

# Research on Solving Assignment Problem Based on Linear Programming Modeling

**Zijun Zhang**

*Shanghai Guanghua Cambridge International School, Shanghai 201315, China;  
2028435326@qq.com*

**ABSTRACT.** *The optimal allocation of resources plays an important role in improving the competitiveness of modern enterprises, but the unbalanced and uncoordinated allocation of resources still restricts the realization of the ideal state of production activities. The research of assignment problem can improve the product design and production efficiency of enterprises. In this paper, through linear programming modeling, basic transformation of matrix and Hungarian algorithm, the optimal assignment method was obtained. It provided the basis for further programming. Then, taking the time arrangement of Chinese manual translation as a basic example, the results showed that using the model and solving has a good effect, and could make the most reasonable allocation in a short time. Based on the traditional Hungarian method, this paper improved the solution of the maximum assignment problem, which makes the solution more direct. It is also of great practical significance to study the solution of optimization assignment problem to improve the efficiency of completing tasks.*

**KEYWORDS:** *Assignment problem, linear programming, matrix transformation, optimal method.*

## 1. Introduction

At present, the progress of science and technology promotes the perfection of contemporary material life. However, the consumption rate of earth energy is also growing rapidly. The matching of capacity and demand is one of the most basic problems in the production operation management at this stage. "Assignment problem" appears frequently in our life [1,2]. If the assignment problem is described in mathematical language, the basic requirement of this kind of problem is to make the overall effect of the assignment scheme the best under satisfying the specific assignment requirements. For example, a certain unit needs to complete  $n$  tasks, and there are exactly  $n$  individuals who can undertake these tasks. However, due to the different expertise of each person, each person has different tasks (or time spent) and

different efficiency. The problem of which person should be assigned to accomplish which task, which makes the total efficiency of  $n$  tasks the highest (or the minimum total time required) arises. This kind of problem is called assignment problem [3,4]. In fact, the study of assignment problem is of certain significance. The development of assignment problem and the support of theory and algorithm can improve product design and production efficiency. It is also of great practical significance to study the solution of optimization assignment problem to improve the efficiency of completing tasks.

Some scholars took personnel allocation as an example and regarded single efficiency as independent variable [5]. They constructed objective function with cost. Hungarian algorithm was used to solve and come to an ideal mode of efficient utilization of human resources, which helps the development of modern enterprises [6]. In order to improve the quality of decision-making in the practical work, some scholars studied the optimal solution of this kind of assignment problem by elaborating the basic idea and steps of Hungarian algorithm. Some scholars also studied the assignment problem of sending many people to perform many tasks in their work [7]. By elaborating the basic idea and steps of Hungarian algorithm, they showed the optimal solution to this kind of assignment problem, so as to improve the decision-making quality of the management decision-maker in the actual work [8].

The main purpose of this paper is to build a mathematical model to assign tasks efficiently. First, we build the model and use Hungarian algorithm and program. After importing data, the most reasonable allocation can be made in a short time. Based on the traditional Hungarian method, this paper improves the solution of the maximum assignment problem, which makes the solution more direct. Section 2 assumes the model and explains the meaning of some symbols. Section 3 establishes the model and uses the simplified Hungarian algorithm to solve it. Section 4 will summarize and further improve the program.

## 2. Models

Suppose there are  $X$  items ( $X_1, X_2, \dots, X_m$ ) and  $Y$  companies ( $Y_1, Y_2, \dots, Y_n$ ) bidding ( $n \geq m$ ), because each company is unique, the time to complete the same project is not necessarily the same for different companies. Let  $T_{ij}$  indicate the time required for the  $i$ th company to complete the  $j$ -th project, then  $T_{ij} = 0$ , if the  $i$ -th company does not do the  $j$ -th project, then the  $i$ -th company does the  $j$ -th project. The specific task schedule is shown in Table 1.

Table 1 Company task schedule

	$Y_1$	$Y_2$	$Y_3$	...	$Y_n$
$X_1$	$T_{11}$	$T_{21}$	$T_{31}$	...	$T_{n1}$
$X_2$	$T_{12}$	$T_{22}$	$T_{32}$	...	$T_{n2}$
...	...	...	...	...	...
$X_m$	$T_{1m}$	$T_{2m}$	$T_{3m}$	...	$T_{nm}$

In order to make the allocation most efficient, the total completion time should be minimized, so the objective function is as follows:

$$\min z = \sum_{i=1}^n \sum_{j=1}^m T_{ij} X_{ij}$$

The requirements of the objective function are as follows:

$$\sum_{j=1}^n X_{ij} = 1, j = 1, 2, 3, 4, \dots, n$$

$$\sum_{j=1}^m X_{ij} = 1, j = 1, 2, 3, 4, \dots, m$$

$$X_{ij} = 0 \text{ or } 1$$

The Hungarian algorithm is described in mathematical language as: Let  $G = (V, E)$  be an undirected graph, if vertex  $v$  can be divided into two disjoint subsets (A, B), and the two vertices  $i$  and  $j$  associated with each edge  $(i, j)$  in the graph belong to the two different vertex sets ( $i$  in A,  $j$  in B), then graph  $G$  is a bipartite graph. The goal of this method is to find as many pairs as possible for the most points in  $X$  [9,10]. For specific problems, the solution is: when each row of elements is subtracted from the minimum value of each row of elements, then each column of elements is subtracted from the minimum value of this column on this basis. If the number of straight lines is less than  $m$ , the optimal solution can be obtain by the following deformation.

### 3. Results and Discussions

Through the above methods, the optimal solution of the hypothesis matrix could be obtained, and the optimal assignment scheme can be obtained. We took the following example as an application, and used the Hungarian algorithm to realize the specific scheme. There was a Chinese manual which needs to be translated into English, Japanese, German and Russian, which were recorded as E, J, G and R

respectively. Now there are four persons, namely, A, B, C and D, who need the following days to translate the Chinese manual into the manual in different languages. How to arrange the work so as to minimize the time required?

Table 2 schedule of Chinese manual translation

	English	Japanese	German	Russian
A	2	15	13	4
B	10	4	14	15
C	9	14	16	13
D	7	8	11	9

After calculation, matrix can be obtained

$$C = \begin{array}{c|cccc} \hline & 0 & 13 & 7 & 0 \\ \hline & 6 & 0 & 6 & 9 \\ \hline & 0 & 5 & 3 & 2 \\ \hline & 0 & 1 & 0 & 0 \\ \hline \end{array}$$

Fig. 1 Calculation results of Hungarian algorithm

According to the model, we can see the optimal method: the shortest time is

$$4 + 4 + 9 + 11 = 28$$

When the  $X_i$  project is completed by  $Y_j$  company, it takes the least time. That is to say, Russian is translated by a, Japanese is translated by B, English is translated by C and German is translated by D in the shortest time.

#### 4. Conclusion

In this paper, we built the model and used Hungarian algorithm and program. After importing data, the most reasonable allocation could be made in a short time. Based on the traditional Hungarian method, the solution of the maximum assignment problem was improved, which makes the solution more direct. In fact, the assignment problem can not only create benefits, but also directly affect the enthusiasm of workers and greatly improve production efficiency. It is an important problem in our life.

This paper only considered the case of single objective assignment. But in real life, it is not so simple. For example, we should save time and money. Moreover, this model is invalid for Multi-objective Assignment problem, which needs further modification and improvement. The improved algorithm can not only solve the transportation problem, but also be applied to the field. Then, it can not only get the optimal solution directly, but also has a good application in the problem of restricted conditions.

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