

Comparative Study of KCS between Pre-service Teachers and In-service Teachers--Based on the Perspective of Problem Posing

Yangkai Zhang¹, Yali Zhu²

¹Chongqing Second Foreign Language School of Sichuan Institute of foreign languages, Chongqing Nanan, 400065, China

²School of Mathematics and Information Science, Henan Normal University, Henan Xinxiang 453002, China

Abstract: The author defines the KCS of mathematics teachers from the perspective of the problem posing, constructs a comparative framework between pre-service teachers and in-service teachers, investigates the current situation of pre-service teachers and in-service teachers KCS, and examines the differences between pre-service teachers and in-service teachers KCS from the perspectives of existing knowledge structure, acceptance ability and mathematical thinking. The study found that in-service teachers had a better understanding of students' existing knowledge structure than pre-service teachers, but their understanding of students' acceptance ability and mathematical thinking was slightly better than that of in-service teachers.

Keywords: KCS, Problem Posture, Pre-Service Teachers, In-Service Teachers

1. Introduction

In 1986, Shulman proposed disciplinary content knowledge (PCK) [1]. Subsequently, ball and other researchers applied it to the field of mathematics education, proposed "mathematical knowledge for teaching (MKT) and proposed knowledge of content and students (KCS) as a lower concept of PCK [2]. KCs is teachers' knowledge about how students learn specific content, and it is the organic combination of content knowledge and students' knowledge [3]. In the process of teaching, teachers should not only understand students' existing knowledge structure and acceptance ability, but also understand students' existing mathematical thinking level.

Problem posing is a cognitive process in which the questioner forms and expresses questions based on a specific problem situation [4]. Behind the question is the reflection of students' mathematical thinking; Question raising can not only evaluate students' thinking, but also an effective teaching strategy. The more information teachers get about what students know and how to think, the more learning opportunities they create for students [5].

Problem posing has not only been used to assess students' thinking, but also as an effective instructional strategy to create more learning opportunities [6]. In the past decades, most studys focus on the mathematical problem posing. The more information teachers can obtain about what students know and how they think, the more opportunities they can create for student success. [7] The purposes of this study, therefore, were three aspects: (1) What are the different grades in the Chinese sixth-, seventh-, eighth-graders' mathematical thinking when problem posing; (2) What are the differences about the pre-service teachers and in-service teachers' KCS Related to the problem posing? (3) With the increase of the teaching time, do teachers have a better understanding of students' mathematical thinking?

2. Theoretical bases

Based on the actual performance of students, this paper discusses the differences between pre-service teachers and in-service teachers in KCS from three aspects: teachers' understanding of students' existing knowledge structure, receptivity and mathematical thinking. Table 1 shows the comparison framework and the comparison method of each indicator in it.

Table 1: Comparison framework of KCS between pre-service and in-service teachers and calculation method of each indicator

| Level indicators | The secondary indicators | | measure |
|--|--------------------------|--------------|--|
| To the student Understanding of existing knowledge structure | similarity | | Similarity is to compare the degree to which the average percentage of each type of question predicted by teachers is close to the average percentage of each type of question raised by students; The matching rate is the comparison of the matching degree between the questions predicted by the teacher and the corresponding questions raised by the students. |
| | The matching rate | | |
| To the student Understanding of receptivity | Difficulty level | | Under the same difficulty, the types of questions predicted by teachers were compared with the types of questions raised by students. |
| | complexity | The semantic | The encoding of semantic complexity can be divided into five semantic types: restatement, combination, comparison, change and replacement. |
| | | structure | Structural complexity consists of two indexes, i.e., beginning unknown and result unknown. |
| To the student Understanding of mathematical thinking | Depth of thought | | This paper analyzes the profundity of mathematical thinking based on regular problems and compares the closeness between the number of teachers predicting regular problems and the number of students proposing regular problems. The regularity problem is divided into the character description law and the symbol description law. |
| | Flexibility of thinking | | The flexibility of mathematical thinking was analyzed based on the quantitative analysis of problem types and the closeness between the number of teachers predicting regular problems and the number of students proposing regular problems was compared. Problems can be classified into extensibility problems, non-extensibility problems, and other problems. |
| | Originality of thought | | Based on the creative problem analysis of the originality of mathematical thinking, compare the number of teachers predicting creative problems with the number of students proposing creative problems. |

