

The Law of Thermal Radiation and Mathematical Model for Different Surfaces of Objects

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Abstract: In this paper, the effect of the surface characteristics on the thermal radiation absorption and emission capacities of cubes with different surface materials and colors was studied by conducting heating experiments. Furthermore, a thermal radiation mathematical model was established based on the Stefan-Boltzmann law, which can control the thermal radiation capacity and heating temperature change of objects by adjusting their absorption rate and emissivity. The positive correlation between the radiation capability and absorption capability of an object has been revealed.

Keywords: Thermal Radiation, Mathematical Model, Material, Color, Temperature

1. Introduction

Thermal radiation is one of the three basic forms of the energy transfer, which is generated by the thermal motion of particles and characterized by the radiation of electromagnetic waves[1]. Thermal radiation has its strong individuality in terms of physical essence and research methods. Firstly, it is the only way of heat transfer that does not require any medium, because electromagnetic waves can propagate in vacuum. Secondly, all the objects in nature with temperature above absolute zero are always radiating outward. Based on the basic theory of thermal radiation, many practical instruments have been invented, such as solar cells, infrared thermometers, spectrometers, and infrared thermal imagers, etc[2-5]. As a crucial component of heat transfer, thermal radiation shows promising in various areas including aerospace, energy conversion, industrial manufacturing, and biomedicine[6].

All objects continuously absorb energy from other objects while radiating outward, which depends on factors such as temperature, color, material, and shape, etc. In this study, a stable heat source was used to heat radiation boxes with different surface characteristics. The effect of the surface structural characteristics on the absorption and emission of thermal radiation was studied and analyzed with the temperature of different surfaces measured. Then, a thermal radiation mathematical model was established based on experimental data and the fundamental theory of thermal radiation, which can analyze the emissivity and absorptivity of various objects.

2. Experimental Equipment and Scheme Design

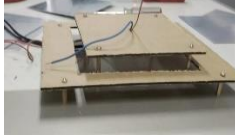
2.1 Composition of Experimental equipment

The composition of experimental equipment for thermal radiation tests was introduced below. The primary experimental instruments were listed in Table 1, which include a 100W incandescent bulb (a radiant heat source), two 4-channel real-time temperature measuring devices, aluminum plates, colored papers, and a self-made Acrylic base.

The thermal radiation experimental apparatus consists of a cube, two temperature measuring devices, a heating power source, and a computer processing system, as shown in Figure 1. The cube has four radiative surfaces, each of which is made of 15 cm×15 cm aluminum plate.

Firstly, four aluminum plates with different materials, namely black, white, rough, and smooth, were selected to study the effect of surface material properties on thermal radiation. Then, four pieces of paper with different colors, i.e. Purple, blue, yellow, and red, were used to cover the aluminum plates (of the same material), so as to investigate the effect of different colors on thermal radiation.

Table 1: List of experimental instruments.

Equipment Name	Specifications	Real Object Image
Incandescent bulb	100W	
Temperature measuring device	4-channel/2 units	
Aluminum Plates	15cm×15cm multiple pieces	
Colored Paper	Various colors	
Acrylic Base	Self-designed and assembled	

