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Research on Dynamic Trajectory Planning and Collision Detection Model Based on Numerical Difference Method and Genetic Algorithm

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Abstract: *This paper presents a dynamic model based on the Archimedean spiral, focusing on the determination of the positions of the faucet and dragon body, collision detection, and pitch optimization. First, in the polar coordinate system, the geometric characteristics of the spiral and the numerical difference method are used to calculate the dynamic position of the faucet through the relationship between arc length and angle. Taking the faucet as the reference, the positions of each segment of the dragon body are derived, and the numerical difference method is used for iterative solution considering the interaction of velocity variables. Second, benches are regarded as line segments, and the convex hull method is used for collision detection. The Runge-Kutta method is employed to solve the faucet trajectory, and the collision threshold is determined by judging the distance between line segments and the intersection of convex hulls. Finally, a genetic algorithm optimization model is constructed with the goal of minimizing the pitch. By setting constraints such as collision detection and radial distance, and through encoding, fitness calculation, and genetic operations, the optimal pitch is obtained. This model achieves accurate calculation of motion trajectories and collision detection through the combination of multiple algorithms, providing an effective framework for dynamic trajectory planning.*

Keywords: *Archimedean Spiral, Numerical Difference Method, Genetic Algorithm*

1. Introduction

This paper focuses on the application of Archimedean spiral in dynamic trajectory planning, aiming to solve the problems of position calculation, collision detection and parameter optimization of the dragon body movement system through constructing a multi-algorithm coupling model [1]. First, a faucet movement model is established based on the polar coordinate system. By using the geometric characteristics of the spiral to derive the dynamic relationship between position and angle, and combining with the numerical difference method, the iterative update of the positions of each segment of the faucet and dragon body is realized, fully considering the complex interaction of velocity variables [2][3]. Second, in order to avoid collision risks during movement, the benches are abstracted into a set of line segments. The convex hull method is used for collision detection, and the Runge-Kutta method is introduced to solve the faucet trajectory equation. Starting from the distance constraints of adjacent and non-adjacent line segments, a collision threshold determination mechanism is constructed [4][5]. Finally, taking the minimum spiral pitch as the optimization goal, a genetic algorithm model is constructed. Through operations such as real-number encoding, fitness function design, selection, crossover and mutation, the optimal solution is sought under the conditions of satisfying radial distance constraints and no collision [6].

The experimental results show that the model can effectively realize the trajectory planning and parameter optimization of the dragon body movement system, providing an engineering-valued solution for the modeling and analysis of similar dynamic systems.

2. Dynamic position calculation model based on Archimedean spiral

2.1 Polar coordinate difference method for faucet position determination

The Archimedean spiral, also known as the arithmetic spiral, describes the relative position of a point that moves away from a fixed point along a line rotating at a constant speed with a constant velocity over

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time. The basic characteristic of the Archimedean spiral is that in the polar coordinate system, the distance r of each point on the spiral from the origin increases or decreases linearly with the angle θ.

First, a polar coordinate system is established, with the origin as the center of the spiral (i.e., the path center), and the polar axis coinciding with the positive direction of the $x$axis. It is assumed that the initial direction is the direction of the starting position of the spiral. The position of the faucet can be expressed as:

$$r_{\text{head}}(\theta) = r_0 - \frac{p}{2\pi}\theta \tag{1}$$

Among them, p is the pitch of the spiral, which is 55 cm. For θ, it is determined according to time by using the faucet speed of 1 m/s. First, in each time step, the arc length moved by the faucet is calculated. According to the geometric characteristics of the spiral, the value of the angle change ($\Delta\theta$) can be obtained through the following formula:

$$\Delta s = v_{\text{head}} \times dt \text{ , } \Delta\theta = \frac{\Delta s}{\sqrt{r(\theta)^2 + \left(\frac{p}{2\pi}\right)^2}} \tag{2}$$

The position of the faucet changes continuously with time. In the polar coordinate system, its position information is represented by θ and r. The formula for the position change of the faucet is:

$$\theta_{\text{head}}(t+\Delta t) = \theta_{\text{head}}(t) + \Delta\theta \text{ , } r_{\text{head}}(t+\Delta t) = r_0 - \frac{p}{2\pi}\theta_{\text{head}}(t+\Delta t) \tag{3}$$

