metro system.

References

- [1] Liu Zhixiang. Research on robustness of weighted network for urban rail transit. Lanzhou Jiaotong University. 2017.06.
- [2] Qu Yingchun, Xu Zhongzhi, Gong Hang, Huang Zhiren and Wang Pu. Vulnerability analysis of urban rail transit network. Journal of Railway Science and Engineering Volume 13, No. 11.2016.11.
- [3] Lai Liping. Study on the Complex Characteristics of Urban Rail Transit Network. Journal of Chifeng University (Natural Science Edition), Vol. 33, No. 4 (I). 2017.04.
- [4] Li Qian. Importance evaluation of urban rail transit network nodes and cascade failure survivability study. Beijing Jiaotong University.2017.03
- [5] Jinguang, Masson Flower. Based on complex theory, the network vulnerability of rail transit is analyzed. Agricultural machinery of the times. Volume 44, No. 6.2017.06.
- [6] Casey P. Shannon, Virginia Chen, Mandeep Takhar, Zsuzsanna Hollander, Robert Balshaw, Bruce M. McManus, Scott J. Tebbutt, Don D. Sin, Raymond T. Ng. SABRE: a method for assessing the stability of gene modules in complex tissues and subject populations. BMC Bioinformatics, 2016,, Vol.17 (1).2016.12.
- [7] Laurie A. Schintler, Rajendra Kulkarni, Sean Gorman, Roger Stough. Using Raster-Based GIS and Graph Theory to Analyze Complex Networks. Networks and Spatial Economics, 2007, Vol.7 (4), pp.301-313. 2007.12.
- [8] Kentaro Katahira, Kazuo Okanoya, Masato Okada. A neural network model for generating complex birdsong syntax. Biological Cybernetics,2007,Vol.97 (5-6), pp.441-448.2007.12.
- [9] J. Reichardt, D. R. White. Role models for complex networks. The European Physical Journal B,2007,Vol.60 (2), pp.217-224.2007.11.
- [10] S. Shabbih. U. H. Jafri, P. Johnson, A. T. Bendiab. Modeling complex network systems architecture and growth. SPIE-the International Society for Optical Engineering. 2007.10.
- [11] Glenn Lawyer. Measuring the potential of individual airports for pandemic spread over the world airline network. BMC Infectious Diseases, 2015,Vol.16 (1). 2015.12.
- [12] Wu Jianjun. Study on the Complexity of Urban Traffic Network Topology [D]. Beijing Jiaotong University. 2008.