A Novel Game Modeling of Multi-Criteria Decision-Making Based on Generalized Fuzzy Data Envelopment Analysis: An Empirical Study of Traditional Chinese Medicine Hospitals

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Abstract: In order to scientifically evaluate the changes in game efficiency of regional traditional Chinese medicine hospitals in China, optimize the resource allocation of Traditional Chinese Medicine (TCM) hospitals, and improve their operational efficiency. This paper analyzes with game theory and a generalized fuzzy Data Envelope Analysis (DEA) model on TCM hospitals. Based on the visual analysis of the overall interval efficiency, cluster, and group according to the hospital scale, we study the efficiency changes before and after cooperation and competition among TCM hospitals. And then each hospital's optimal partner and competitor are obtained to provide a theoretical basis and reasonable suggestions for the decision-making of cooperation and competition of relevant hospitals. The results show that the cooperation and competition strategy among TCM hospitals can significantly change their operation efficiency. In the case of limited resources, TCM hospitals should choose the optimal cooperation and competition strategy according to the changes in their input-output efficiency to promote the optimal allocation of resources and improve operational efficiency.

Keywords: Generalized fuzzy DEA model, Game theory, Decision-making, Fuzzy systems, TCM hospital

1. Introduction

Globally, traditional medicine has long been used to treat relatively common disease [1]. As an essential representative of Chinese traditional medicine, TCM has significantly contributed to safeguarding and promoting people's life and health [2-5]. Moreover, TCM hospitals have developed TCM, and their services and operational efficiency are of great relevance. However, along with China's aging, urbanization, and other socioeconomic transformation processes, the current healthcare delivery system remains irrational in areas such as healthcare and resource allocation [6]. The government and health care providers must improve the productivity, efficiency, and quality of health care providers to achieve a comprehensive and balanced development of the public service system [7].

In 2019, the "Opinions of the Central Committee of the Communist Party of China and the State Council on Promoting the Inheritance, Innovation, and Development of Traditional Chinese Medicine" clearly emphasized: 'Improve the coverage of county-level TCM hospitals and improve the TCM service system' [8]. Although the scale of TCM hospitals in China has been expanding and the service volume has been rising in recent years, problems exist, such as insufficient high-quality medical resources, unreasonable structure, and uneven distribution of resources [9]. It is urgent to improve the operation efficiency of TCM hospitals and optimize the allocation of resources.

In the medical field, Beijing has always been plagued by 'big city disease,' while Tianjin and Hebei face the problem of relatively insufficient superior medical resources [10]. Promoting the coordinated development of Beijing, Tianjin, and Hebei is a major national strategy. One of the important contents is to uphold the "improvement of people 's well-being " and achieve equalization in the region [11]. Through the division of labor, medical institutions in the region can achieve close cooperation, effectively improving the allocation of medical and health resources and the actual utilization rate of medical and health resources. So that residents in the region can enjoy better medical services while ensuring the level of medical services and saving medical and health resources [12]. Moreover, through the coordination of medical and health resources, medical institutions within the region can narrow the overall gap in...
resource allocation, promote the coordinated development of traditional Chinese medicine in Beijing, Tianjin, and Hebei, and realize the equalization of traditional Chinese medicine services in Beijing, Tianjin, and Hebei.

Also in 2019, the "Beijing-Tianjin-Hebei TCM Hospital Cooperative Development Community" was established by the TCM societies of Beijing, Tianjin, and Hebei to actively integrate into the Beijing-Tianjin-Hebei cooperative development strategy and build a highland for TCM hospital development. However, there are significant differences in medical resources in Beijing, Tianjin, and Hebei, so how to improve the internal operation efficiency of TCM hospitals and optimize external cooperation and competition strategies under uneven resource distribution has become an important research issue. Based on this, this paper combines game theory and generalized fuzzy DEA theory based on collecting data related to 12 TCM hospitals in the Beijing-Tianjin-Hebei region. It investigates three questions: (1) What is the input-output efficiency of TCM hospitals in the Beijing-Tianjin-Hebei region? (2) What is the prospect of cooperation among TCM hospitals in the Beijing-Tianjin-Hebei region? (3) What is the prospect of competition among TCM hospitals in Beijing, Tianjin, and Hebei regions?

The remainder of the paper is organized as follows: Section 2 provides an overview of the relevant research literature. Section 3 describes the methods and data in detail. Section 4 presents the study results. Section 5 reports the discussion. Conclusions and limitations are presented in Section 6.

2. Literature review

To emphasize this paper's research value and relevance, we review relevant studies on the hospital efficiency in Section 2.1 and then introduce the concept of efficiency measure in Section 2.2.

