

# Remaining Useful Life Prediction of Nonlinear Wiener Process-Based Degradation Model with Fusing Failure Data Time

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**Abstract:** Aiming at the imperfection of historical degradation data for equipment, a remaining useful life (RUL) prediction method with fusing failure time data is proposed. Firstly, the nonlinear Wiener process-based degradation mode is used to model the degradation process of equipment. Then, based on the failure time data of congeneric equipment, the expectation maximization (EM) algorithm is used to estimate the unknown parameters in the model, in which the fixed parameters are calculated based on the field degradation data of the evaluated equipment. Finally, the degradation data of lithium-ion batteries are used to verify the proposed RUL prediction method. The experimental results show that for degradation data of equipment with imperfect prior information, the RUL prediction method with fusing failure time data is better than the traditional RUL prediction method.

**Keywords:** Fusing failure time data, Remaining useful life prediction, Parameter estimation, Wiener process, Nonlinear

## 1. Introduction

With the progress of society and the development of science and technology, electromechanical equipment such as subway, automobile and aircraft are widely used in daily civil and military fields. However, due to the increase of precision, complexity of production requirements and the lack of natural resources, the manufacturing cost of electromechanical equipment increases significantly, the whole life cycle of electromechanical equipment from design, manufacturing, sales, use, maintenance to scrapping needs to be optimized [1]. For some key electromechanical equipment in major fields, once the parts fail during operation, it would produce significant economic losses, and even cause major accidents such as casualties. For example, on June 22, 2009, due to track circuit failure, two southbound Washington subway trains collided in the northeast of Washington, D.C., killing 9 people and injuring 10 others [2].

The study found that in the actual operation process, due to the influence of complex working environment and surrounding random impact interference, the parts of electromechanical equipment would undergo a gradual deterioration process, resulting in the gradual decline of their performance [3]. In this process, if the maintenance personnel can predict the remaining useful life (RUL) during operation according to the performance degradation status or historical failure data of parts, track the health status of electromechanical equipment parts in real time, and formulate corresponding maintenance strategies to achieve planned maintenance or replacement of spare parts, it can greatly prevent the sudden occurrence of accidents. At the same time, it can reduce the preventive maintenance cost of replacing parts in advance. Prognostic and health management (PHM) technology is an effective technology to solve this kind of problem [4].

PHM technology uses monitoring information, expert knowledge and maintenance support information, with the help of artificial intelligence learning and reasoning model to realize the monitoring and prediction of equipment operation status, and then carry out intelligent maintenance. PHM technology is mainly divided into two parts [5]. One is prediction, which determines the performance status and the RUL of the evaluated equipment parts according to the historical degradation data, failure time data and the field degradation data of the evaluated equipment parts. The second is health management, that is to make the optimal maintenance decision according to the predicted performance status of equipment parts, the RUL and product instructions, so as to achieve the

lowest maintenance cost and minimum failure risk, mainly including determining the optimal maintenance time, formulating spare parts ordering strategy and providing a scheme to prolong service life [6,7]. Engineering practice shows that PHM technology can reduce maintenance costs, improve equipment reliability and safety, and reduce the risk of failure events. It is very important for military, aerospace and other fields with high safety and reliability requirements.

RUL prediction is one of the important components in PHM [8,9]. Its purpose is to evaluate the failure probability of equipment parts after running for a period of time according to the relevant degradation data, the failure time data of congeneric equipment parts and the field degradation data of the evaluated equipment [10]. In recent years, with the improvement of the reliability of complex electromechanical equipment, the research on RUL prediction has attracted great attention of scholars [11,12]. The traditional RUL prediction heavily depends on the historical degradation data of electromechanical equipment. However, under the existing conditions, it often occurs imperfect prior information, i.e., the historical degradation data of congeneric equipment are often inaccurate, incomplete [13,14] or even non-existent [5] in practical application. Therefore, this paper attempts to fuse the failure time data to predict the RUL of equipment.

The remainder is as follows. Section 2 develops a nonlinear Wiener process-based model and obtains the parameters estimation results by the expectation maximization (EM) algorithm with fusing failure time data. A case study of lithium-ion battery is provided in Section 3; and Section 4 draws the main conclusions.

