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Research on the development of students' intuitive imagination literacy based on the '5E' teaching mode—Taking elliptic curve as an example

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Abstract: *In order to realize the optimization of conic curve teaching, so that students can really learn conic curve. Combined with the background of the new era and the requirements of the new curriculum standard, following the ' 5E ' teaching mode, using GeoGebra software, taking the teaching of ' elliptic curve ' as an example, a teaching mode with conic curve as the teaching content is constructed. Help students develop their own core literacy such as intuitive imagination in the concept of systematically constructing conic curves, in order to provide reference for basic mathematics teaching.*

Keywords: *' 5E ' teaching mode; mathematics teaching; GeoGebra software*

1. Background of the study

In the context of the new era, the curriculum reform of basic education in China pays more and more attention to the development of students' mathematical core literacy, and in the basic concepts of high school mathematics education put forward in the General High School Mathematics Curriculum Standards (2017 Edition 2020 Revision) (hereinafter collectively referred to as the Curriculum Standards), it is explicitly pointed that it is necessary to condense the students' mathematical core literacy, and therefore the implementation of the core literacy has an important research value [1]. Conic section, as the core content of high school analytic geometry, is the focus and difficulty of the college entrance examination on the one hand, and accounts for a large score in the college entrance examination, but the score rate is relatively low; on the other hand, it is the focus and difficulty of teaching, which has a great impact on the development of students' mathematical thinking, ability, and geometric intuition represented by mathematical combination of numbers and shapes, but students also have the problem of "difficult to understand and poor application" [1]. However, students also have the problem of "difficult understanding and poor application" [2].

Against this background, educational researchers have done a great deal of research on conic curves, mainly focusing on the following three aspects: firstly, the teaching of conic curves is explored by means of modern information technology. For example, Chen Feng proposed the use of GeoGebra software in the literature [3]; Jia Xuan et al. proposed the teaching of visualisation exercises with the help of Geometry Drawing Board in the literature [4]. Secondly, the teaching theory is used as a support to study the teaching of conic curves. For example, Ran Xiao proposed in the literature to design teaching using PBL teaching model [5]; Zheng Yanjie proposed in the literature to design teaching for unit teaching based on ADDIE model [6]. Thirdly, teaching strategies are proposed by studying the current situation of problem solving, teaching and learning. For example, Wang Haiqing and Cao Guangfu proposed in the literature to reconstruct the conceptual teaching of conic curves in view of the problems of difficulty in understanding, recognising and mastering in the teaching of conic curves [2]; Geng Shilan et al. proposed to make use of the concept of the "three teachings" - teaching, thinking, experiencing and expressing, and put forward the concept of teaching and learning based on the ADDIE model [7].

Through the literature study, it can be found that, with the development of the teaching of conic curves is gradually cumbersome and complex, and while emphasising the further improvement of the development of students, it ignores the acceptance of students and the current situation of learning, which deviates from the goal of implementing real and effective teaching for all students, and on the

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basis of enabling students to master the knowledge of conic curves, forming an intuitive understanding of the mathematical ideas of combining mathematics with figures and shapes, and further developing the mathematical abilities and literacy of the students. and literacy. Based on the problems existing in the current study, this paper carries out a student-centred 5E teaching model and assists the GeoGebra software to design the teaching of "number and shape" combination, in order to enable students to really learn the knowledge of the process, and better develop students' intuition and imagination and other core literacy, so as to provide references for the teaching of basic mathematics.

2. The basic theory of the "5E" teaching model

"5E" is the acronym of the five teaching links of Engage, Explore, Explain, Elaborate and Evaluate, which is an inquiry teaching mode based on the constructivist theory, emphasising student-centred learning, arousing students' interest, creating problematic situations based on students' existing knowledge and experience and real life, and guiding students to understand and construct knowledge in inquiry learning. It is an inquiry-based teaching model based on constructivist theory, which emphasises student-centred learning, advocates arousing students' interest, creates problematic scenarios based on students' existing knowledge and experience and real life, and guides students to understand and construct knowledge in inquiry learning [7]. It was developed by the Biology Curriculum Study (BSCS) in the United States and was initially applied to the teaching of biology courses mainly in western countries.