First of all, the understanding of the existing knowledge structure of the students is reflected by observing the similarity and matching the problems raised by students themselves and the problems which teachers predicted that students would propose. Secondly, the understanding of students' receptivity includes teachers' cognition of the difficulty and complexity of predicting the types of questions raised by students and the types of questions raised by students themselves. The complexity of mathematical problems is not a single concept, but a complex synthesis. [8] Therefore, complexity is divided into semantic complexity and structural complexity. The encoding of semantic complexity refers to the standard of complexity of the problem by ZhouRuohong, CAI Jinfang and others. [9-10] It can be divided into five semantic types: restatement, combination, comparison, change and replacement. Among other things, all problems can be classified according to semantic complexity or the number of relationships required to solve the problem, and the semantic type of the problem ranges from 1 to 5. There are two indicators of structural complexity: start unknown and end unknown. The unknown quantity is not the unknown quantity at the end of the question. For example, "175 guests entered the meeting room when the bell rang." The unknowns are stated at the end of the problem, such as "how many black spots are there in the graph (10)". Thirdly, the comprehensive evaluation of the understanding level of pre-service teachers and in-service teachers on students' mathematical thinking was conducted from the depth of mathematical thinking, flexibility, originality. [10] Problems can be divided into extensibility problems, non-extensibility problems and other problems. Extensibility problems refer to mathematical problems that go beyond the initial graphics or quantities given in the task situation. Non-extensibility problems are limited to a given task situation and do not jump out of a previously given graph or quantity. Invalid problems refer to non-mathematical problems, unsolved problems, unclear problems and meaningless problems irrelevant to the topic. Originality is defined as the number of questions a student asks that does not make up more than 10% of the total.

3. Method

3.1. Tool

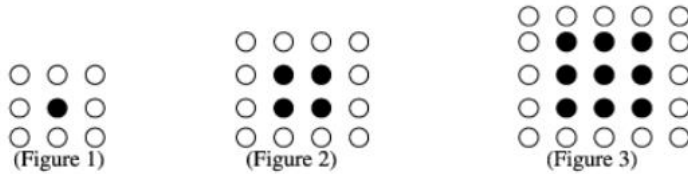
The test papers are divided into three categories: for students, for pre-service teachers and for in-service teachers. The test questions are composed of two open question situations, namely, the dot situation (figure 1) and the doorbell situation (figure 2). The two situations are selected from the research

on mathematical problems put forward by Cai Jinfa and others, and have good reliability and validity. In the test paper, students are required to ask three mathematical questions with different difficulty levels according to the given situational information: simple questions, medium difficulty questions and more difficult questions; without knowing the results of the students' questions, pre-service teachers and in-service teachers are required to predict which mathematical problems students will ask at different difficulty levels based on their own understanding of the students and based on the given task situation.

J. Cai, S. Hwang / Journal of Mathematical Behavior 21 (2002) 401–421

Dots Problem-Posing

Mr. Miller drew the following figures in a pattern, as shown below.



For his student's homework, he wanted to make up three problems BASED ON THE ABOVE SITUATION: an easy problem, a moderate problem, and a difficult problem. These problems can be solved using the information in the situation. Help Mr. Miller make up three problems and write these problems in the space below.

Figure 1: Dot situation

J. Cai, S. Hwang / Journal of Mathematical Behavior 21 (2002) 401–421

Doorbell-Posing

Sally is having a party, the first time the doorbell rings, 1 guest enters.
 The second time the doorbell rings, 3 guests enter.
 The third time the doorbell rings, 5 guests enter.
 The fourth time the doorbell rings, 7 guests enter.

Keep on going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

For his student's homework, Mr. Johnson wanted to make up three problems BASED ON THE ABOVE SITUATION: an easy problem, a moderate problem, and a difficult problem. These problems can be solved using the information in the situation.

Help Mr. Miller make up three problems and write these problems in the space below.