## 2. RUL Prediction of Nonlinear Wiener Process-Based Model with Fusing Failure Time Data

### 2.1. Degradation Model

The nonlinear Wiener process-based model can be expressed as follows

$$X(t) = \lambda \Lambda(t; \theta) + \sigma_B W(\Lambda(t; \theta)) \quad (1)$$

Where  $\lambda$  denotes the drift coefficient,  $\theta$  is the nonlinear parameter,  $\sigma_B$  denotes the diffusion coefficient and  $W(t)$  is the standard Brownian motion. Let  $w$  denote the failure threshold of equipment. The probability density function (PDF) of lifetime can be written as [15]

$$f_T(t | \lambda) \approx \sqrt{\frac{w^2}{2\pi\sigma_B^2\Lambda(t; \theta)^3}} \exp\left(-\frac{(w - \lambda\Lambda(t; \theta))^2}{2\sigma_B^2\Lambda(t; \theta)}\right) \quad (2)$$

### 2.2. Parameters Estimation with Fusing Failure Time Data

Suppose that the field degradation data of the evaluated equipment at time  $[t_1, t_2, \dots, t_k]$  are  $x_{1:k} = [x_1, x_2, \dots, x_k]$ , let  $\Delta x_j = x_j - x_{j-1}$  and  $\Delta v_j = \Lambda(t_j; \theta) - \Lambda(t_{j-1}; \theta)$ , then,  $\Delta x_j$  follows the normal distribution, i.e.,  $\Delta x_j \sim N(\lambda \Delta v_j, \sigma_B^2 \Delta v_j)$ . In addition, it is assumed that there are  $n$  sets of failure time data  $T_{1:n} = [T_1, T_2, \dots, T_n]$  and the failure time data of the  $i$ th unit is  $T_i$ .

Inspired by Tang et al [16], we give a two-step parameters estimation method with fusing failure time data, which are as follows:

**Step 1:** Calculating the fixed parameter  $\theta$  and  $\sigma_B^2$  based on the field degradation data of the evaluated equipment

The log-likelihood function based on the field degradation data of the evaluated equipment can be expressed as

$$\ln L(\theta, \lambda, \sigma_B^2) = -\frac{k}{2} \ln 2\pi - \frac{k}{2} \ln \sigma_B^2 - \frac{1}{2} \sum_{j=1}^k \ln \Delta v_j - \frac{1}{2\sigma_B^2} \sum_{j=1}^k \frac{(\Delta x_j - \lambda \Delta v_j)^2}{\Delta v_j} \quad (3)$$

The estimation of  $\lambda$  and  $\sigma_B^2$  can be obtained by maximizing Equation (3), as shown in Equations (4) and (5).

$$\hat{\lambda}(\theta) = \frac{x_k}{t_k} \quad (4)$$

$$\hat{\sigma}_B^2(\theta) = \frac{1}{k} \sum_{j=1}^k \frac{(\Delta x_j - \hat{\lambda} \Delta v_j)^2}{\Delta v_j} \quad (5)$$

The estimation of  $\theta$  can be obtained by maximizing Equation (6) through the “FMINSEARCH” function of MATLAB.

$$\ln L(\theta | x_{1:k}) = -\frac{k}{2} \ln 2\pi - \frac{k}{2} \ln \sigma_B^2 - \frac{1}{2} \sum_{j=1}^k \ln \Delta v_j - \frac{k}{2} \quad (6)$$

Then, the estimation of  $\lambda$  and  $\sigma_B^2$  can be obtained by bringing  $\hat{\theta}$  into Equations (4) and (5).

**Step 2:** Calculating the random coefficient  $\mu_\lambda, \sigma_\lambda^2$  with fusing failure time data

The log likelihood functions of  $\Theta = (\mu_\lambda, \sigma_\lambda^2)$  based on failure time data can be obtained as

$$\begin{aligned} \ln L(\Theta | T_{1:n}, \lambda) = & -\frac{n \ln 2\pi}{2} + n \ln w - \frac{n}{2} \ln \sigma_B^2 - \frac{3}{2} \sum_{i=1}^n \ln \Lambda(T_i; \hat{\theta}) - \sum_{i=1}^n \frac{(w - \lambda_i \Lambda(T_i; \hat{\theta}))^2}{2\sigma_B^2 \Lambda(T_i; \hat{\theta})} - \frac{n \ln 2\pi}{2} \\ & - \frac{n}{2} \ln \sigma_\lambda^2 - \frac{1}{2\sigma_\lambda^2} \sum_{i=1}^n (\lambda_i - \mu_\lambda)^2 \end{aligned} \quad (7)$$

It is assumed that  $\hat{\Theta}^{(k)} = \{\hat{\mu}_\lambda^{(k)}, \hat{\sigma}_\lambda^{2(k)}\}$  is the estimation of  $\Theta$  in the  $k$ th step based on  $T_{1:n}$ . Then, the EM algorithm is implemented.

