ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

Cultivating Senior High School Students' Mathematical Core Competencies Based on Thinking Visualization

Shuangshuang Li[1,a], Jihui Wang[1,*]

[1]*School of Mathematical Sciences, University of Jinan, Jinan, China*
[a]*17686533863@163.com*
[*]*Corresponding author: wangjh@ujn.edu.cn*

Abstract: *Against the backdrop of the ongoing deepening of curriculum reform guided by core competencies, high school mathematics instruction urgently needs to break free from the entrenched limitation of "prioritizing knowledge transmission over the development of thinking skills". As a new teaching approach, visual thinking instruction externalizes implicit thought processes, providing students with logical thinking paths guiding the natural development of their thinking. This study primarily explores strategies for applying thinking visualization in high school mathematics instruction and analyzes its impact on fostering students core mathematical competencies, with the aim of providing practical guidance for classroom teachers.*

Keywords: *Visualization Thinking; Core Mathematical Competencies; High School Mathematics*

1. Introduction

The *General High School Mathematics Curriculum Standards (2017 Edition, Revised in 2020)* emphasizes that the cultivation of core competencies is the central objective and key guiding principle of current educational practice [1]. Mathematical thinking represents the internalization of core competencies in mathematics; while sharing commonalities with general human thinking, it also embodies the rigor and abstract nature characteristic of the discipline [2]. Therefore, for the discipline of mathematics, core mathematical literacy not only embodies the educational value of mathematics but is also deeply intertwined with the development of students mathematical thinking. However, current mathematics instruction suffers from the pitfall of overly abstract and generalized thinking; traditional teaching methods often impose cognitive load on students, fostering a fear of mathematics. Visual thinking instruction guides students to express concepts through language, graphics, and actions, transforming the static transmission of mathematical knowledge into a dynamic process of text-and-image collaboration. Abstract mathematical concepts become clear and intuitive through visual communication, reducing students cognitive load and fostering the development of their mathematical thinking.

2. Connotation and Theoretical Foundations of Thinking Visualization

Core competencies refer to the essential character traits and key abilities that students should possess to meet the needs of their own lifelong development and societal progress. They focus not only on the accumulation of knowledge and skills but also emphasize the thinking skills, innovative abilities, problem-solving skills, and positive emotional attitudes and values required to navigate a complex and ever-changing real world. Mathematical core competencies are the specific manifestation of core competencies within the discipline of mathematics; they refer to the fundamental abilities and qualities that students should possess during the process of learning mathematics. These include mathematical abstraction, logical reasoning, mathematical modeling, visual spatial imagination, data analysis, mathematical expression and communication, and the mathematical application. These competencies help students understand and master mathematical knowledge, apply mathematical methods to solve real-world problems, and cultivate their comprehensive abilities in independent thinking, collaborative communication, and innovative practice, thereby laying a solid foundation for their lifelong learning and all-round development.

ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

In China, Professor ZhuoyuanLiu was the first to propose the concept of "visualization of thinking". He defines it as the process of making previously invisible thought processes, structures, methods, and strategies clearly visible through diagrams or combinations of diagrams [3]. Furthermore, Professor Liu emphasizes that the diagrams referred to here are not ordinary images, but rather clearly structured knowledge frameworks composed of elements and relationships between them. In actual teaching practice, two types of diagrams are commonly used for visualizing thinking: the "knowledge organization type", such as subject-specific mind maps, and the "problem-solving type", which primarily includes subject-specific fishbone diagrams, problem-solving strategy models, and diagrams illustrating quantitative relationships. At the same time, in his article *Visualizing Thinking: A New Lever for Reducing Academic Burden and Enhancing Efficiency*, ZhuoyuanLiu argues that applying thinking visualization to subject-specific instruction can help systematize fragmented knowledge, make implicit thinking explicit, and model problem-solving patterns. He emphasizes both the explicit and implicit values of thinking visualization, focusing on how it enhances students' learning efficiency and promotes the coordinated functioning of the left and right hemispheres of the brain. This approach enables students to discover the joy of learning through reflection, thereby achieving the goal of reducing academic burden and enhancing learning efficiency.