2.1. Hospital efficiency

Hospital resource utilization has always been an essential factor in the healthcare industry's development; therefore, more and more researchers have been analyzing hospital efficiency. Scholars worldwide have gradually paid more attention to hospital efficiency and published many research articles, laying the foundation for the study in this paper [13-17]. However, many scholars have focused on general hospitals in a particular country or province [18-22]. Fewer studies have been conducted on TCM hospitals in different regions, provinces, and cities. Specific studies such as Linna, [23] compared the cost efficiency of public hospitals in each country by analyzing the input and output data of public hospitals in four Nordic countries, citing the DEA model. [24] used 92 county-level public general hospitals in Shanxi Province as a study object. They measured the efficiency of medical services using the super-efficient SBM-DEA model to examine the impact of reform on the county-level public general hospitals. [6] evaluated the relative efficiency of hospital operations using the DEA-BCC model with a sample of all municipal TCM hospitals in Gansu Province from 2017-2019. They used the Manny-Whitney test to conduct a comparative analysis of input and output variables for technical efficient and inefficient hospitals. [25] examined the efficiency of medical services by collecting 143 public hospitals in Henan Province from 2005 to 2017. The efficiency and revenue of public hospitals were evaluated using the Barrow Economic Growth (BEG) model, stochastic frontier analysis (SFA), and vector autoregressive (VAR) model. Previous studies have focused on examining hospital efficiency and influencing factors, and no studies have explored the game decided on the efficiency of TCM hospitals.

Regarding China's medical and health infrastructure, most studies mainly establish an index system from the government's perspective to evaluate hospital efficiency [26-29]. In addition, [30] selected the input and output data of 10 adjacent county-level public hospitals in Henan Province from 2017 to 2019. Based on the traditional DEA model, a generalized fuzzy DEA cooperative decision-making model with better applicability to fuzzy indicators and optional decision-making units was constructed to calculate the cooperation efficiency interval of different hospitals to select the best partners in different decision-making units.

2.2. Efficiency measure

There are many methods available to assess hospital efficiency. Sherman points out that the most commonly used methods to assess efficiency are as follows: ratio analysis, balanced scorecard (BSC), total factor productivity (TFP), regression analysis, production frontier approach (PFA), and data envelope analysis (DEA) [31]. Ratio analysis can only assess a single input-output factor, while balanced scorecard, TFP, and PFA apply to multiple input-single output problems. DEA can simultaneously assess
multiple input and output factors, making it more suitable for application in complex healthcare organizations and healthcare industry environments [32-35].

The DEA method is an evaluation method to calculate the efficiency of decision units of the exact nature proposed by [36], which has been widely used in the healthcare field. However, the decision set of the traditional DEA method is all decision units, and it is not possible to select some of them for evaluation, which causes partial inaccuracy of evaluation. In contrast, the generalized DEA method can reasonably select decision units according to the actual problem, which is more advantageous when dealing with complex real-world problems [37-40]. Meanwhile, fuzzy DEA theory solves the problem that traditional DEA models cannot handle when the input-output data are fuzzy numbers [41-44]. Although the generalized DEA and fuzzy DEA theories greatly enrich and expand the application fields of DEA methods, they cannot evaluate the efficiency of cooperation and competition between decision units. 2020, a generalized game fuzzy DEA model was proposed based on the combination of game theory and a generalized fuzzy DEA model [29, 45]. The model derives the efficiency situation after the game from two perspectives of cooperation and competition, respectively, and then determines the optimal partners and competitors of the decision unit, which can effectively analyze the cooperation and competition strategies in the case of complex relationships between decision units.

Through searching and reading a large amount of related literature, we found that although international scholars have increased their research on hospital efficiency in recent years, there are relatively few studies on the efficiency of specialty hospitals. Furthermore, the scholars mainly apply the traditional DEA method when evaluating the efficiency of hospitals, but fewer of them improve the DEA model and combine it with other models. The efficiency evaluation by the traditional DEA method can only evaluate the relative efficiency. The efficiency value obtained is only related to the "excellent unit," which may cause problems for some decision units. For example, it cannot fully reveal the degree of excellence of this unit. Also, most of the literature only analyzes from the game theory perspective or discusses health care policy, but not from the broad perspective of the game DEA-based efficiency evaluation method.

Based on the above research, this paper combines game theory and the DEA model to study the efficiency of TCM hospitals in the Beijing-Tianjin-Hebei region and analyzes the prospects of cooperation and competition between hospitals, providing suggestions for the development of related hospitals. This paper differs from other studies by using similar specialty hospitals for comparative analysis instead of general hospitals, which reduces the errors caused by different focuses of various hospitals, improves the accuracy, and is more informative. Moreover, selecting the Beijing-Tianjin-Hebei region to study cooperation and competition can improve hospital efficiency and drive the comprehensive development of Beijing-Tianjin-Hebei hospital integration.

3. Methods and indicators

3.1. Research method

Firstly, a brief introduction of the generalized fuzzy DEA model and its related definitions under cooperation-oriented and competition-oriented is used in this paper.