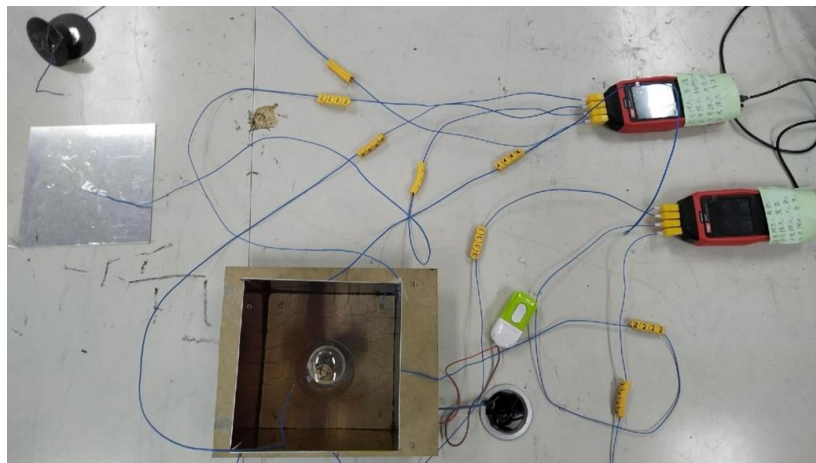


Figure 1: Thermal radiation experimental apparatus.

2.2 Experimental Scheme Design

The overall experimental process is divided into three stages: experimental design and preparation, experimental operation, and data processing, as shown in Figure 2. According to the experimental design, the required experimental equipment was selected and prepared, and the experimental platform was set up. Then, a custom-made radiation enclosure was used to study the laws governing the thermal radiation properties of materials from the two perspectives of material and color. Experimental operations were performed to acquire experimental data. Finally, data processing was carried out to present the physical laws.

This experiment was conducted in two groups. The first group studied the effect of different materials on the absorption capacity of thermal radiation. Four surface materials were selected: black, white, frosted, and smooth. The second group studied the effect of different colors of paper on the thermal radiation capacity of objects. Four colors of paper were selected: purple, blue, yellow, and red.

Additionally, a digital experimental system was designed to dynamically present the physical laws related to thermal radiation. The system utilized temperature sensors, a four-channel data acquisition

device, and a computer system for image processing. The data acquisition process for this experiment is shown in Figure 3.

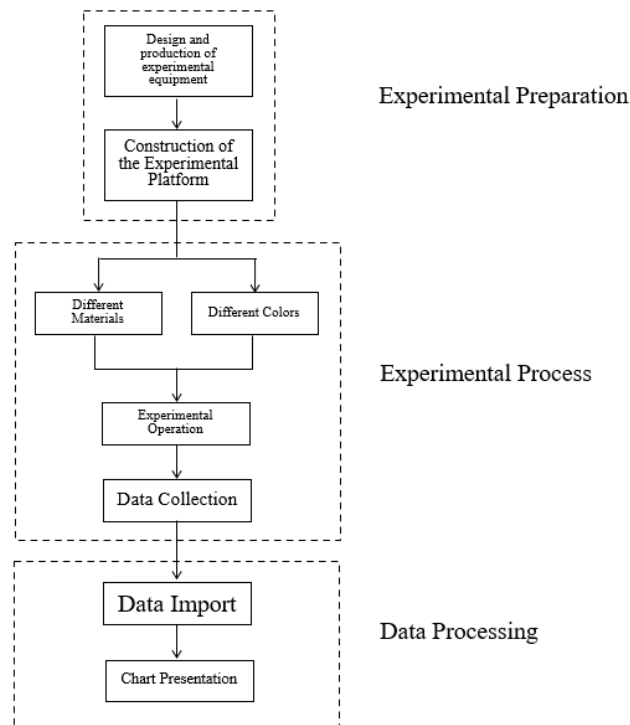


Figure 2: Overall experimental process.

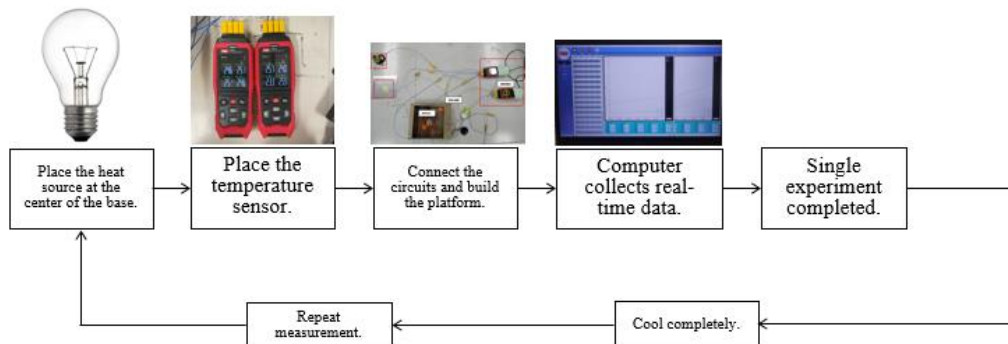


Figure 3: Data acquisition process.