According to brain science theory, the human brain is divided into two hemispheres—the left and the right—each of which performs distinct functions: the left hemisphere is primarily responsible for logical thinking, while the right hemisphere focuses more on visual thinking. Compared to traditional teaching methods, visual thinking instruction incorporates right-brain elements such as imagery, imagination, color, and rhythm, making abstract concepts more concrete and intuitive. Once thoughts are visualized, the body's comprehension and memory capabilities improve, thereby enhancing the efficiency of information processing and transmission and boosting the brain's performance. Therefore, theories of brain science play a crucial role in the application of visual thinking in education. Specifically, teachers should incorporate more vivid, visual information into school curricula, adding illustrations and other materials that help stimulate visual thinking to promote students' comprehensive development.

Constructivist theory posits that knowledge is the result of active construction by learners, who process and construct new information based on their existing knowledge and experience to form new cognitive schemas. It emphasizes the active, contextual, and socially interactive nature of learning. Thinking visualization tools present complex knowledge in a structured manner through graphics, symbols, and diagrams, helping students actively establish connections between new and existing knowledge and achieve the integration of knowledge. At the same time, by engaging in hands-on practice and collaborative communication using these tools, students construct their own unique knowledge systems. This demonstrates that learning is a process of meaning-making, which aligns closely with the principles of constructivist theory.

3. Application Strategies of Thinking Visualization in Senior High School Mathematics Teaching

3.1. Visualization in Instructional Design

Visualizing teaching concepts contributes to the clarification of instructional approaches. First, concretizing learning objectives enables both teachers and students to quickly grasp the intended outcomes and clarify the direction of teaching and learning. Second, structuring instructional content allows teachers to identify key points more intuitively and organize their teaching strategies accordingly. On this basis, the visualization of learning objectives and instructional content helps teachers clarify "what to teach and how to teach", while guiding students to understand "what to learn and to what extent". Taking these as benchmarks, teachers can further enhance the precision and objectivity of instructional assessment.

Visualizing Thinking Clarifies Unit-Level Core Competencies. Against the backdrop of the new curriculum reform, high school mathematics instruction is gradually shifting from a knowledge-oriented approach to one focused on enhancing competencies. How to effectively implement core competencies in the classroom has become a key focus of teachers research during the instructional design phase. By visualizing thinking, teachers can present the overall framework of the unit and the corresponding core competencies before the unit begins, thereby gaining a holistic understanding of the instructional content and ensuring the implementation of core competencies in the classroom. As shown in Figure 1, take the unit "Planar Vectors and Their Applications" as an example.

ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

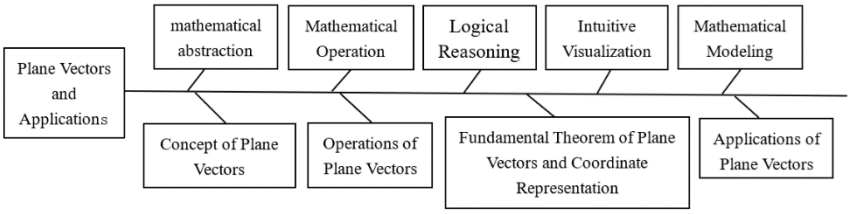

Figure 1: Core Competencies Framework of the Plane Vector Unit

3.2. Visualization in Concept-Based Instruction

Mathematical concepts typically have multiple representations, and differences in the representations of a given concept often lead learners to form different thought processes [4]. In teaching, guiding students to construct differentiated representations of concepts across various representational systems helps reinforce their understanding of the intrinsic connections within concepts. The visualization of mathematical concepts can be summarized as follows:

Intuitive diagrams to introduce concepts. In concept instruction, teachers use real-life examples to demonstrate relevant mathematical concepts, sparking interest in mathematics. Students come to appreciate the close connection between mathematics and daily life, understanding that mathematics originates from and serves life.

Dynamic representations to clarify essence. As an intuitive and vivid teaching method, dynamic representations transform abstract thought processes into visual forms through hands-on practice, shifting students from a passive to an active role. Through practice, students gradually master abstract knowledge, laying the foundation for deep learning.

Dynamic Demonstrations for Analyzing Mathematical Concepts. If abstract knowledge is conveyed solely through static images or textual explanations, students must construct the process through imagination, which can easily lead to cognitive biases. Dynamic demonstrations directly present the process of knowledge generation, transforming the abstract into the concrete and the static into the dynamic. This reduces students cognitive load and facilitates a deeper understanding of concepts.