Figure 2: Doorbell situation

3.2. Participants

The participants included students, pre-service teachers and in-service teachers. The total number of the students and in-service teachers is 372 including sixth to eighth graders and corresponding class mathematics teachers in three public schools in Xinxiang City, Henan Province. The schools are above average in terms of educational quality and educational resources. The pre-service teachers were junior students of a normal university in Henan province, of whom 52 were normal students majoring in primary education and 122 were normal students majoring in mathematics and applied mathematics in the School of Mathematics. The distribution of valid test samples finally recovered from them is shown in Table 2.

Table 2: Distribution of subjects in the study

| | students | Pre-service teachers' | In-service teachers' | A total of |
|------------|----------|-----------------------|----------------------|------------|
| Grade 6 | 136 | 52 | 12 | 200 |
| Grade 7 | 114 | 62 | 7 | 183 |
| Grade 8 | 122 | 60 | 10 | 192 |
| A total of | 372 | 174 | 29 | 575 |

3.3. Data Analysis

Data coding mainly analyzes the questions raised by the subjects from two aspects of problem type and problem complexity. Firstly, the questions were divided into expansion questions, non-expansion questions and other questions. Next, each extensibility or non-extensibility problem is further classified according to the nature of the problem, as shown in Table 3. Based on the above coding system, all the questions raised by the subjects were individually coded and analyzed, and the coding results were input into Excel software.

Table 3: Dot situation problem type coding table

| | coding | Type of problem | Type description | For example |
|---------------------------|--------|--|---|--|
| Non-extensibility problem | 1 | The number of points in a single figure | Calculate the number of dots in a graph according to the previous given graphs | How many black spots are there in figure (3)? |
| | 2 | The number of points in multiple figures | Calculate the total number of dots in multiple sets of figures according to the previous given figures | How many white dots are there in figures (1) to (3)? |
| | 3 | Comparison of graph points | According to the first few given graphs, compare the number of black and white dots in different graphs or the same graph, including the calculation of difference, multiple, percentage, etc | Figure (2) What percentage of the number of black dots in the number of white dots? |
| Scalability problem | 4 | The number of points in a single figure | The number of dots in a graph other than a given graph | How many white dots and how many black dots are there in figure (10)? |
| | 5 | The number of points in multiple figures | Calculation of the total number of dots in a given graph | How many black dots are there from figure 1 to figure 100? |
| | 6 | Comparison of graph points | In addition to a given graph, compare the number of black and white dots in the same graph, or the number of black and white dots between different graphs, including difference, multiple, percentage, etc | Figure (6) What is the ratio of the number of black dots to the number of white dots? |
| | 7 | The serial number of a graph | Given the number of dots of a graph outside the given graph, calculate the serial number of the graph | Which graph has 49 black spots? |
| | 8 | Draw something | Draw a figure other than the given figure | Please draw the figure of figure (7); |
| | 9 | General rules | There are no general rules for specific objects | What's the pattern? |
| | 10 | An algebraic expression expresses a law | The number of dots and its change rule are expressed by mathematical expression | How many white dots and how many black dots are there in graph (n)? |
| | 11 | Add the condition | Add to, change, or create a new situation | In one picture, there are 625 black dots, so how many white dots are there in this picture? |
| other | 12 | Other problems | Non-mathematical problems, unsolvable problems, unclear problems, and nonsense problems | Why is Figure (2) better than figure (1)? |
| | 11 | Add the condition | Add to, change, or create a new situation | If only 100 people can sit in the meeting room, then after what time the bell rings no guests can be admitted? |
| other | 12 | Other problems | Non-mathematical problems, unsolvable problems, unclear problems, and nonsense problems | What colour are the lights in the venue? |

3.4. Reliability

To ensure the reliability of data coding, 20% students samples and 30% pre-service teachers samples

were randomly selected from all grades. Two raters independently coded their questions, and three researchers independently coded the questions predicted by all in-service teachers. Among both students and pre-service teachers, the agreement between raters reached 90%; the coding consistency of in-service teachers is above 95%.

4. Results

4.1. Pre-service teacher test results

(1) In terms of students' existing knowledge structure, pre-service teachers believe that specific numerical calculation and the application of algebra play a dominant role.