Suppose $DMU_1 = (X_1, Y_1)^T$ and $DMU_2 = (X_2, Y_2)^T$ are two sample decision units, and the decision units after the cooperation between the two decision units $DMU_1$ and $DMU_2$ are defined as follows:

$$(X_c, Y_c) = (X_1 \land X_2, Y_1 \lor Y_2)$$

The two decision units $DMU_1$ and $DMU_2$ after competition are defined as follows:

$$(X_f, Y_f) = (X_1 \lor X_2, Y_1 \land Y_2)$$

Where $\land$ and $\lor$ denote change operators, $\land$ means to take the smallest number of the two sets of indicators, and $\lor$ means to take the largest number of the two sets of indicators.

The cooperation-oriented generalized fuzzy DEA model is:
\[
GFCR_{PC} \begin{cases}
\max Y^T u, \\
\text{s.t.} \quad X^T v - Y^T u, (1 \leq j \leq n) \\
\quad X^T v = 1, \\
\quad v \geq 0, u \geq 0
\end{cases}
\] (1)

The efficiency evaluation value \( E_{ik} \), under cooperative relationship

\[
E_{ik} = E_{ic} = \frac{Y^T u_i}{X^Tv_i}
\]

The competition-oriented generalized fuzzy DEA model is:

\[
GFCR_{PF} \begin{cases}
\max Y^T u, \\
\text{s.t.} \quad X^T v - Y^T u, (1 \leq j \leq n) \\
\quad X^T v = 1, \\
\quad v \geq 0, u \geq 0
\end{cases}
\] (2)

The efficiency evaluation value \( E_{ik} \), under competitive relationship

\[
E_{ik} = E_{ic} = \frac{Y^T u_i}{X^Tv_i}
\]

where \( n \) is the number of decision units, \( DMU_i = (X_i, Y_i)^T \) \((i \neq j, 1 \leq i \leq n)\) is any sample decision unit to be evaluated, \( X_i = (x_{i1}, x_{i2}, \ldots, x_{im}) \) is the input indicator, \( Y_i = (y_{i1}, y_{i2}, \ldots, y_{is}) \) is the output indicator, \( m \) is the number of input indicators for each sample decision unit number of indicators, \( s \) is the number of output indicators for each decision unit, \( v = (v_1, v_2, \ldots, v_m) \) is the input indicator weight, \( u = (u_1, u_2, \ldots, u_s) \) is the output indicator weight, \((X_j, Y_j)^T\) is the selected reference system, and \( u_i^* \) and \( v_i^* \) are the optimal input indicator weight and optimal output indicator weight of decision unit \( DMU_i \) weights.

**Definition 1** (Optimal partner of a generalized fuzzy sample decision unit) \((X_1, Y_1)^T\) is a generalized fuzzy sample decision making unit seeking partners. \((X_{H_1}, Y_{H_1})^T\) is the potential partner of \((X_1, Y_1)^T\), and \((X_j, Y_j)^T\) is the selected reference frame. The decision unit \((X_{C_1}, Y_{C_1})^T\) in \((X_{H_1}, Y_{H_1})^T\) that makes the optimal efficiency improvement of \((X_1, Y_1)^T\) after cooperation with \((X_1, Y_1)^T\) is called the optimal partner of \( DMU_1 \). One of the optimal partner selection models based on the CCR model is shown in model (3).

\[
GFCR_{PCopt} \begin{cases}
\max \max \limits_{l \neq i \in n} (Y_l, Y_{H_l})^T u, \\
\text{s.t.} \quad X^T v - Y^T u, (1 \leq j \leq n) \\
\quad (X_1, Y_{H_1})^T v = 1, \\
\quad v \geq 0, u \geq 0
\end{cases}
\] (3)

**Definition 2** (Optimal cooperation efficiency of generalized fuzzy sample decision making units) The optimal cooperation efficiency of two generalized fuzzy sample decision making units \((X_1, Y_1)^T\) and \((X_2, Y_2)^T\) is defined as the efficiency value of all decision making units with the smallest input and the largest output among all possible decision making units in \((X_{C_1}, Y_{C_1})^T\).

**Definition 3** (worst cooperation efficiency of generalized fuzzy sample decision making units) The worst cooperation efficiency of two generalized fuzzy sample decision making units \((X_1, Y_1)^T\) and \((X_2, Y_2)^T\) is defined as the efficiency value of all the decision making units with the largest input and the smallest output in \((X_{C_1}, Y_{C_1})^T\).