2.2 Spiral-based recursive algorithm for dragon body positioning

After determining the position of the faucet, the position and velocity of the dragon body can be obtained using the faucet as a reference point. Therefore, the angle difference between each segment of the dragon body and the faucet is first calculated. Similar to the method for determining the faucet position, taking the faucet as the base point, the arc length distance L_i of each segment relative to the faucet is:

$$L_i = (i-1) \times l \tag{4}$$

Where $i = 2,3,\ldots,223$ represents the i th segment, and $l = 2.2$ m is the length of each segment.

Since each segment follows the previous one, the polar angle of the i th segment is equal to the polar angle of the faucet minus the angular offset. After obtaining the polar angle, the radius of each segment of the dragon body can be derived from the spiral equation.

$$\theta_i = \theta_{\text{head}} - \frac{L_i}{r_{\text{head}}} \text{ ,} r_i = r_0 - \frac{p}{2\pi}\theta_i \tag{5}$$

Where, θ_{head} is the polar angle of the faucet, and L_i / r_{head} is the ratio of the arc length of the i th segment relative to the faucet to the radius, which gives the polar angle difference of the i th segment.

2.3 Calculation by numerical difference method

Since the expression for velocity involves complex interactions of multiple variables, the numerical difference method is introduced to obtain the results. The calculation steps of the numerical difference method are as follows:

Step 1: Constant Definition and Array Initialization: After defining a series of constants, initialize the array "positions" for storing positions, which is used to store the x and y coordinates of each section at different time points. Initialize the array speeds used to store the speeds of each section at different points in time.

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Step 2: Setting Initial Conditions: Set the initial Angle of the faucet to 0 radians. The initial position is set to $(r0, 0)$, that is $xhead = r0, yhead = 0$.

Step 3: Simulation Loop: Simulation time from start to finish:

1) Update the Faucet Position:

Calculating radius and arc length differentiation ds . The formula is as follows:

$$ds = \sqrt{r_{\text{head}}^2 + \left(\frac{\text{pitch}}{2\pi}\right)^2} \tag{6}$$

This arc length differential reflects the infinitesimal arc length the faucet moves in the polar coordinate system. By tapping the travel speed, time step and arc length differentiation to approximate the angular variation $d\theta$ .Equation is as follows:

$$d\theta = \frac{\text{speed} \times dt}{ds} \tag{7}$$

Here, the numerical difference method is used to approximate the angular change.

2) Store Faucet Position: Save the faucet coordinates and velocity into the corresponding arrays.

3) Update Positions of Other Segments: For each segment:

For each section, first calculate the arc length distance $section\_distance$. It is calculated based upon the serial number of the section, the length of each section, and a fixed value indicating the arc length of the section from the faucet. The formula is:

$$\text{section\_distance} = (\text{i-1}) \times \text{section\_length} + 1.21 \tag{8}$$

$$\theta_{\text{section}} = \theta - \frac{\text{section\_distance}}{r_{\text{head}}} \tag{9}$$

Here, the idea of numerical difference is used again, determining the angular change of each segment relative to the faucet through the proportional relationship between the arc length distance and the current faucet radius. Calculate the radius of each section r_{section} with the formula:

$$r_{section} = r_0 - \frac{pitch}{2\pi} \times \theta_{section} \tag{10}$$

Where r_0 is the initial radius and $pitch$ is the spiral pitch.

Through step-by-step calculation, the positions of each segment of the dragon body in the Cartesian coordinate system are updated.

4) Calculate the velocity of each segment.

5) Update the angle: Update the angle for the next time step.