3. Experimental Measurement and Analysis

The experiment was conducted in two groups to investigate the thermal radiation properties of samples with different materials and colors. The radiation enclosure was heated, and temperature sensors were used to measure the surface temperature changes in real time.

3.1 Effect of Different Materials on Thermal Radiation Absorption

A radiation enclosure with four surfaces made of different materials has been shown in Figure 4, corresponding to the first group of experiments.

From Figure 5, it can be observed that the temperature change rate of the four surfaces decreases over time, with a faster change in the initial stage than in the later stage. After the temperature reaches a specific value, the rate of change of the temperature curve slows down until it stabilizes. The stable temperature is defined as the saturation temperature. Based on the data comparison from the four channels of the temperature sensor, the saturation temperatures observed in the experiment follow the

order: the black surface > the white surface > the frosted (or Sanding) surface > the smooth surface.



Figure 4: Radiation enclosure with different materials.

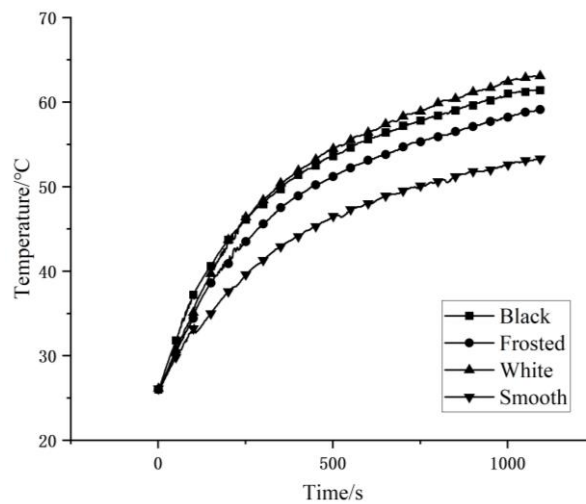


Figure 5: Variation of surface temperature with time for different materials.

3.2 Effect of Different Colors on Thermal Radiation Absorption

Figure 6 shows a radiation enclosure with different colored surfaces, which corresponds to the second group of experiments.

Based on a comparison of data from the four channels of the temperature sensor, the saturation temperatures of different colored papers observed in the experiment followed the order: purple paper > blue paper > yellow paper > red paper, as shown in Figure 7.

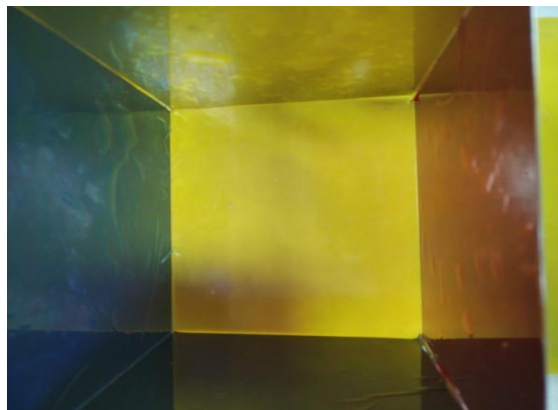


Figure 6: Radiation enclosure with different colors.