Whether in dot situation or doorbell situation, most pre-service teachers predicted that the problems which would be proposed most frequently by students were single graph points (type 4), multiple graphic points type (5) and algebraic expression rule type (10), which showed that pre-service teachers think detailed numerical calculation and the application of algebraic expression in the students' knowledge structure plays an important role. In addition, few pre-service teachers predicted the problems of reverse questioning (type 5-7) and creating new situations (type 11 adding conditions), indicating that pre-service teachers thought students' performance in reverse and creative thinking was relatively weak.

In short, although pre-service teachers have a certain understanding of the existing knowledge structure of students, in-service teachers' understanding of students is closer to the actual situation of students. The in-service teachers corresponding to Grade 6 students have a slightly lower grasp of the existing knowledge structure of students, and there is no significant difference between Grade 7 and Grade 8 in-service teachers. In the three grades, the rank of in-service teachers' understanding of the existing knowledge structure of students is: Grade 8 > Grade 6 > Grade 7.

(2) In terms of understanding of students' acceptance ability, pre-service teachers tend to predict non-expansibility problems and specific numerical calculation problems in simple problems, while regularity problems and problems containing multiple semantic types in difficult problems.

First of all, preservice teachers tend to predict non-extensibility problems and specific numerical calculation problems in simple problems, and put forward regularity problems in difficult problems.

Non-extensibility problems are not the primary choice of pre-service teachers at any difficulty level, and the proportion of such problems decreases sharply with the increase of difficulty. In particular, no preservice teachers predicted non-extensibility problems among the more difficult problems. In general, most of the problems predicted by pre-service teachers are expansibility problems, and the proportion of non-expansibility problems decreases rapidly with the increasing difficulty of the problems.

In the dot problem, type 9 accounts for a large proportion of the three difficulties, while in the doorbell problem, type 5 is more, indicating that pre-service teachers tend to take revealing the essential law of the situation as the evaluation standard of the difficulty of the problem. No matter what grade the students are in, the pre-service teachers always predict specific numerical problems in simple problems, that is, calculate a certain graph or a certain bell ringing, and predict regular problems in medium or more difficult problems.

Secondly, preservice teachers tend to predict problems involving multiple semantic types in difficult problems.

Table 4 and Table 5 respectively show the proportion distribution of problems with multiple semantics in different difficulty levels predicted by pre-service teachers in the dot situation and doorbell situation:

Table 4: The proportion of problems with multiple meanings in dot scenarios at different difficulty levels

| | The sixth grade | | | The seventh grade | | | The eighth grade | | |
|-----------------------|-----------------|------|-------|-------------------|-------|-------|------------------|-------|-------|
| | P1 | P2 | P3 | P1 | P2 | P3 | P1 | P2 | P3 |
| A kind of semantic | 100.0 | 90.0 | 90.70 | 98.31 | 85.48 | 91.67 | 100.0 | 92.73 | 92.16 |
| Two kinds of semantic | 0.0 | 10.0 | 9.30 | 1.69 | 14.52 | 8.33 | 0.0 | 7.27 | 7.84 |

Table 5: Proportion of doorbell situation questions with multiple semantics in different difficulty levels

| | The sixth grade | | | The seventh grade | | | The eighth grade | | |
|-------------------------|-----------------|-------|-------|-------------------|-------|-------|------------------|-------|-------|
| | P1 | P2 | P3 | P1 | P2 | P3 | P1 | P2 | P3 |
| A kind of semantic | 88.89 | 73.47 | 59.09 | 88.06 | 76.56 | 56.72 | 90.48 | 82.54 | 48.44 |
| Two kinds of semantic | 11.11 | 26.53 | 38.64 | 11.94 | 21.88 | 41.79 | 9.52 | 17.46 | 48.44 |
| Three kinds of semantic | 0.0 | 0.0 | 2.27 | 0.0 | 1.56 | 1.49 | 0.0 | 0.0 | 3.13 |

Looking at the table above, preservice teachers tend to predict problems with only one semantic type, regardless of the difficulty level. Nevertheless, the proportion of questions involving one semantic type is decreasing, and the number of questions involving both increases with difficulty. In other words, pre-service teachers believe that students will judge the difficulty of a problem according to the semantics of the problem.