Through the above steps, the positions and velocities of the front handles of the faucet, dragon body, and dragon tail, as well as the center of the rear handle of the dragon tail, are obtained for each full second, and the following graph is drawn:

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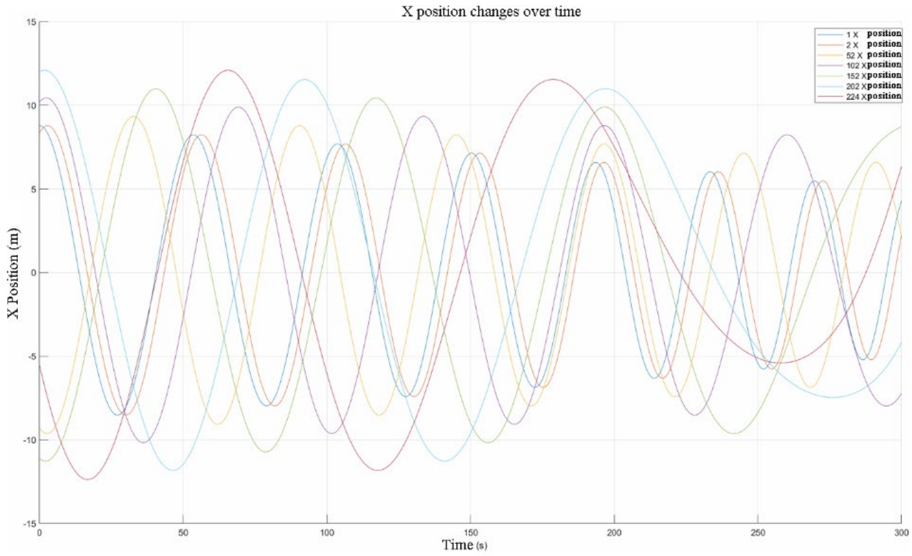

Figure 1 Position over time graph (x)

The Figure 1 shows the changes in the x coordinates of key segments, helping to understand how they move over time.

3. Collision detection model based on convex hull method and Runge-Kutta solution

3.1 Geometric modeling and constraint conditions for collision detection

In order to prevent the bench from colliding, in the solving process, the geometric shape of the bench is not considered, and the bench is regarded as a line segment formed by connecting the centers of the front hole and the rear hole. Therefore, the spiral can be regarded as a figure composed of 223 lines segments. During the marching process of the bench dragon, if two adjacent or nonadjacent line segments intersect, a collision is determined to have occurred.

According to the motion model, the position of the faucet at time t can be expressed as:

$$r(t) = r_0 - \frac{p}{2\pi} \cdot \theta(t) \tag{11}$$

Starting from the dragon head, the position of each dragon body's board relative to the previous section can be represented by a polar angle offset. For the ith section of the plateen, the polar angle is:

$$\theta_i(t) = \theta(t) - \frac{i \cdot L_{body}}{r_0} \tag{12}$$

Assume that the positions of the i th and $(i+1)$ th segments of the dragon body at time t are $(x_i(t), y_i(t))$ and $(x_{i+1}(t), y_{i+1}(t))$ respectively. It is easy to know that the distance between them can be expressed by the distance formula as:

$$d_{i,i+1}(t) = \sqrt{\left(x_{i+1}(t) - x_i(t)\right)^2 + \left(y_{i+1}(t) - y_i(t)\right)^2} \tag{13}$$

It should be noted that the collision threshold is divided into two parts: one is the collision between two adjacent benches, and the other is the collision between nonadjacent benches. For the former, it is only necessary to satisfy that for any i:

$$d_i(t) < L_{\text{body}} \tag{14}$$

For the latter, since nonadjacent benches are not arranged in the same direction, the collision threshold should be the width of the bench.

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3.2 Convex hull algorithm and Runge-Kutta iteration for model solution

In the process of solving the model, the Runge-Kutta method is used to solve the trajectory of the head bench and calculate the coordinates and angle of the front hole of the bench [2]. When performing collision detection, the convex hull method is used, which involves calculating the convex hull of the benches and checking whether the convex hulls intersect.

The specific steps are as follows:

Step 1: Parameter Definition and Initialization

First, the range of values for pitch is defined. Initialize the minimum angle array of values to take.

Step 2: Draw Part of the Spiral

Based on the range of defined angles, calculate the solenoidal polar diameter and right angle coordinates and plot a portion of the helix.