**Definition 4** (the strongest competitor of generalized fuzzy sample decision making unit) \((X_1, Y_1)^T\) is a generalized fuzzy sample decision making unit in a competitive position. \((X_{H_1}, Y_{H_1})^T\) is a potential competitor of \((X_1, Y_1)^T\), \((X_j, Y_j)^T\) is the selected reference frame. After competing with \((X_j, Y_j)^T\), the decision-making unit \((X_{F_1}, Y_{F_2})^T\) that minimizes the efficiency of \((X_1, Y_1)^T\) is called the strongest competitor of \( DMU_1 \). The selection model of the strongest competitor based on CCR model is shown in model (4).
\[
GFCCEP_{\text{com}} = \min_{\text{Theta}} \max_{\text{Theta}} \left( \frac{\sum_{i=1}^{n} (Y_i, Y_H) u_i}{\sum_{i=1}^{n} (X_i, X_H) v_i} \right) \quad \text{s.t.} \quad \begin{align*}
X^T_j v - Y^T_j u_i (1 \leq j \leq n) \\
(X_i, X_H) v = 1,
\end{align*} \\
v \geq 0, u \geq 0
\]

\[ (4) \]

**Definition 5** (Optimal competitive efficiency of generalized fuzzy sample decision making units) The optimal competitive efficiency of two generalized fuzzy sample decision making units \((X_1, Y_1)^T\) and \((X_2, Y_2)^T\) is defined as the efficiency value of all decision making units with the smallest input and the largest output in \((X_F, Y_F)^T\).

**Definition 6** (worst competitive efficiency of generalized fuzzy sample decision making units) The worst competitive efficiency of two generalized fuzzy sample decision making units \((X_1, Y_1)^T\) and \((X_2, Y_2)^T\) is defined as the efficiency value of all the decision making units with the largest input and the smallest output in \((X_F, Y_F)^T\).

### 3.2. Indicators and data

In terms of indicator selection, based on relevant literature [43-45], the various input-output indicators that affect the operational efficiency of hospitals are summarized and summarized by combining the essential characteristics of TCM hospitals in the Beijing-Tianjin-Hebei region [46-48]. The input indicators are selected from the financial resources, human resources, environment, and scale of the hospital: total number of employees, total number of beds, and total expenditure; the output indicators are selected from the medical level and social influence of the hospital: annual number of outpatient and emergency visits, number of discharges, and social evaluation. Because of the fuzzy nature of the social evaluation indicator, "very good, good, good, average" is recorded as "4, 3, 2, 1," respectively, which are converted into precise data.

On the basis of literature research, the purposive sampling method is used to select 12 municipal TCM hospitals in the Beijing-Tianjin-Hebei region from 2017 to 2019 according to the distribution of high-quality medical resources, the areas where patients are concentrated, the influence of hospitals, the integrity and availability of data, etc. represent hospital names. Due to the fluctuation of hospital data in three years, such as the new entry or retirement of employees, the increase or decrease of medical equipment investment, etc., all the hospital data from 2017 to 2019 are regarded as fuzzy data. The maximum and minimum input-output data of each index in three years are taken as the interval endpoint values of fuzzy data. The interval values of each index are obtained, as shown in Table 1. The data in this paper are derived from the 'Health Statistics Yearbook' of each region and the official website information of each hospital.

**Table 1: Interval values of input-output indicators of 12 TCM hospitals in Beijing-Tianjin-Hebei region.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Total number of employees</th>
<th>Total number of beds</th>
<th>Total expenditure (Million yuan)</th>
<th>Annual number of outpatient and emergency visits (Million people)</th>
<th>Number of discharges (Thousands)</th>
<th>Social evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>(2518,2542)</td>
<td>(2600,2600)</td>
<td>(215462,253441)</td>
<td>(358,401)</td>
<td>(67,75)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>H2</td>
<td>(1119,1132)</td>
<td>(600,710)</td>
<td>(202173,253411)</td>
<td>(124,140)</td>
<td>(28,49,31,58)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>H3</td>
<td>(835,1310)</td>
<td>(365,480)</td>
<td>(47156,63961)</td>
<td>(37,9,56,2)</td>
<td>(5,254,6,021)</td>
<td>(2,4)</td>
</tr>
<tr>
<td>H4</td>
<td>(1119,1479)</td>
<td>(649,702)</td>
<td>(210583,223481)</td>
<td>(259,290)</td>
<td>(14,36,21,21)</td>
<td>(3,4)</td>
</tr>
<tr>
<td>H5</td>
<td>(882,908)</td>
<td>(1030,1400)</td>
<td>(210382,223481)</td>
<td>(124,3,135,6)</td>
<td>(8,225,12,975)</td>
<td>(3,4)</td>
</tr>
<tr>
<td>H6</td>
<td>(2045,2361)</td>
<td>(1806,2017)</td>
<td>(80817,102983)</td>
<td>(89,83)</td>
<td>(25,6,42,2)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>H7</td>
<td>(1200,1256)</td>
<td>(1000,1000)</td>
<td>(102540,125436)</td>
<td>(72,83)</td>
<td>(6,71,11)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>H8</td>
<td>(996,1018)</td>
<td>(583,620)</td>
<td>(31083,51729)</td>
<td>(21,2,27)</td>
<td>(27,8,31,5)</td>
<td>(2,3)</td>
</tr>
<tr>
<td>H9</td>
<td>(486,536)</td>
<td>(280,410)</td>
<td>(36485,53291)</td>
<td>(21,9,27,3)</td>
<td>(10,283,12,156)</td>
<td>(2,3)</td>
</tr>
<tr>
<td>H10</td>
<td>(2056,2356)</td>
<td>(1800,2100)</td>
<td>(2057,2257)</td>
<td>(70,2,75,2)</td>
<td>(18,45,20,1)</td>
<td>(1,2)</td>
</tr>
<tr>
<td>H11</td>
<td>(359,382)</td>
<td>(299,355)</td>
<td>(18159,25818)</td>
<td>(28,32,8)</td>
<td>(0,71,0,82)</td>
<td>(1,2)</td>
</tr>
<tr>
<td>H12</td>
<td>(127,132)</td>
<td>(75,156)</td>
<td>(16534,24936)</td>
<td>(10,22,6)</td>
<td>(0,56,0,92)</td>
<td>(1,2)</td>
</tr>
</tbody>
</table>
4. Results