Preservice teachers thought that the students of their predicted grade did not perform well in the structural complexity, and few preservice teachers predicted the "initiation unknown" type of problems. In fact, pre-service teachers usually state unknowns at the end of a question when they express a question. Even if the question is based on reverse thinking, they will put unknowns at the end of the question.

(3) The pre-service teachers believe that the thinking depth of senior students is obviously better than that of junior students, and junior students are slightly better than senior students in thinking flexibility and originality.

According to the comprehensive analysis of the data in the table, with the increase of grade, more students can put forward regular questions, which indicates that senior students have good depth of mathematical thinking. Most of the preservice teachers predicted the regularity problems, which indicated that the preservice teachers thought the students of the grade they predicted had better depth of mathematical thinking.

In addition, most of the pre-service teachers believe that the students in their predicted grade can raise different types of questions from at least two different perspectives. One is to simply imitate the original situation. The other is to ask¹ questions by distilling situational information. The types of questions raised by the lower grade students mainly focus on three types, while the higher grade students focus on two types, which is consistent with the prediction results of the corresponding pre-service teachers.

Finally, it is not difficult to see that the pre-service teachers think that the originality of thinking of junior students is relatively good, while the originality of thinking of senior students needs to be improved.

4.2. Post-Service teacher test results

(1) In terms of the existing knowledge structure of students, in-service teachers believe that the application of specific numerical calculation and algebra gives an absolute advantage in the existing knowledge structure of students.

Study shows whether in the dot situation or doorbell situation, one of the most serious problems teachers forecast is nothing more than the following two kinds: one pair of graphics or one's situation and the change rule of a given pattern, bell indicates that teachers think detailed numerical calculation and the application of algebraic expression in the students' knowledge structure occupies an extremely important position, and in Grade three, in-service teachers predict the performance of no significant difference.

(2) In terms of understanding of students' acceptance ability, in-service teachers tend to predict specific numerical calculation problems in simple problems, while regular problems and problems containing multiple semantic types in difficult problems.

First of all, in-service teachers tend to predict specific numerical calculation problems in simple problems, and put forward regularity problems in difficult problems.

Most of the problems predicted by in-service teachers at any difficulty level are extensibility problems. In other words, the in-service teachers believed that the students in their grade did not pay much attention

¹ Note: IN this study, P1, P2 and P3 refer to simple, medium and difficult problems respectively.

to the internal relationships of the existing situations when they asked questions. In general, in-service teachers believe that students in their grade often pay attention to the development trend of the follow-up situation in the process of asking questions. They usually think from a given situation, excavate mathematical relationships in the situation, and then propose extensibility problems.

Secondly, in-service teachers tend to predict difficult problems involving multiple semantic types.

Table 6 and Table 7 respectively show the proportion distribution of problems with multiple semantics in different difficulty levels predicted by in-service teachers in the dot situation and doorbell situation:

Table 6: Percentage of problems with multiple meanings in dot scenarios at different difficulty levels

| | The sixth grade | | | The seventh grade | | | The eighth grade | | |
|-----------------------|-----------------|-------|-------|-------------------|-------|-------|------------------|-------|------|
| | P1 | P2 | P3 | P1 | P2 | P3 | P1 | P2 | P3 |
| A kind of semantic | 100.0 | 100.0 | 63.64 | 100.0 | 57.14 | 66.67 | 100.0 | 100.0 | 60.0 |
| Two kinds of semantic | 0.0 | 0.0 | 36.36 | 0.0 | 42.86 | 33.33 | 0.0 | 0.0 | 40.0 |

Table 7: Proportion of doorbell situation questions with multiple semantics in different difficulty levels

| | The sixth grade | | | The seventh grade | | | The eighth grade | | |
|-------------------------|-----------------|-------|-------|-------------------|-------|-------|------------------|------|------|
| | P1 | P2 | P3 | P1 | P2 | P3 | P1 | P2 | P3 |
| A kind of semantic | 100.0 | 69.23 | 45.45 | 100.0 | 57.14 | 42.86 | 100.0 | 80.0 | 40.0 |
| Two kinds of semantic | 0.0 | 23.08 | 54.55 | 0.0 | 42.86 | 57.14 | 0.0 | 20.0 | 60.0 |
| Three kinds of semantic | 0.0 | 7.69 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

According to the table above, for simple problems, in-service teachers of all grades predicted problems that only contained one semantic type, while for medium-difficulty problems, teachers predicted problems that contained two or three semantic types. Serving teachers believe that students in their grade levels perform less well on structural complexity. In-service teachers in both situations did not predict the "initial unknown" type problems, and all the problems were expressed in the structure of "outcome unknown".