For example, when teaching the definition of an ellipse, teachers use visual aids to demonstrate ellipses found in everyday life, thereby reinforcing students understanding of the concept and stimulating their interest in learning and curiosity. Building on this foundation, students work in groups to draw ellipses using the two-pin method. Following this inquiry-based activity, the teacher uses GeoGebra software to demonstrate the process of forming an ellipse, as shown in Figure 2. Through this dynamic demonstration, students experience the process of knowledge construction and cultivate their ability for visual imagination through intuitive perception.

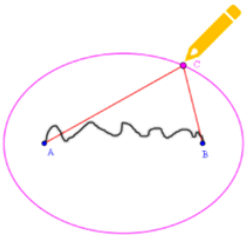

Figure 2: Ellipse Generation Trajectory

3.3. Visualization in the problem-solving process

In the context of the new college entrance examination system, the requirement for examination questions to explicitly assess problem-solving processes necessitates that teaching breaks away from the conventional practice of “prioritizing results over processes” and transform implicit cognitive activities into observable cognitive trajectories [5]. As a bridge connecting abstract mathematical knowledge with concrete cognitive processes, the visualization of thinking plays a crucial role in high school mathematics education by constructing a cognitive pathway from problem representation to strategy generation.

When solving complex problems, flowcharts can clearly illustrate the thought process, helping

ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

students quickly and accurately apply their knowledge to solve the problem. As shown in Figure 3, in the quadrangular pyramid $P-ABCD$ shown in the figure, $PA \perp$ plane $ABCD$, $BC \parallel AD$,and $AB \perp AD$.

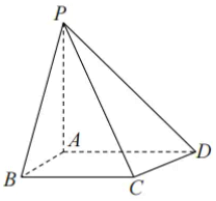

Figure 3: Geometric Diagram of Quadrangular Pyramid P-ABCD

(1) Prove that plane $PAB \perp PAD$

(2) If $PA = AB = \sqrt{2}, AD = \sqrt{3}+1, BC = 2,$ and points P, B, C, D lie on the same spherical surface, let the center of this sphere be O.

(*a*) Prove that Olies on plane $ABCD$.

(*b*) Find the cosine value of the angle formed by the straight lines AC and PO.

Solid geometry is one of the key topics on the college entrance exam. For most students, Part (1) is a relatively basic question that primarily assesses their proficiency in identifying and applying the properties of the positional relationships between lines and planes. Part (2) is relatively more challenging. If students rely solely on geometric methods, they may struggle to visualize the spatial configuration of the sphere circumscribed around a tetrahedron, particularly the position of the center of the circumscribed circle of the right trapezoidal base and the perpendicular relationship between the center of the sphere and the circumcenter of the base. Therefore, attempting to visualize the textual content and using GeoGebra to display the solid figures can help enhance students' spatial imagination ability. The difficulty of using the vector method lies in the fact that errors can easily occur when determining the coordinates of point C, and the simplification and solution of parameters may be inaccurate. When solving problems, students often encounter interruptions in their proofs due to inconsistent logic; therefore, flowcharts can be used to visually represent their thought process.

In question (1), a flowchart is used to clarify how to prove surface perpendicularity through line and plane perpendicularity. By analyzing concepts and deriving problem-solving steps, the entire thought process is visually represented, which helps students of different levels in their learning. In question (2), the solution methods are first analyzed, including vector methods and geometric methods, and the appropriate solution method is selected through comparison. This problem-solving approach makes the previously invisible thought process visually clear, which reduces the cognitive difficulty for students and cultivates their logical reasoning abilities through the reasoning process of the entire problem.

These visualization methods not only help students better understand and remember each step and logical relationship in the problem-solving process, but also enhance their ability to learn independently and improve their problem-solving efficiency. In teaching, teachers can combine specific mathematical problems and flexibly use these visual methods to help students establish clear problem-solving ideas, cultivating their logical thinking and analytical abilities.

3.4. Visualization in Review Sessions

Review lessons are not merely a list or summary of key concepts; they are a crucial component for enhancing students' key competencies and fostering their core mathematical literacy [6]. Teachers can use visual thinking tools to guide students through an efficient review process.

First, use concept maps to plan your study schedule. The high school mathematics curriculum is vast and complex, encompassing multiple modules such as functions, geometry, algebra, and probability and statistics. Knowledge points within each module are closely interlinked, and there are cross-references between modules. A weakness in a single knowledge point can easily lead to gaps in understanding across an entire subject area. Therefore, designing a well-structured review plan and establishing a clear logical framework are essential prerequisites for improving review efficiency. In the initial stages of review sessions—particularly at the outset of unit reviews, module reviews, or the first round of senior year review—teachers can guide students to independently construct mathematical concept maps. By

ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

empowering students to take the initiative in their learning rather than simply presenting a ready-made knowledge framework, teachers can cultivate students' ability to organize information independently and foster their mathematical abstract thinking Competencies.