4.1. Overall efficiency evaluation

The generalized DEA model is solved by using MATLAB software. The highest efficiency value is obtained by selecting the data with the minimum input and the maximum output of each index from 2017-2019 for 12 municipal TCM hospitals in the Beijing-Tianjin-Hebei region as the input and output data of the new decision unit. Similarly, the minimum efficiency value is obtained by taking the data with the most considerable input and the smallest output as the input and output data of the decision unit. Finally, the maximum and minimum efficiencies are averaged to obtain the average efficiency value. In order to more clearly visualize the different trends of average, minimum, and maximum efficiency, a radar chart is drawn, as shown in Fig 1.

![Radar Chart](Figure 1: Comparison of three efficiency values of 12 traditional Chinese medicine hospitals in Beijing-Tianjin-Hebei region.)

As seen in Fig 1, the efficiency values of the hospitals in the generalized DEA model ranged from 0.6 to 1.5, with significant variability between the highest and lowest efficiency. The minimum and maximum efficiency values are used as the endpoints of the efficiency change interval for analysis. Suppose the efficiency of Hospital A after competing with Hospital B is significantly higher than the maximum efficiency of hospital A. In that case, it indicates that under the optimism criterion, Hospital A has a reason to cooperate with hospital B. On the contrary, if the efficiency of Hospital A after competing with Hospital B is significantly lower than the minimum efficiency of Hospital A, then under the pessimism criterion, hospital A is more likely to make the strategy of giving up competing with Hospital B.

4.2. Cooperation and competition efficiency analysis

For efficiency evaluation, if there is a large difference in size between decision units, the results may not reflect the impact of size differences. In practice, it is also difficult for hospitals with large size differences to be willing to cooperate or compete with each other. Therefore, selecting TCM hospitals of equal size for cooperative competition research is more realistic. In this paper, the 12 TCM hospitals are classified into three categories according to the size difference of each hospital using cluster analysis so that each category is of equal size. The first category is H3, H8, H9, H10, H11, and H12, the second category is H6 and H7, and the third category is H1, H2, H4, and H5. The results of the cluster analysis are shown in Fig 2.
Based on the generalized fuzzy DEA model, the input-output data of two decision units after cooperation is substituted into the model instead of the original arbitrary sample decision unit data, separately deriving the optimal and the worst cooperation efficiency. Generally, the inputs of the two sample decision units become smaller, and the outputs become more after cooperation. Similarly, instead of the original arbitrary sample decision unit data, the input and output data of the two decision units after the competition are substituted into the generalized fuzzy DEA model to derive the optimal and worst competitive efficiencies. Usually, competition causes an increase in inputs and a decrease in outputs.

The following is the cooperation efficiency analysis with the first category of TCM hospital as a representative. The post-cooperation data are substituted into the cooperation-oriented generalized fuzzy DEA model using MATLAB software, and the post-cooperation efficiency is derived, as shown in Table 2.

Table 2: Efficiency interval values after the first category of hospital cooperation.