The five links of the "5E" teaching model are both independent of each other and complementary to each other, forming a closed-loop teaching model, and each link can be adjusted in order according to the actual situation, and the implementation process is shown in Figure 1 [8].

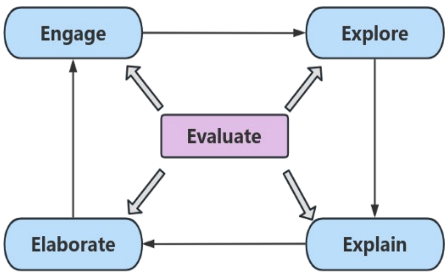

Figure 1: Flowchart of the "5E" teaching model

3. Teaching design of elliptic curve under the guidance of "5E" teaching mode

3.1. Teaching key points

3.1.1. Teaching key points

Understand the definition of ellipse and its standard equation, and relate the "shape" of the ellipse to the "numbers" in it.

3.1.2. Teaching Difficulties

Derivation, understanding and application of the standard equation of an ellipse.

3.2. Teaching objectives

3.2.1. Problems and scenarios

Starting from the origin of conic curve, based on the real situation of building flower beds, the definition of ellipse is observed and summarised through hands-on operation.

3.2.2. Knowledge and Skills

To understand the definition and standard equation of ellipse, to understand the geometrical significance and relationship of the coefficients through investigation, and to summarise the basic steps of finding the equation of a trajectory.

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3.2.3. Thinking and Expressing Objectives

Through revealing the intrinsic connection between "number" and "shape", we can cultivate the idea of combining numbers and shapes, and improve the core mathematical skills of logical reasoning, intuitive imagination and the sense of mathematical application.

3.2.4. Communication and Reflection Objectives

To appreciate the connection between mathematics and life, and to develop teamwork and active communication through hands-on investigations and group work.

3.3. Examples of teaching design

3.3.1. Intuitive feeling, know the ellipse

Teaching process 1: draw a straight line through the origin in the plane, so that the straight line rotates around the axis of one week, to get the conic surface in space. After obtaining the conic surface, do the perpendicular axis of the plane to intercept the conic surface, to get the circle, as shown in Figure 2; the plane is slightly inclined to intercept the conic surface, to get the ellipse, as shown in Figure 3.

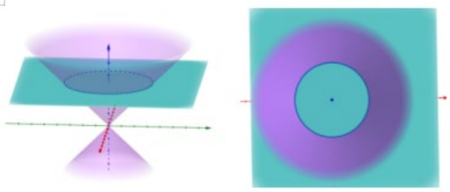

Figure 2: Intercept and top view of circle

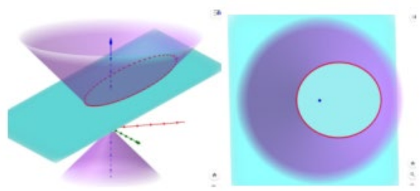

Figure 3: Intercept and top view of ellipse

Teaching process 2: Teacher question: according to the shape of the just intercepted ellipse, think of life we will encounter what objects are oval? Students discuss and answer; the teacher shows the example.

Design intention: (1) Based on the introduction link of 5E, draw students into the class. (2) Demonstrate the formation of conic surfaces using GGB software, so that students can understand how to get conic curves and recognise the shape of ellipse. Let students understand that conic curves originated from the mathematician Apollonius more than 2000 years ago to expand their horizons. (3) Combined with real life, students will be made aware of elliptic curves in life and deepen their understanding of ellipses.

3.3.2. Realistic inquiry, the formation of concepts

Teaching activity 1: create a realistic situation: workers want to build round and oval flower beds, how to draw the circle and oval? Ask students to take out the prepared cardboard, fine line, pegs, pencils and other tools, first draw a circle, the teacher demonstrated that the two ends of the thin rope is fixed at the same point when the circle is drawn, as shown in Figure 4; Teacher further questions, how to get the ellipse? Teachers prompted students to separate the two ends of the thin rope fixed, let the students draw a hand, you can get the ellipse, the teacher demonstrated with GGB software, as shown in Figure 5.