**E-step:** Calculating the expectation of the complete log-likelihood function

$$\begin{aligned} L(\Theta | \hat{\Theta}^{(k)}) = & E_{\lambda | T_{1:n}, \hat{\Theta}^{(k)}} \ln L(\mu_\lambda, \sigma_\lambda^2 | T_{1:n}, \lambda) \\ = & -\frac{n \ln 2\pi}{2} + n \ln w - \frac{n}{2} \ln \sigma_B^2 - \frac{3}{2} \sum_{i=1}^n \ln \Lambda(T_i; \hat{\theta}) - \sum_{i=1}^n \frac{(w - \lambda_i \Lambda(T_i; \hat{\theta}))^2}{2\sigma_B^2 \Lambda(T_i; \hat{\theta})} \\ & - \frac{n \ln 2\pi}{2} - \frac{n}{2} \ln \sigma_\lambda^2 - \frac{1}{2\sigma_\lambda^2} \sum_{i=1}^n [(\lambda_i - \mu_{\lambda,i})^2 + \sigma_{\lambda,i}^2] \end{aligned} \quad (8)$$

where

$$\mu_{\lambda,i} = E(\lambda_i | T_i, \hat{\Theta}^{(k)}) = \frac{w\sigma_\lambda^{2(k)} + \mu_\lambda^{(k)}\sigma_B^2}{\Lambda(T_i; \hat{\theta})\sigma_\lambda^{2(k)} + \sigma_B^2}, \sigma_{\lambda,i}^2 = \text{var}(\lambda_i | T_i, \hat{\Theta}^{(k)}) = \frac{\sigma_B^2\sigma_\lambda^{2(k)}}{\Lambda(T_i; \hat{\theta})\sigma_\lambda^{2(k)} + \sigma_B^2} \quad (9)$$

**M-step:** Maximizing  $L(\Theta | \hat{\Theta}^{(k)})$

$$\hat{\Theta}^{(k+1)} = \arg \max_{\Theta} L(\Theta | \hat{\Theta}^{(k)}) \quad (10)$$

Taking the first partial derivatives  $L(\Theta | \hat{\Theta}^{(k)})$  with respect to  $\mu_\lambda$  and  $\sigma_\lambda^2$ , and setting these derivatives to zero, the MLE of  $\mu_\lambda$  and  $\sigma_\lambda^2$  can be obtained as follows

$$\hat{\mu}_\lambda^{(k+1)} = \frac{1}{n} \sum_{i=1}^n \mu_{\lambda,i} \quad (11)$$

$$\hat{\sigma}_\lambda^{2(k+1)} = \frac{1}{n} \sum_{i=1}^n [(\mu_{\lambda,i} - \hat{\mu}_\lambda^{(k+1)})^2 + \sigma_{\lambda,i}^2] \quad (12)$$

Then, the above E-step and M-step are iterated until  $\|\Theta^{(k+1)} - \Theta^{(k)}\|$  is sufficiently small.

**2.3. RUL Prediction**

In order to further adapt to the degradation characteristics of a single individual, the random coefficient need to be updated based on Bayesian theory. Given the field degradation data of the evaluated equipment  $x_{1:k} = [x_1, x_2, \dots, x_k]$  and the prior information of  $\lambda$ , the posterior distribution of random coefficient can be obtained as

$$\lambda | x_{1:k} \sim N\left(\frac{x_k \sigma_\lambda^2 + \mu_\lambda \sigma_B^2}{\Lambda(t_k; \theta) \sigma_\lambda^2 + \sigma_B^2}, \frac{\sigma_\lambda^2 \sigma_B^2}{\Lambda(t_k; \theta) \sigma_\lambda^2 + \sigma_B^2}\right) \tag{13}$$

Then, the PDF of the RUL can be obtained as [17]

$$f_{L_k}(l_k) = \sqrt{\frac{(w - y_k)^2}{2\pi \Delta v(l_k)^3 (\sigma_{\lambda,k}^2 \Delta v(l_k) + \sigma_B^2)}} \exp\left(-\frac{(w - x_k - \mu_{\lambda,k} \Delta v(l_k))^2}{2\Delta v(l_k) (\sigma_{\lambda,k}^2 \Delta v(l_k) + \sigma_B^2)}\right) \frac{d\Lambda(l_k + t_k; \theta)}{dl_k} \tag{14}$$

Where  $\Delta v(l_k) = \Lambda(l_k + t_k; \theta) - \Lambda(t_k; \theta)$ .