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H3</th>
<th>H8</th>
<th>H9</th>
<th>H10</th>
<th>H11</th>
<th>H12</th>
</tr>
</thead>
<tbody>
<tr>
<td>H3</td>
<td>(0.693,1.102)</td>
<td>(1.402,1.550)</td>
<td>(1.371,1.510)</td>
<td>(1.298,1.331)</td>
<td>(1.404,1.412)</td>
<td>(1.302,6.631)</td>
</tr>
<tr>
<td>H8</td>
<td>(1.402,1.550)</td>
<td>(0.868,1.304)</td>
<td>(1.537,1.580)</td>
<td>(1.237,1.582)</td>
<td>(1.425,1.555)</td>
<td>(1.361,6.581)</td>
</tr>
<tr>
<td>H9</td>
<td>(1.371,1.510)</td>
<td>(1.537,1.580)</td>
<td>(0.814,1.310)</td>
<td>(1.249,1.377)</td>
<td>(1.417,1.418)</td>
<td>(1.535,6.639)</td>
</tr>
<tr>
<td>H10</td>
<td>(1.298,1.331)</td>
<td>(1.237,1.582)</td>
<td>(1.249,1.377)</td>
<td>(0.912,1.082)</td>
<td>(1.297,1.422)</td>
<td>(0.488,2.055)</td>
</tr>
<tr>
<td>H11</td>
<td>(1.404,1.412)</td>
<td>(1.425,1.555)</td>
<td>(1.417,1.418)</td>
<td>(1.297,1.422)</td>
<td>(0.842,1.190)</td>
<td>(0.784,2.401)</td>
</tr>
<tr>
<td>H12</td>
<td>(1.302,6.631)</td>
<td>(1.361,6.581)</td>
<td>(1.535,6.639)</td>
<td>(0.488,2.055)</td>
<td>(0.784,2.401)</td>
<td>(0.733,4.493)</td>
</tr>
</tbody>
</table>

The efficiency interval value on the diagonal in Table 2 is the highest and lowest efficiency of the hospital. Taking H3 hospital as an example, the efficiency interval of H3 hospital is (0.693, 1.102) under non-cooperative competition. The efficiency interval of H3 hospital increased after cooperation with other hospitals. After comparison, it is found that the efficiency interval of H3 hospital changed to (1.302, 6.631) after cooperation.
6.631) after cooperation with H12 hospital, among which the highest efficiency increased the most, which indicated that H12 hospital is the optimal partner of H3 hospital under the optimism criterion. Under the pessimism criterion, the efficiency interval between H3 hospital and H11 hospital became (1.404,1.412), and the lowest efficiency increased the most, indicating that H11 hospital is the optimal partner of H3 hospital under the pessimism criterion. The efficiency change of H3 hospital after cooperation is shown in Fig 3.

Similarly, the cooperation efficiency interval values for the second and third categories of hospitals are shown in Tables 3 and 4.

**Table 3: Efficiency interval values after the second category of hospital cooperation.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H6</th>
<th>H7</th>
</tr>
</thead>
<tbody>
<tr>
<td>H6</td>
<td>(0.901,1.131)</td>
<td>(1.122,1.237)</td>
</tr>
<tr>
<td>H7</td>
<td>(1.122,1.237)</td>
<td>(1.000,1.063)</td>
</tr>
</tbody>
</table>

**Table 4: Efficiency interval values of the third category of hospitals after cooperation.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H1</th>
<th>H2</th>
<th>H4</th>
<th>H5</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>(0.936,1.077)</td>
<td>(1.079,1.129)</td>
<td>(1.092,1.169)</td>
<td>(1.081,1.086)</td>
</tr>
<tr>
<td>H2</td>
<td>(1.079,1.129)</td>
<td>(0.987,1.124)</td>
<td>(1.184,1.212)</td>
<td>(1.088,1.132)</td>
</tr>
<tr>
<td>H4</td>
<td>(1.092,1.169)</td>
<td>(1.184,1.212)</td>
<td>(0.954,1.112)</td>
<td>(1.132,1.265)</td>
</tr>
<tr>
<td>H5</td>
<td>(1.081,1.086)</td>
<td>(1.088,1.132)</td>
<td>(1.132,1.265)</td>
<td>(0.865,1.041)</td>
</tr>
</tbody>
</table>

Due to the small number of hospitals in the second category, the competition efficiency analysis is next performed with the third category of hospitals as a representative. Using MATLAB software, the post-competition data of the third category of hospitals are substituted into the competition-oriented generalized fuzzy DEA model, and the obtained interval efficiency values are shown in Table 5.

**Table 5: Efficiency interval values after the third type of hospital competition.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H1</th>
<th>H2</th>
<th>H4</th>
<th>H5</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>(0.936,1.077)</td>
<td>(0.405,0.950)</td>
<td>(0.776,0.878)</td>
<td>(0.625,0.738)</td>
</tr>
<tr>
<td>H2</td>
<td>(0.405,0.950)</td>
<td>(0.987,1.124)</td>
<td>(0.387,0.586)</td>
<td>(0.772,0.822)</td>
</tr>
<tr>
<td>H4</td>
<td>(0.776,0.878)</td>
<td>(0.387,0.586)</td>
<td>(0.954,1.112)</td>
<td>(0.391,0.788)</td>
</tr>
<tr>
<td>H5</td>
<td>(0.625,0.738)</td>
<td>(0.772,0.822)</td>
<td>(0.391,0.788)</td>
<td>(0.865,1.041)</td>
</tr>
</tbody>
</table>