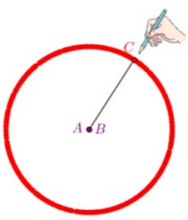

Figure 4: Drawing of circle

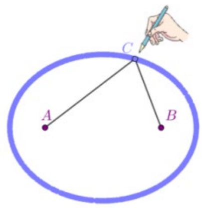

Figure 5: Drawing of ellipse

Design intention: (1) Based on the inquiry session of 5E, set up problem scenarios with real-life scenarios so that students can cooperate and do hands-on inquiry and make bold thinking. (2) Deepen students' knowledge of moving points on a circle, so as to understand the geometric conditions met by moving points on an ellipse by analogy, which is conducive to the understanding of the concept of an ellipse, pave the way for the induction of the concept of an ellipse, and improve students' sense of

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inquiry and inductive ability.

Teaching process 2: Teachers ask, by the circle and ellipse drawing just now, summarise the factors that affect the circle and ellipse respectively? Students discuss and the teacher summarises. Teacher continues to ask, how to combine the definition of circle and give the definition of ellipse according to the characteristics of ellipse? Students discuss and the teacher gives the definition of an ellipse in relation to an elliptical trajectory.

Design intention: (1) let the students intuitively feel the factors affecting the shape of the ellipse, the formation of self-knowledge, for the later specific definition of the ellipse to make preparations; (2) let the students make clear the definition of the ellipse: the trajectory of the point in the plane and the sum of the distances from the two fixed points (F_1, F_2) is equal to a constant $2a$ (greater than $|F_1F_2| = 2c$) is called an ellipse, these two fixed points are called the focal points of the ellipse, and the distance between the two focuses is called the focal length of the ellipse; (3) based on the interpretation of the 5E session, thinking and summarising to form concepts.

Teaching process 3: Teacher asks why the definition of ellipse must be defined in the plane? Students discuss their answers and the teacher demonstrates the space ellipsoid plane. The teacher asks why the sum of the distances from any point to two fixed points is equal to the constant 2a,and $2a > |F_1F_2|$?Students Students discuss their answers and the teacher illustrates. Finally, the teacher points out the three key points of the definition of ellipse.

Design intention: (1) Let students deeply understand the definition of ellipse through observation and reflection, and develop students' combination of mathematics and shapes and logical thinking; (2) Make it clear to students that the trajectory is an ellipse when $2a > |F_1F_2|$; the trajectory is a line segment when $2a = |F_1F_2|$; and the trajectory does not exist when $2a < |F_1F_2|$; (3) Abstract the existence of the intuitive reality into mathematical concepts, and guide students to discover the geometric characteristics of the ellipse, and to form an intuitive feeling for the elements within the ellipse. (4) Based on the 5E's explanation session, conceptual discernment to deepen understanding.

3.3.3. Deepen the study and construct the equation

Teaching process 1: Teachers guide students to the focus of the line for the x-axis, to the focus of the perpendicular bisector of the line for the y-axis to establish a right-angled coordinate system, set the coordinates of the focal point for the $F_1(-c, 0)$ and $F_2(c, 0)$ as shown in Figure 6, and then the use of ellipse definition of $|PF_1| + |PF_2| = 2a$, after the simplification of the ellipse equation can be obtained.

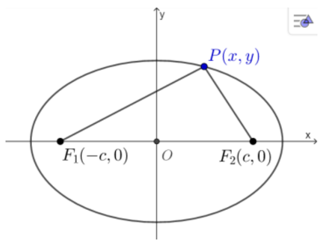

Figure 6: Ellipse trajectory and focus coordinates

Design intent: (1) by the teacher to guide students to complete the ellipse equation to push to, can get the ellipse equation is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1 (a > c > 0)$$

(2) Give students more time to think, emphasise the students' subjective position, and develop students' logical thinking and arithmetic ability; (3) Based on the inquiry link of 5E, guide students to deduce the formula and construct the equation initially.