During the actual implementation process, teachers first identify the core review topics—such as "Properties and Applications of Functions" or "Spatial Relationships in Solid Geometry". They then guide students to start from these core concepts and systematically break down sub-topics, derived knowledge points, relevant formulas and theorems, as well as common pitfalls and points of confusion. By using lines, arrows, and hierarchical relationships, they clearly mark the hierarchical relationships, connections, and logical derivations between concepts, thereby forming a comprehensive conceptual map network. For example, when reviewing the "Planar Vectors" module, "Planar Vectors" serves as the central node, branching out into six major categories: definition, representation methods, linear operations, dot product, coordinate representation, and applications. Each branch is further broken down into specific knowledge points; for instance, linear operations include addition, subtraction, and scalar multiplication, along with their corresponding rules, geometric interpretations, and special conclusions. By independently creating concept maps, students move beyond passively absorbing knowledge to actively organizing its structure. This allows them to precisely identify review priorities and knowledge gaps, clarify the sequence and core focus of their revision, and simultaneously strengthen their mathematical abstraction skills and solidify foundational logical reasoning abilities through the process of conceptual abstraction and logical organization.

Second, promoting the structuring of knowledge through knowledge framework diagrams. While concept maps focus on organizing concepts hierarchically within a single module, knowledge framework diagrams emphasize the vertical depth of knowledge connections and horizontal cross-module relationships. They serve as core visualization tools for structuring and systematizing knowledge, helping students break down module barriers, establish a comprehensive mathematical cognitive system, and further enhance their ability to apply knowledge holistically and think systematically. In traditional review classes, students often have a solid grasp of individual knowledge points but struggle when faced with cross-module comprehensive problems or final exam questions. The core issue lies in the lack of a structured knowledge framework, which prevents them from quickly mobilizing related knowledge to solve complex problems. Knowledge framework diagrams are precisely designed to address this shortcoming.

When guiding students in constructing a knowledge framework diagram, teachers must balance vertical logic with horizontal connections: Vertically, following the sequence of knowledge development and logical reasoning, they should build a knowledge chain that progresses from basic definitions, formulas, and theorems to properties, applications, and typical problem types, and finally to extensions, summaries of methods—moving from simple to complex and from easy to difficult; Horizontally, they should transcend the boundaries of textbook chapters to connect common knowledge points across different modules. For example, they might link the monotonicity of functions with applications of derivatives, the monotonicity of sequences, and changes in lines, planes, and angles in solid geometry; or connect similar triangles in plane geometry with the parallelism of lines and planes in solid geometry and the geometric properties of conic sections. Through this dual-directional organization, the knowledge framework diagram weaves scattered knowledge points into a comprehensive network, helping students build a structured cognitive framework. This enables students to clearly grasp the origins, development, and internal connections of knowledge, allowing them not only to quickly identify and fill in knowledge gaps but also to swiftly pinpoint key concepts and mobilize related knowledge when facing comprehensive problem types, thereby achieving flexible knowledge transfer and integrated application. At the same time, the process of constructing this framework deeply hones students' logical reasoning and systematic thinking skills, gradually fostering a holistic mathematical mindset and laying a solid foundation for the enhancement of core competencies.

4. The Impact of Visualizing Mathematical Thinking on the Development of Core Mathematical Competencies Among High School Students

The Curriculum Standards for General Senior High School Mathematics (2017 Edition, Revised in 2020) explicitly identifies six core competencies: "mathematical abstraction, logical reasoning, visual-spatial reasoning, mathematical operations, data analysis, and mathematical modeling". As a bridge connecting abstract knowledge with concrete thinking, mathematical visualization serves as a vehicle for cultivating these core competencies in students. This is manifested in the following six aspects:

ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

Cultivating mathematical abstraction ability by virtue of visual tools. Through a progressive process of "concrete semi-abstract abstract", help students gradually master abstract methods. For example, by presenting function scenarios with line graphs and bar charts, students can intuitively feel the correspondence between two variables, guiding them to abstract the concept of function.