Considering the space reason, take H5 hospital as an example for the analysis. In the non-cooperative competition state, the efficiency interval of the H5 hospital is (0.865,1.041), and the efficiency interval of the H5 hospital decreases after competing with other hospitals. Its maximum and minimum efficiency values have decreased significantly after competing with H1 and H4 hospitals, indicating that H5 hospitals should avoid competing with H1 and H4 hospitals. However, the efficiency interval value of the H5 hospital changed to (0.772,0.822) after competing with the H2 hospital. The change in the minimum efficiency is only 0.093, which did not significantly impact the H5 hospital. Moreover, for the H2 hospital, the minimum efficiency change is 0.215, which effectively reduces the minimum efficiency of the H2 hospital. In terms of maximum efficiency, the maximum efficiency of the H5 hospital is reduced after the competition with the H2 hospital. However, the decrease is the smallest, while the maximum efficiency of H2 hospital is decreased more. Therefore, H5 hospitals should choose H2 hospitals as a competitor under both optimistic and pessimism criteria. The efficiency change of H5 hospital after cooperation is shown in Fig 4.

**Figure 4: H5 hospital efficiency change interval after competition.**

Similarly, the competitive efficiency interval values for the first and second categories of hospitals
are shown in Table 6 and Table 7.

**Table 6: Efficiency interval values after the first category of hospitals competition.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H3</th>
<th>H8</th>
<th>H9</th>
<th>H10</th>
<th>H11</th>
<th>H12</th>
</tr>
</thead>
<tbody>
<tr>
<td>H3</td>
<td>(0.693,1.102)</td>
<td>(0.139,0.716)</td>
<td>(0.237,0.670)</td>
<td>(0.447,0.747)</td>
<td>(0.371,0.828)</td>
<td>(0.234,0.471)</td>
</tr>
<tr>
<td>H8</td>
<td>(0.139,0.716)</td>
<td>(0.868,1.304)</td>
<td>(0.641,0.855)</td>
<td>(0.270,0.373)</td>
<td>(0.062,0.255)</td>
<td>(0.147,0.174)</td>
</tr>
<tr>
<td>H9</td>
<td>(0.237,0.670)</td>
<td>(0.641,0.855)</td>
<td>(0.814,1.310)</td>
<td>(0.353,0.382)</td>
<td>(0.254,0.397)</td>
<td>(0.348,0.730)</td>
</tr>
<tr>
<td>H10</td>
<td>(0.447,0.747)</td>
<td>(0.270,0.373)</td>
<td>(0.353,0.382)</td>
<td>(0.912,1.082)</td>
<td>(0.579,0.820)</td>
<td>(0.042,0.402)</td>
</tr>
<tr>
<td>H11</td>
<td>(0.371,0.828)</td>
<td>(0.062,0.255)</td>
<td>(0.254,0.397)</td>
<td>(0.579,0.820)</td>
<td>(0.842,1.190)</td>
<td>(0.244,0.522)</td>
</tr>
<tr>
<td>H12</td>
<td>(0.234,0.471)</td>
<td>(0.147,0.174)</td>
<td>(0.348,0.730)</td>
<td>(0.042,0.402)</td>
<td>(0.244,0.522)</td>
<td>(0.733,4.493)</td>
</tr>
</tbody>
</table>

**Table 7: Efficiency interval values after the second category of hospitals competition.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H6</th>
<th>H7</th>
</tr>
</thead>
<tbody>
<tr>
<td>H6</td>
<td>(0.901,1.131)</td>
<td>(0.496,0.918)</td>
</tr>
<tr>
<td>H7</td>
<td>(0.496,0.918)</td>
<td>(1.000,1.063)</td>
</tr>
</tbody>
</table>

Based on the above analysis of cooperation and competition within the three categories of hospitals, this paper summarizes the optimal partners and competitors under the optimism and pessimism criterion for 12 TCM hospitals in the Beijing-Tianjin-Hebei region, using the same analysis method as above. The results are shown in Table 8. It is important to note that the optimism criterion and the pessimism criterion are only considered for two extreme cases to give hospitals a more thorough game analysis. When the pessimism and optimism criteria are the same, the hospital can directly take the current decision plan. When the pessimism criterion and the optimism criterion are different, the hospital can choose the pessimism criterion to plan the game behavior if the hospital decides to seek stability or choose the optimism criterion to plan the game behavior if the hospital decides to seek progress.