(3) In terms of depth of thinking, in-service teachers all predicted that students of the grade they taught would perform well; in terms of the flexibility and originality of thinking, the in-service teachers think that the performance of the junior students is better, while the senior students need to be improved.

First, in-service teachers predicted that their grade students had good depth of thought.

According to the number of regular problems predicted by in-service teachers, in-service teachers of senior grades think that students of the grade they teach are good at deep thinking and are good at revealing the essential laws of things and using mathematical symbol language to express the laws. The in-service teachers of the lower grade think that the students of the grade will not pay too much attention to the general law of the development of things, but dig out the internal connection from the existing information, and the depth of thinking is relatively low.

Secondly, the in-service teachers of the lower grade thought that the students' thinking was more flexible, while the in-service teachers of the higher grade thought that the students' thinking flexibility should be improved.

Based on the in-depth analysis of the number of types of problems predicted by in-service teachers, in-service teachers of Grade 6 think that students in the grade they teach have a strong strain ability of mathematical thinking, and that students can get rid of the constraints of existing models and have relatively high degree of flexibility in thinking. The in-service teachers of Grade 7 and 8 think that students of the grade will be affected by the thinking pattern and directly regard it as a regular problem. In the process of asking questions, they will not pay attention to the relationship between the internal structure of things, and the flexibility of thinking is relatively low.

Finally, the junior in-service teachers think the students of their grade are relatively good in thinking originality, while the senior in-service teachers think the students of their grade are relatively poor in thinking originality.

As can be seen from the above table, in-service teachers of senior grades believe that students of their grade will be affected by stereotyped thinking and have unsatisfactory performance in creative thinking. The in-service teachers of the lower grade think that the students of the grade can get rid of the stereotyped thinking, break through the original way of thinking to produce new ideas, put forward some novel and unique problems, think actively, with a good sense of innovation.

4.3. Comparison of pre-service and in-service teacher test results

4.3.1. Post-service teachers have a better understanding of students' existing knowledge structure than pre-service teachers

First of all, the predicted performance of in-service teachers is closer to the actual level of students from the perspective of problem types. However, both pre-service and in-service teachers underestimated students' ability of imagination and innovation consciousness, and overestimated students' ability of abstraction and generalization. Secondly, from the matching rate, the average matching rate predicted by in-service teachers is significantly higher than that of pre-service teachers. This shows that in-service teachers' understanding of students' knowledge structure is relatively close to the actual situation of students, but further development is needed.

4.3.2. Pre-service teachers have a better understanding of students' acceptance ability than in-service teachers

Firstly, both pre-service teachers and in-service teachers overestimated the students' grasp of the difficulty of the problem, but the prediction of pre-service teachers was closer to the students' understanding of the difficulty of the problem. Secondly, from the semantic type of the problem, the prediction of pre-service teachers is closer to the actual level of students. Based on the previous analysis, it can be seen that in both situations, most of the questions raised by students are of the "outcome unknown" type, and few are of the "initial unknown" type. However, the two types of teachers almost did not involve the "initial unknown" type in the prediction questions.

In the complexity of the problem, the pre-service teachers' understanding of students' acceptance ability is closer to the actual situation of students. There was no significant difference between pre-service teachers and in-service teachers in their understanding of students' acceptability, and in-service teachers in all three grades underestimated students' understanding of various semantic types.

4.3.3. Pre-service teachers have a slightly better grasp of students' mathematical thinking than in-service teachers

First, both pre-service teachers and in-service teachers have a poor understanding of the depth of students' mathematical thinking, but pre-service teachers perform better. The rank of pre-service teachers' understanding of the depth of students' mathematical thinking from high to low is: Grade 7 > Grade 8 > Grade 6; the rank of in-service teachers' understanding of the depth of students' mathematical thinking from high to low is: Grade 7 > Grade 8 > Grade 6.