Step 3: Iterative Simulation and Collision Detection

Solve for the faucet handle angle, using the Ronger Kuta method. Specifically for: Define the first handle angle evolution function of the faucet. The function expression is:

$$\text{mydtheta} = @(t,\theta) - 1./(k \times \sqrt{1+\theta.^2}) \tag{15}$$

Solve the angle evolution of the head handle.with the expression:

$$[tt,\theta] = \text{ode45}(\text{mydtheta}, \text{tspan}, \Theta(1,1)) \tag{16}$$

This gives the angle evolution of the first handle of the faucet starting from the initial angle $\Theta(1,1)$ within the given time interval tspan . Collision detection is performed by calculating the positional information of the dragon body and the dragon tail stool holes. If any collision has been detected, the minimum angle is recorded and the result is plotted.

Collision Detection Using Convex Packet Method to Find the Closest Outer Layer Points to the Front and Back Handles. For each bench, first find the three points closest to the front handle and the three points closest to the rear handle to determine the current range of outermost handle points.

With the above algorithm, the termination time of disk entry is determined to be 412s, and the following results are also obtained:

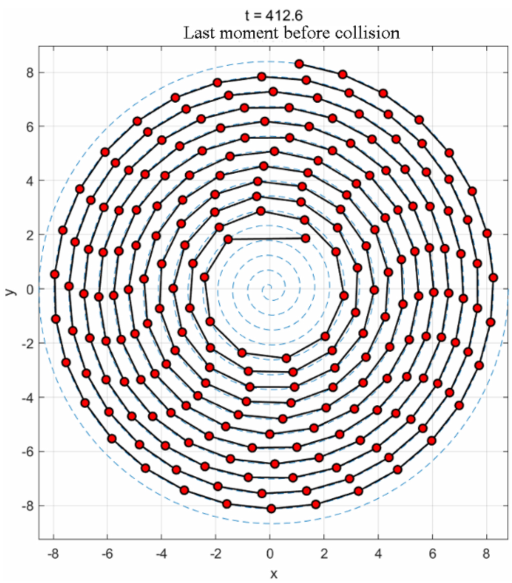

Figure 2 Last moment before collision

In the Figure 2, small dots represent the positions of each handle at the current time, straight lines represent the benches, and the dashed line is the coiling spiral.

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4. Genetic algorithm-driven optimization model for spiral parameters

4.1 Construction of Constraint Conditions and Objective Function for Optimization Model

In order to determine a minimum pitch that allows the front handle of the faucet to smoothly rotate along the corresponding spiral into the boundary of the turning space, it is necessary to determine the minimum pitch a such that the front handle of the faucet coils along a spiral into a turning space with a radius 4.5m. The constraint conditions are:

1) No collision occurs between benches (detected based on the convex hull method);

2) The radial distance r of the front handle of the faucet satisfies $r \leq 4.5m$;

3) Head tail followability (the polar angle of the dragon tail $\theta_{tail} \leq \theta_{head}$).

4.2 Construction of genetic algorithm model

(1) Encoding and Population Initialization

During the population initialization phase, each individual is directly mapped to a pitch parameter a. This parameter is encoded using a real number coding scheme with values in the range [0.3, 0.6].

(2) Fitness Function Design

The fitness function needs to balance the minimization of pitch and constraint satisfaction, defined as:

$$\text{fitness}(a) = \frac{1}{a + \lambda_1 \cdot f_{\text{collision}}(a) + \lambda_2 \cdot f_{\text{radius}}(a)} \tag{17}$$

$f_{\text{collision}}(a)$: Collision penalty term. Based on the convex hull method for detecting intersection of bench line segments, it takes 1 if a collision occurs, otherwise 0.

$f_{\text{radius}}(a)$: Radial distance penalty term. If the maximum radial distance of the faucet $r_{\max} > 4.5\text{m}$, then $f_{\text{radius}} = (r_{\max} 4.5) / 0.5$; otherwise, 0.

Penalty coefficients λ_1=100 and λ_2 = 50 ensure that constraints have higher priority than pitch optimization.

(3) Spiral Motion and Constraint Verification Model

1) Faucet Trajectory Calculation: Based on the polar coordinate equation of the Archimedean spiral $r(\theta) = r_0 + a\theta$, where the initial radius r_0 is determined by the 16th circle. The polar angle \(\theta\) changes with time, solved by the numerical difference method, and converted to Cartesian coordinates.