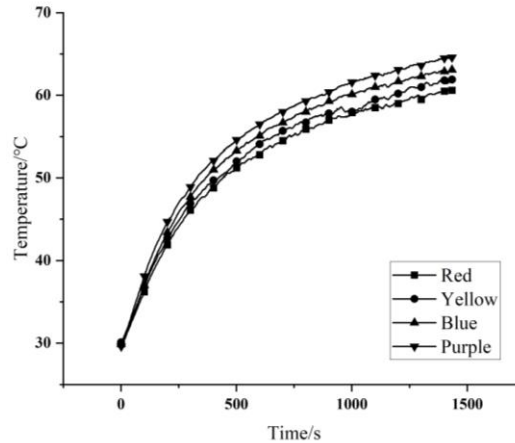


Figure 7: Variation of surface temperature with time for different colors.

Through analysis, the following conclusion can be drawn: the material and color of an object's surface have a significant effect on its ability to absorb thermal radiation. Darker colors and rougher surfaces were found to be more effective at absorbing thermal radiation. Additionally, during the initial phase of heating, the absorbed thermal radiation energy is significantly greater than the emitted energy, resulting in a rapid increase in temperature. Over time, as the temperature rises, the emitted thermal radiation energy gradually increases, which slows the rate of temperature rise until a steady-state temperature is reached. At this point, the energy absorbed by the surface of the radiation enclosure is approximately equal to the energy it emits.

4. Establishment of a Theoretical Model for Thermal Radiation

Based on the Stefan-Boltzmann law, we refined and developed a thermal radiation model suitable for this experiment. Assuming that the different surfaces of the radiation enclosure both absorb radiant energy from the heat source and emit thermal radiation based on their temperature, the temperature variation over time can be determined, which is described by the following equation:

$$\frac{dT}{dt} = \frac{Q_{\text{absorbed}} - Q_{\text{emitted}}}{m \cdot c} \quad (1)$$

Where, dT/dt is the rate of temperature change with time, m is the mass of the object, c is the specific heat capacity of the object, Q_{absorbed} is the thermal radiation energy absorbed by the surface of the enclosure, Q_{emitted} is the thermal radiation energy emitted by the surface of the enclosure. Both types of thermal radiation energy are functions of temperature.

$$Q_{\text{absorbed}} = \alpha \cdot P \quad (2)$$

$$Q_{\text{emitted}} = \varepsilon \cdot \sigma \cdot A \cdot T^4 \quad (3)$$

Where, P is the radiation power of the heating bulb, α is the absorptivity of the radiation enclosure surface, ε is the emissivity of the enclosure surface, σ is the Stefan-Boltzmann constant, A is the surface area of the enclosure sides.

Substituting Equations (2) and (3) into Equation (1), it yields

$$\frac{dT}{dt} = \frac{\alpha \cdot P - \varepsilon \cdot \sigma \cdot A \cdot T^4}{m \cdot c} \quad (4)$$

Equation (4) is a nonlinear differential equation. Specific initial conditions for the equation (e.g., the initial temperature, surface area, mass of the object, and specific heat capacity) can be obtained through experimental measurements and calculations. Then, by using the L-BFGS-B optimization algorithm and specific analytical techniques, equation (4) is solved to derive the relationship between temperature and time. This solution can be used to predict the experimental data, interpret the results, and analyze the temperature variation over time until a steady state is reached.

To make the model more consistent with reality, Equation (1) is modified as follows:

$$\frac{dT}{dt} = \frac{\alpha \cdot P - \varepsilon \cdot \sigma \cdot A \cdot T^4 \cdot S(T) + h \cdot A \cdot (T_{env} - T)}{m \cdot c} \quad (5)$$

Where, h is the cooling coefficient, T_{env} is the ambient temperature, $S(T)$ is the sigmoid function which can be expressed as

$$S(T) = \frac{1}{1 + \exp\left(-\frac{T - T_{mid}}{\delta T}\right)} \quad (6)$$

Where, T_{mid} represents the midpoint temperature of the sigmoid function, and δT controls the heat absorption rate of the sigmoid function.