Secondly, pre-service teachers are more aware of the flexibility of students' mathematical thinking. The pre-service teachers corresponding to Grade 6 students had a weak understanding of the flexibility of students' mathematical thinking, and there was no significant difference between Grade 7 and Grade 8. The in-service teachers in Grade 8 have a poor understanding of the flexibility of students' mathematical thinking, and there is no significant difference between Grade 6 and Grade 7.

Thirdly, pre-service teachers have a better understanding of the originality of students' thinking in mathematics than in-service teachers. The pre-service teachers corresponding to the sixth grade students had a relatively good understanding of the originality of students' mathematical thinking, and there was no obvious difference between the seventh and eighth grades. Teachers in Grade 8 had the worst understanding of students' creative thinking, while there was no significant difference between Grade 6 and Grade 7.

5. Recommendations

5.1. Recommendations for pre-service teachers

The training of pre-service teachers should integrate the conditional and ontological knowledge of teacher trainees with each other. With the help of PBL teaching mode, teacher trainees can be guided to conduct exploratory learning based on real situations and construct their knowledge system and deepen their understanding in completing projects [11]. Secondly, universities and primary and secondary schools cooperate to build a "progressive and consistent" practical teaching system. Through the guidance of instructors, teacher trainees can gain experience and reflect on classroom teaching and interdisciplinary teaching based on real situations in their practice positions, and finally promote teacher trainees to acquire a complete knowledge system for professional development of teachers [11].

5.2. Recommendations for in-service teachers

In-service teachers first establish their own "problem-posing teaching case library". This will help teachers understand students' cognitive structure and mathematical thinking, and improve their own KCS level. Secondly, paying attention to reading materials such as the research results in frontier areas of education and professional teaching journals helps in-service teachers understand the cognitive level of students at all stages, and understand students' receptivity. Thus in-service teachers' KCS level can also be developed.

To sum up, it is suggested that question proposal, both from teachers and students in classroom teaching, should occur more often. Problem formulation is a powerful tool to understand students' mathematical thinking. Using problem formulation in classroom teaching can help in-service teachers to understand students' mathematical thinking and promote the continuous development of in-service teachers' KCS. At the same time, establishing a case library based on the timely record of unpredicted problems before class can gradually accumulate and enhance their understanding of students.

References

- [1] Shulman, L.S.. *Educational Researcher*, 1986, 15(2): 4--14. (in Chinese)
- [2] Ball, D.L., et al. *Content Knowledge for Teaching: What Makes It Special?* [J] *Journal of Teacher Education*, 2008, 59 (5): 389 -- 407.
- [3] Pang Yali. *Research on MKT Status and Development of Pre-service Mathematics Teachers* [D]. Shanghai: East China Normal University, 2011.
- [4] Li Xinlian, Song Naiqing et al. *Journal of mathematics education*, 2019, 28(02): 1-6.
- [5] Cai, J.. *Mathematical Thinking and Learning*, 2005, 7(2): 135 -- 169.
- [6] Cai, J., & Hwang, S. (2019). *Learning to teach mathematics through problem posing: Theoretical considerations, methodology, and directions for future research*. *International Journal of Educational Research*. <https://doi.org/10.1016/j.ijer.2019.01.001> Online First.
- [7] Cai, J. (2005). *U.S. and Chinese Teachers' Constructing, Knowing, and Evaluating Representations to Teach Mathematics* 7 (2), 135-169.
- [8] Li Huaijun, ZHANG Weizhong. *Journal of mathematics education*, 2019, 28(05): 2-8.
- [9] Zhou Ruohong, LU Chuanhan. *Journal of Guizhou Normal University*, 2002(02): 24-30. (in Chinese)
- [10] Silver, E.A., et al. *An Analysis of Arithmetic Problem Posing by Middle School Students* [J], *Journal for Research in Math Education*, 1996, 27(5): 521-539.
- [11] Zhang Huirong, Wu Jingyu. *Journal of Southwest University (Social Science Edition)*, 2011, 47(02): 118-127.