2) Radial Distance Constraint: During coiling, calculate the maximum radial distance of the faucet front handle to ensure that it does not exceed $r(\theta) \leq 4.5m$ to avoid interference with surrounding objects.

3) Collision Detection: Consider each bench as a line segment connecting the center of the front and back holes. Use the convex hull method to determine if any two line segments intersect; if intersection occurs, trigger collision penalty.

4.3 Solution process of genetic algorithm

1) Population Initialization: Generate random spacings a and compute the coordinates of the faucet trajectories corresponding to each a.

2) Constraint Condition Verification:

The intersection of the line segments is detected using the convex hull method to verify that it is beyond the boundary of the turning space.

3) Fitness Evaluation: The adaptation degree of an individual is calculated. The higher the fitness, the smaller the spacing and the higher the constraint satisfaction.

4) Genetic Operations:

Selection: Selection of better adapted individuals for the next generation, using methods such as tournament selection or roulette wheel selection.

Crossover: A crossover operation is performed on the selected individuals with a certain probability (e.g., arithmetic crossover with real number coding) to generate a new sample of offspring.

Mutation: Random variation is introduced into the offspring with low probability to maintain population diversity.

5) Result Output: At the end of the iteration, the pitch with the highest fit is taken as the optimal solution and its stability is verified with different initial conditions.

After the above steps, the calculated minimum pitch is: 0.425 meters. The Figure 3 is generated:

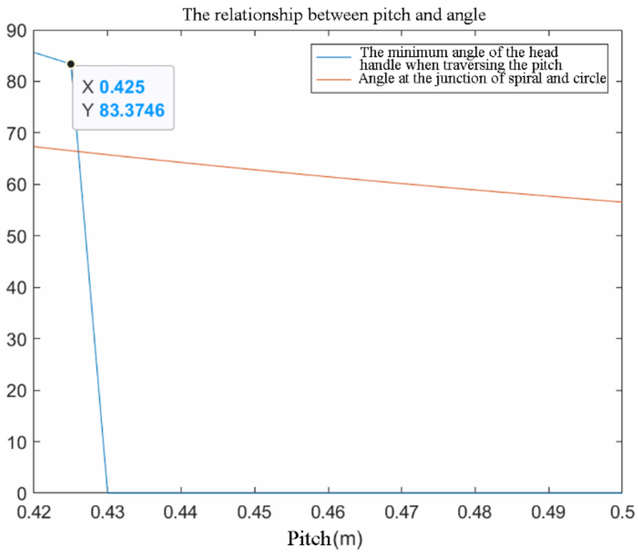

Figure 3 The relationship between pitch and angle

5. Conclusions

This paper proposes a multi-dimensional dynamic model integrating the geometric characteristics of Archimedean spiral, numerical difference method, convex hull collision detection and genetic algorithm. Through interdisciplinary algorithm coupling, the model realizes the trajectory planning and parameter optimization of complex motion systems, providing a solution with both theoretical rigor and engineering practicability for dynamic system modeling. First, the position calculation model of the faucet and dragon body constructed based on the polar coordinate system uses the linear distance change characteristic of the spiral to transform the motion in the time dimension into the solution of the geometric relationship between angle and radius, effectively solving the problem of real-time update of dynamic positions. Second, the innovatively introduced convex hull collision detection mechanism and Runge-Kutta trajectory solution method abstract entities into line segment sets and combine geometric topology analysis to accurately define the safety threshold of system operation, providing a quantitative basis for avoiding collision risks. Then, the constructed genetic algorithm optimization model takes the minimum pitch as the objective function, and realizes the optimal configuration of system parameters through adaptive optimization under multiple constraint conditions, improving the adaptability of the model in practical applications. Finally, experiments verify the effectiveness of the model in terms of trajectory calculation accuracy, collision detection efficiency and parameter optimization effect. Future research can further explore the extended application of the model under irregular space constraints, or combine machine learning algorithms to enhance the adaptive adjustment ability of dynamic systems.

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