**3. Experimental Study**

In this section, we use the lithium-ion battery data provided by the National Aeronautics and Space Administration (NASA) Ames Prognostics Center of Excellence to verify the effectiveness of RUL prediction with fusing failure time data. The original degradation paths of lithium-ion batteries are shown in Figure 1. There are four sets of lithium-ion batteries data. Since the relaxation effect of lithium-ion battery would affect the accuracy of RUL prediction, we use the degradation data after eliminating the moderation effect [18], as shown in Figure 2.

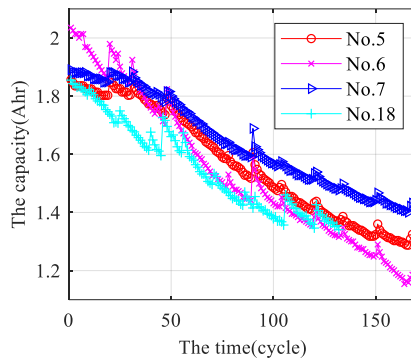


Figure 1: Degradation paths of lithium-ion battery

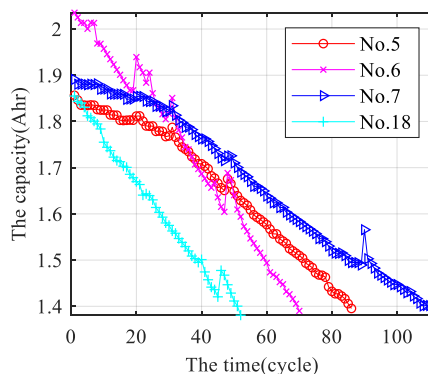


Figure 2: Degradation paths after eliminating the relaxation effect

The failure threshold is 1.4 Ahr. For simplicity, the method of RUL prediction with fusing the failure time data is referred to  $M_0$  and the traditional RUL prediction method is referred to  $M_1$ . NO.5 lithium-ion battery is selected as the evaluated equipment. The failure time data of the remaining

three groups of lithium-ion batteries are 69.5, 110.3 and 51 respectively. Then, the unknown parameters in the model are calculated by using the parameters estimation method with fusing failure time data proposed in Section 2.2. The corresponding RUL prediction results and the mean squared errors (MSEs) at some points are shown in Figure 3 and Figure 4 respectively. It can be observed from Figure 3 and Figure 4 that the RUL prediction results with fusing failure time data is better than those based on the traditional RUL prediction method.

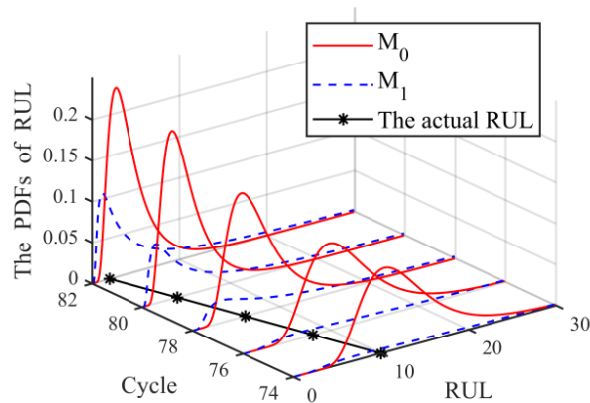


Figure 3: The estimated RULs by  $M_0$  and  $M_1$

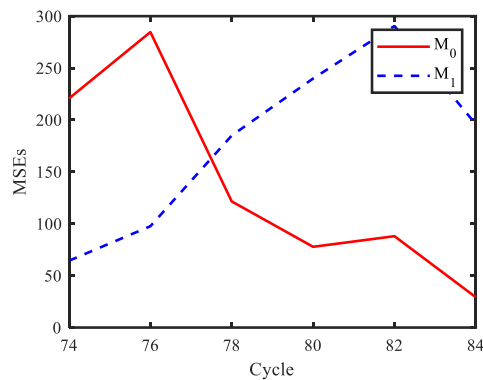


Figure 4: MSEs at some times

#### 4. Conclusions

It is expensive to build a complete historical degradation database of equipment. Therefore, it is necessary to consider using more easily available failure time data to predict the RUL of equipment. In this paper, a RUL prediction method with fusing failure time data is proposed. Firstly, the fixed parameters are calculated based on the field degradation data of the evaluated equipment. Then, the EM algorithm is used to solve the prior information of the random coefficients in the model with fusing the failure time data, and the Bayesian theory is used to update the random coefficients. After that, based on these parameters estimation results, the RUL of the equipment is predicted. The results show that for the degraded data with imperfect prior information, the RUL prediction results obtained by fusing the failure time data is better.

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