Strengthening logical reasoning skills with visual aids. By incorporating visual tools such as mind maps, flowcharts, and dynamic presentations, we make the implicit process of logical reasoning explicit, helping students gradually master reasoning methods and develop solid logical reasoning skills. For example, after introducing a new lesson, teachers can provide semi-structured templates for students to fill in the branches and annotate their reasoning process. By independently constructing fully structured mind maps, students can establish a coherent logical framework for the subject matter.

Developing visual spatial imagination ability through dynamic demonstrations. Professor Ning zhong Shi notes that information technology is transforming our teaching through integration with academic disciplines, giving rise to an entirely new teaching model. The teaching of solid geometry serves as a key resource for enhancing visual-spatial literacy. By creatively combining static and dynamic elements in the Algebra and Graphics panes of GeoGebra, teachers can help cultivate students' spatial visualization skills and foster an understanding of the relationship between numbers and shapes. For example, teachers can use GeoGebra to demonstrate models of inscribed and circumscribed spheres, enabling students to grasp the essence of these models through intuitive perception.

Enhancing mathematical operation ability with concept maps. Teachers guide students to use cross-module concept maps to establish connections between operations. For example, after learning about derivatives, students should understand the differences and commonalities in their applications to determining the monotonicity of functions, solving tangent line equations, and solving problems involving inequalities that hold true under certain conditions. This helps students develop flexible problem-solving thinking within an interconnected knowledge system, enabling them to apply a single method to multiple problems and use multiple methods to solve a single problem.

Students develop data analysis skills by working with statistical charts. Their data analysis literacy is cultivated through hands-on practice, covering the entire process from data collection to drawing conclusions. For example, during the data organization phase, structured statistical charts—such as pie charts, bar charts, histograms, line charts, and scatter plots—are used to standardize data presentation. When analyzing data, students use cause-and-effect diagrams to illustrate causal relationships and box-and-whisker plots to compare differences, thereby uncovering the underlying connections between variables.

Develop mathematical modeling literacy by applying it to real-world problems. Mathematical modeling bridges the gap between mathematics and the real world by abstracting real-world conflicts or needs into mathematical problems. By solving these problems using mathematical tools and then applying the solutions back to real-world contexts, students enhance their ability to solve problems using mathematical knowledge. For example, the "Optimal Time for Drinking Tea model" in the People's Education Press Compulsory Course 1 textbook uses a problem closely related to students' daily lives to assess their ability to apply functions comprehensively, thereby helping students improve their mathematical modeling literacy throughout the problem-solving process.

5. Conclusion

In summary, visual thinking is not merely a teaching aid, but rather a path to pedagogical innovation that aligns with the characteristics of high school mathematics. It not only reduces the abstract nature of mathematical knowledge through concrete representations, but also builds a bridge from theory to practice for core mathematical literacy by leveraging structured thinking frameworks. In future research and teaching practice, we can further develop the tiered instructional pathways centered on the application of thinking visualization to foster disciplinary thinking, refine strategies for systematically integrating thinking visualization into classroom instruction, enhance classroom quality, and achieve the educational goal of fostering students' development through the subject.

References

[1] Ministry of Education of the People's Republic of China. (2020) Curriculum Standards for Senior High School Mathematics (2017 Edition, 2020 Revision) . Beijing: People's Education Press.

ISSN 2663-8169 Vol. 8, Issue 3: 59-65, DOI: 10.25236/IJNDE.2026.080309

[2] Wu, L.S., Zhu, L. M. (2021) Core Competencies of Senior High School Mathematics: Generation Mechanism and Cultivation Paths. Journal of Mathematics Education, 4, 25-29.
[3] Liu, Z.Y. (2017) Visualization of thinking: Making learning truly happen. Contemporary Educator, 11, 69-71.
[4] Zhang, Z.Y. (2018) Visualization Teaching of Senior High School Mathematics: Principles, Approaches and Strategies—Based on GeoGebra Platform. Bulletin of Mathematics, 7, 21-24+28.
[5] Zhang, X.L. (2025) A Study on Thinking Visualization Teaching Strategies of Senior High School Mathematics under the Background of the New College Entrance Examination. College Entrance Examination, 18, 30-32.
[6] Tan, C. (2025) Construction and Practice of High-quality Review Lessons for Senior High School Mathematics under the Background of College Entrance Examination Reform. Reference for Middle School Teaching, 23, 32-35.