Teaching process 2: Teacher asks, what does $a^2 - c^2$ stand for? Students discuss, make a diagram, and the teacher makes a demonstration of Figure 7 to give the standard equation of the ellipse.

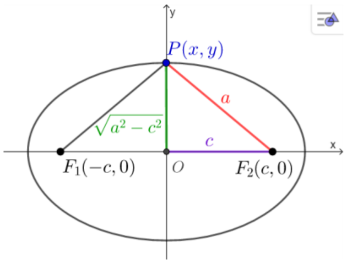

Figure 7: Relationship diagram in ellipse

Design intention: (1) Make it clear to the student that $|OP|^2 = a^2 - c^2$, so that $|OP| = b$,then the standard equation of the ellipse can be obtained

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$$

(2) Based on the inquiry session of 5E, the question leads to simplify the equation.

Teaching process 3: The teacher asks what a represents on a graph in the standard equation of an ellipse. Students discuss and the teacher demonstrates, as shown in Figure 8, and gives the concepts of the long and short axes of an ellipse, explaining that a is represented as the length of the half-long axis and b as the length of the half-short axis.

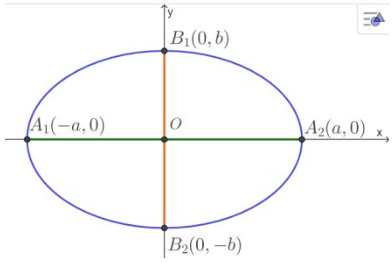

Figure 8: Illustration of Ellipse Long and Short Axes

Design intention: (1) To make the geometrical significance of a and b clear to students, to help them establish the connection between numbers and shapes, to strengthen their understanding and mastery of the coefficients in the standard equation of an ellipse, and to reinforce their awareness of the combination of numbers and shapes and their geometrical intuition; (2) based on the explanation of the link of the 5E, the definition of discernment, and to deepen the understanding of the equation.

Teaching process 4: The teacher asks what the standard equation of an ellipse is if the line at the focus is the y-axis? Students discuss, teacher summarises; Teacher asks, if $a = b$ in the elliptic equation, what curve will the elliptic curve become at this time? Students discuss, the teacher sums up.

Design intention: (1) Let students make clear that when the line where the focus is located is the y-axis (that is, the long axis is on the y-axis), the standard equation of the ellipse is

$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1(a>b>0);$$

(2) Let students understand that when a = b,the two focus of the elliptic curve will merge into one and become a circular curve; (3) Cultivate students' ability to summarise and reason by analogy; (4) Based on the inquiry session of 5E, reason by analogy and deepen understanding.

3.3.4. Application development, ability improvement

Example 1 Write the standard equation of an ellipse that fits the following conditions:

(1)$a=4, b=1$, the focus is on the x-axis;

(2)$a=4, c=\sqrt{5}$, and the focus is on the y-axis;

(3)$a+b=10$，$c=\sqrt{5}$.

(4) The coordinates of the two focus are $(-3,0)$、$(3,0)$,and pass through the point $(1,1,5)$.

Teaching activities: let the students according to what they have learnt, independent hands to solve

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the problem, after the teacher points to analyse.

Design intention: to enable students to master the definition of the ellipse method, the method of coefficients to be determined to seek the standard equation of the ellipse, to further understand the position of the focus of the ellipse and the standard equation of the relationship between the ellipse to deepen the understanding of the equation of the ellipse.

Example 2 Knowing that, F_1 and F_2 are the two focus of the ellipse $C:\frac{x^2}{9}+\frac{y^2}{4}=1$ and that the point M is on C, what is the maximum value of $|MF_1|\cdot|MF_2|$?

Teaching process: let students think, hands-on problem solving. Teachers point out what are $|MF_1|$ and $|MF_2|$ related to?