In the modified model, the introduction of h effectively explains the characteristic that the radiation enclosure surface absorbs and emits heat simultaneously. The sigmoid function accurately reflects the trend of change in the surface temperature, which rises rapidly at first and then gradually slows down. The model represented by Equation (5) reveals the variation of the enclosure's surface temperature over time, considering factors such as the object's absorptivity, emissivity, and the environment.

The initial conditions in this model (e.g., initial temperature, surface area, object mass, specific heat capacity, etc.) can be obtained through experimental measurement or calculation. With this model and the L-BFGS-B optimization algorithm combined, a Python program was used to calculate theoretical temperature values at various time points.

The theoretical curve was then plotted and compared with the training set data curve to analyze the degree of agreement, where the experimental measurement data was split into training and validation sets at an approximate ratio of 3:1. Based on the degree of agreement, the absorptivity and emissivity were adjusted. The adjusted values were used to recalculate the theoretical temperature values, and the theoretical curve was plotted again for comparison with the training set data curve.

This iterative process was repeated multiple times until the theoretical model values aligned with the experimental data with the highest accuracy, resulting in the final determined values for the object's absorptivity and emissivity. The process of establishing the model is illustrated in Figure 8.

Taking the effect of different colors on thermal radiation absorption capacity as an example, the experimental measurement data from Figure 7 in Section 3.2 were substituted into the theoretical model of Equation (5). Using the program flowchart illustrated in Figure 8, the absorptivity and emissivity of the object's surface corresponding to different colors were calculated.

It can be observed that both absorptivity and emissivity exhibit a trend of increasing from low to high with the frequency increased, as presented in Table 2 and Figure 9.

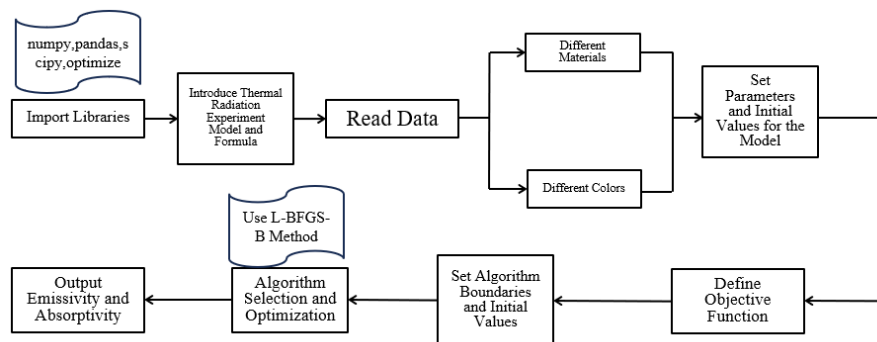


Figure 8: Python program flowchart.

Table 2: The absorptivity and emissivity of surfaces with different colors on the radiation enclosure.

	Red	Yellow	Blue	Purple
Absorptivity	0.0748	0.0778	0.0793	0.0813
Emissivity	0.9314	0.9382	0.9504	0.9671

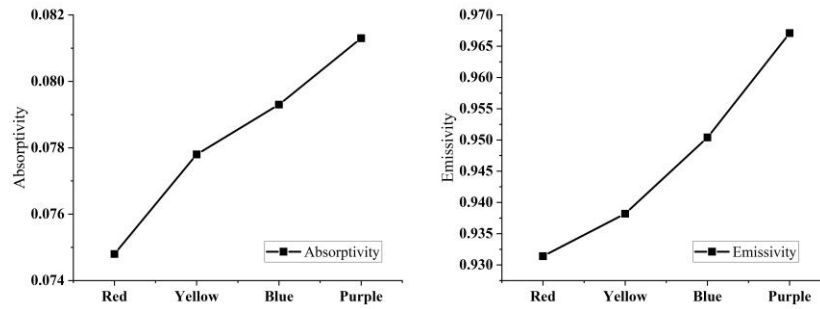


Figure 9: The absorptivity and emissivity of surfaces with different colors on the radiation enclosure.