**Table 8: Partners and competitors of 12 TCM hospitals in Beijing-Tianjin-Hebei region.**

<table>
<thead>
<tr>
<th>Hospital</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>H4</th>
<th>H5</th>
<th>H6</th>
<th>H7</th>
<th>H8</th>
<th>H9</th>
<th>H10</th>
<th>H11</th>
<th>H12</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal Partners</strong></td>
<td>H4</td>
<td>H5</td>
<td>H12</td>
<td>H4</td>
<td>H5</td>
<td>H4</td>
<td>H7</td>
<td>H6</td>
<td>H12</td>
<td>H12</td>
<td>H12</td>
<td>H3</td>
</tr>
<tr>
<td><strong>Optimal Competitors</strong></td>
<td>H4</td>
<td>H5</td>
<td>H10</td>
<td>H10</td>
<td>H1</td>
<td>H2</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

5. Discussion

5.1. Overall efficiency change analysis

The comprehensive study shows that, in general, cooperation and competition between hospitals can generally improve and reduce operational efficiency. However, the decrease in efficiency after the competition is significantly higher than the increase after cooperation, which indicates that the overall competition cost is high. Moreover, some hospitals, such as H3, H6, H7, and H8, did not even find the optimal competitors due to the significant decrease in the efficiency range after the competition between these hospitals and the group hospitals. It also shows that the relative competitiveness of these hospitals’ resource allocation and operation could be stronger. In the case of relatively scarce health resources, competition may lead to a waste of resources and, ultimately, to decreased operational efficiency. Therefore, the strategic choice at this stage for TCM hospitals should be to reduce competitive measures, seek common ground while reserving differences, and improve the utilization of resources through the complementary advantages of medical technology and health resources. In addition, it can also promote the standardization, refinement, and scientific management of the hospital to improve its comprehensive management capability.

5.2. Cooperation analysis

Table 8 shows that some hospitals are optimal partners for each other, such as H3 and H12, H4 and H5, H6 and H7 under the optimism criterion; H8 and H9 under the pessimism criterion. Such hospitals should firmly choose each other as partners in decision-making, regularly communicate and cooperate,
learn from each other's strengths, and strive to form a 'win-win' pattern. For hospitals that are not the best partners for each other, since the efficiency of all hospitals has improved after cooperation. Such hospitals should adopt an open strategy of seeking common ground while reserving differences and cooperating not only with their best partners but also as the best partners of other parties who should be as open as possible to cooperate and complement each other's advantages. Especially in the current context of the new crown epidemic prevention and control, multi-party cooperation is more conducive to improving the efficiency of disease treatment. It can achieve a multi-win situation between hospitals and doctors, and patients.

5.3. Competition analysis

As Table 8 shows, under the optimism criterion, H4 is both the optimal partner and the optimal competitor of H1. In this case, H1 hospital should first find the hospital's position when making decisions. If it is in the leading position among similar hospitals and H4 is the main competitor, it can compete with H4. On the contrary, if an H1 does not have obvious advantages in scale and efficiency among similar hospitals, it is recommended to cooperate with H4 to achieve complementary resources and efficiency improvement. In addition, the competitive strategy should be carefully chosen for hospitals with optimal competitors due to the inevitable decrease in operational efficiency after a competition. Furthermore, it is better to rely on the introduction of scientific and technological talents, improve the level of treatment, improve the hospital environment, and other methods to improve the overall operational efficiency. Hospitals without the best competitors should find their own positioning, improve their differentiation ability, clarify their strengths and weaknesses, and enhance their core competitiveness.

6. Conclusions

This paper combines game theory and the generalized fuzzy DEA method to analyze some TCM hospitals' cooperation and competition efficiency in the Beijing-Tianjin-Hebei region. From the research method, the generalized game fuzzy DEA method can effectively break through the limitation of the traditional DEA method on the selection of decision units and improve the study's accuracy by making reasonable groupings according to the size of TCM hospitals. From the research content, the comparative analysis is of specific reference value because of the close resource endowment, which makes cooperation and competition easier to achieve compared with similar TCM specialty hospitals in general hospitals. Moreover, from a regional perspective, the synergistic development of medical resources in the Beijing-Tianjin-Hebei region is essential to integrating the three regions. The analysis of cooperation and competition among TCM hospitals has solid practical significance due to convenient transportation.

Although this study explains the in-depth investigation of the cooperation and competition efficiency of TCM hospitals in the Beijing-Tianjin-Hebei region, there are some limitations of this paper due to the inevitable sampling error. First, the samples studied in this paper come from some municipal TCM hospitals in the Beijing-Tianjin-Hebei region. The reference value of other counties (district) TCM hospitals need to be reconsidered. Secondly, because only the hospital's data from 2017 to 2019 are selected, the stability and robustness of evaluation results will be reduced. It is still necessary to optimize the research design. Finally, this paper mainly examines the decision-making of improving the operational efficiency of traditional Chinese medicine hospitals in the Beijing-Tianjin-Hebei region. The efficiency of hospitals may be affected by environmental factors, such as population composition, local economic level, and health service costs. Because of this, follow-up studies can further explore the impact of environmental factors on hospital game decision-making.

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References