Design intention: (1) this question is the 2021 college entrance examination question 5, mainly examining the definition of the ellipse, and the use of the basic inequality relationship to solve the problem; (2) practical exercises to improve the application of the students' ability; (3) based on the migration link in the 5E, to improve the application of the students' ability.

Example 3 As shown in Figure 9, take any point P on the circle $x^2+y^2=4$ and make a vertical segment PD of the x-axis through point P. D is the pendant, what is the trajectory of the midpoint M of the segment when the point P is moving on the circle? Why?

Teaching Activity 1: First of all, first let the students think about guessing the trajectory drawn, and then the teacher uses the GeoGebra software to dynamically demonstrate to prove the students' guesses, as in Figure 10.

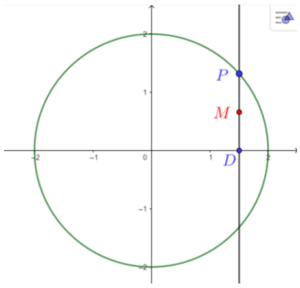

Figure 9: Original diagram of Example 2

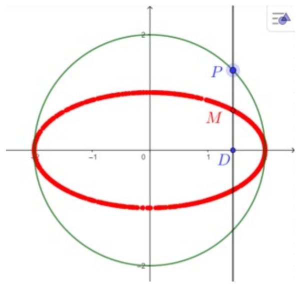

Figure 10: Trajectory diagram of the moving point in Example 2

Teaching activity 2: Let students think about how to use the general steps to find the trajectory equation of the trajectory equation, and try to derive; the teacher guides students to think about the connection between the point P and the point M, and find the conditions met by the point M.

Design intention: (1) With the dynamic demonstration of GGB, develop students' geometric intuition and spatial imagination ability, and strengthen students' understanding and awareness of ellipse; (2) and summarise the general steps of finding the trajectory equations, i.e.: build the system - set up the points - list the equation -simplify; (3) Based on the migration link in 5E, further enhance students' problem-solving ability.

3.3.5. Reviewing and summarising, evaluating and expanding

Teaching Process 1: Review and Summarise

The teacher guides the students to think actively, review what they have learnt in this lesson, and construct a knowledge framework after sorting out to further master the knowledge related to ellipses.

Teaching Process 2: Expanding and Enhancing

After the lesson to expand, let students look up information to understand the Dandelin two-sphere model, and understand how to use light and shadow to derive the definition of an ellipse.

Teaching process 3: Comprehensive evaluation

Divided into three aspects: individual self-evaluation, group evaluation and teacher evaluation, students will be evaluated comprehensively on their class participation, problem identification, problem analysis and problem solving in the middle of the lesson as well as on their class gains and class notes at the end of the lesson.

Design intention: (1) summarise the summary, highlight the key points, consolidate new knowledge, and form a knowledge network; (2) the migration link in the 5E, step by step improvement, help students who have the ability to learn to get further improvement, and then give students space for development; (3) the evaluation link based on the 5E, systematically summarise the knowledge learnt, comprehensively evaluate the students, and enable students to form a correct self-understanding.

4. Summary

The teaching design follows the "5E" teaching model, flexibly using the various links in the "5E" teaching design, and using multimedia and GeoGebra software to assist teaching. Firstly, Apollonius introduced the content of this lesson through the plane cutting cone and the life of the ellipse, and then through the example of investigation and drawing operation to explore, so as to lead to the definition of the conic curve and summarise the explanation. Then the definition of ellipse as the basis to guide students to explore the elliptic equation, and through the question string form to explain the specification of the elliptic equation, through the transfer of consolidation to strengthen and expand, and finally to sum up the evaluation.

The "5E" teaching mode follows the students' cognitive law based on the constructivist theory, breaks through the limitations of traditional teaching, and can better help students construct concepts and form cognition through inquiry-based teaching. In the teaching process, the use of GeoGebrato create a dynamic geometric environment can help students break through the cognitive level of limitations, so that students more intuitive perception of the "shape", and through the "shape" to understand the "number". "This helps students to better appreciate the idea of combining number and shape, and develop the core mathematical literacy of intuitive imagination.

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