Figure 10 illustrates the degree of agreement between the theoretical model curve and the experimental data curve. Figure 11 shows the training and validation set data with the red color as an example. It can be observed that the validation set data are uniformly distributed around the final fitted experimental data curve, further confirming the correctness of the absorptivity and emissivity parameters.

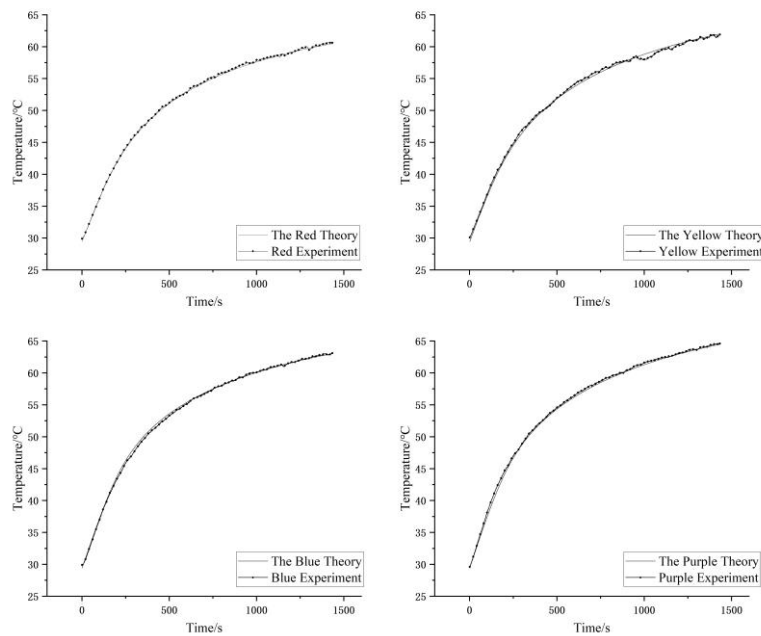


Figure 10: The degree of agreement between the theoretical model curve and experimental data curve.

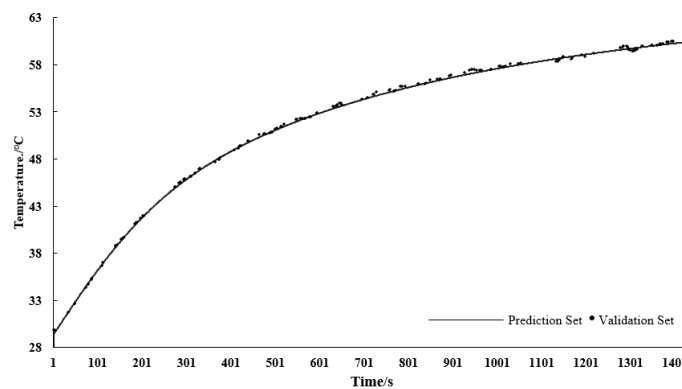


Figure 11: Training Set and Validation Set for red color.

5. Conclusions

This experiment investigated the effect of surface characteristics on the absorption and emission capacities of thermal radiation. It was found that an object's thermal radiation capacity is related to factors such as its surface material and color. While absorbing thermal radiation, an object also emits it, and the surface temperature of a heated object tends to reach saturation at a certain point. During the process of absorbing thermal radiation, as the temperature increases, the object's ability to emit thermal radiation becomes stronger, resulting in an overall trend where the rate of temperature change decreases from fast to slow.

Moreover, a theoretical model for an object's absorptivity and emissivity was proposed. The calculation results demonstrated that the stronger an object's emissivity, the stronger its absorptivity, indicating a positive correlation between the two parameters.

Further studies could introduce more control variables, such as the thickness of the object and the distance between the object and the heat source, to enrich and deepen the understanding of thermal radiation behaviors.

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