

Intelligent Pricing Strategy Analysis for Vegetable Products Using Single-objective Optimization

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Abstract: Vegetables, being an essential part of our daily lives, possess a limited shelf life, rendering the task of stock management crucial for supermarkets aiming to maximize profits. Current research on pricing and replenishment strategies, although existing, lacks the necessary computerization, intelligence, and speed, ultimately resulting in inefficient operations. To address this, the present study utilizes the Nerlove model, incorporates the inventory state transfer equation, and applies multivariate nonlinear regression to elucidate the relationship between price and expected supply. Factoring in storage conditions and expected supply reaction model constraints, a dynamic planning model is formulated. This model offers supermarkets a refined approach to pricing and replenishment decisions for vegetables. Implementing this decision-making framework not only enhances supply chain coordination and efficiency but also optimizes inventory management, ensures supply-demand balance, mitigates inventory wastage and out-of-stock situations, thereby boosting overall economic performance.

Keywords: Nerlove Model; Dynamic Programming; Nonlinear Programming

1. Introduction

Vegetables are daily essentials, and supermarkets are convenient and competitive. Short shelf life and leftover products can result in quality loss and supermarket losses. To reduce losses, supermarkets use demand analysis for replenishment strategies and apply "cost-plus pricing" with discounts for worn-out products. Creating an optimal pricing scheme to maximize benefits and consumer satisfaction remains challenging.

Vegetable freshness is key to pricing, with the supply chain central to cost reduction. Mao Lisha [1] tailored a vegetable production and marketing model for the pandemic era, analyzing supply chain components. Ma Ning [2] optimized small farmers' pricing mechanisms for self-sold fresh produce, addressing quality losses. Tang Run and Li Qianqian [3] integrated quality loss into a dual-channel sales model, exploring optimal discounts and market clearing factors. Despite this, vegetable research lacks in addressing inventory challenges for pricing and replenishment. This study aims to bridge this gap with dynamic programming algorithms, developing a nuanced pricing model for a scientifically grounded framework.

Currently, there's a major gap in models that explore the correlation between aggregate sales volume and cost-plus pricing for a category. These models must establish a functional relationship and offer a daily replenishment and pricing strategy to maximize benefits.

To explore the link between total sales and cost-plus pricing, one could start by plotting their correlation. Then, using sales volume and time data, a binary linear regression model with a lagged sales variable can be built for deeper analysis.

2. Model Building

2.1 Establishment of the Objective Function

2.1.1 Relationship between Selling Price and Cost-plus Pricing

Based on data from the 2023 National Championship of Mathematical Modeling for College Students (http://www.mcm.edu.cn/html_cn/node/c74d72127066f510a5723a94b5323a26.html), for the daily replenishment and pricing strategies, we analyzed variables such as daily sales volume, cost price, and pricing decisions for various vegetable categories. We also considered the iterative linear relationship

between pricing and sales volume, as well as changing product space occupancy. By using a dynamic programming model, our goal is to find the optimal replenishment and pricing strategy to maximize profits for the following week.

The eggplant category is used as a representative example to illustrate the analysis of the sales volume and cost-plus pricing relationship, as the fundamental steps are similar across all categories. Processing the data yielded a scatter plot showing the relationship between category unit price and total sales, Figure 1.

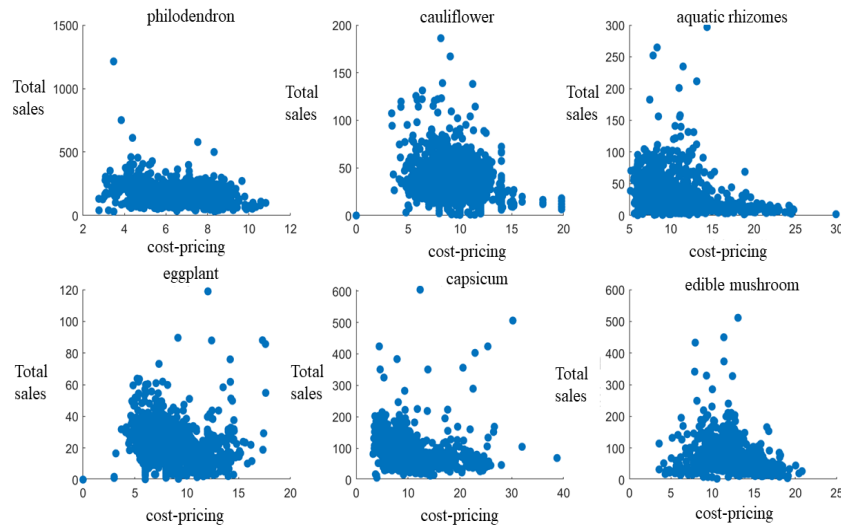


Figure 1: Scatterplot of category unit price and total sales volume

Upon analyzing Figure 1, we find weak correlations between unit sales price and total sales, suggesting unaccounted factors. Time, mentioned in the title's sales volume-time relationship, appears crucial. With multiple independent variables, we chose multiple linear regression. Literature review revealed the *Nerlove* model as the preferred method for estimating agricultural product supply response [4]. This model adjusts daily sales volume expectations based on previous day's actual vs. expected sales volume differences [5-6]. Our model incorporates the previous day's sales volume as an independent variable.

$$Q_t = a_0 + a_1P_t + a_2Q_{t-1} \quad (1)$$

Where Q_t represents the sales volume on day t , and P_t represents the cost-plus pricing on day t .

2.1.2 Dynamic Planning of Daily Replenishment and Pricing for Each Category of Vegetables

The aim is to maximize weekly profits by summing daily profits for the superstore. Profitability usually considers revenue and costs, but for this vegetable analysis, losses and discounts are negligible. Assuming profit is determined by total cost and revenue, the profit function for day t is:

$$profit_t = \sum_{i=1}^6 (s_{ti} \cdot p_{ti} - c_{ti} \cdot x_{ti}) \quad (2)$$

s_{ti} , p_{ti} , c_{ti} , and x_{ti} represent total sales, pricing, cost, and replenishment volume of category i on day t/i , respectively. The model incorporates variables influencing sales volume of category i on day t . Constraints will identify other influencing factors, ensuring model completeness and reliability, and a certain degree of credibility.

The ultimate conclusion is that the primary goal is to maximize the overall profit value.

$$\max \text{imize}_1 = \sum_{t=1}^7 profit_t = \sum_{t=1}^7 \sum_{i=1}^6 (s_{ti} \cdot p_{ti} - c_{ti} \cdot x_{ti}) \quad (3)$$

Subject to specific constraints, maximize the given equation and identify the optimal daily replenishment and pricing strategies for the category that yields the highest profit margin.

2.1.3 Non-linear Programming Model for Daily Replenishment and Pricing Optimization of Individual Vegetable Items

The filtering process selects 48 weekly saleable items (1-48) into six groups: foliar, cauliflower, aquatic root, eggplant, chilli, and mushrooms. To derive the objective function, we follow previous steps, starting with the *j*th profit calc for the first product.

$$\text{profit}_j = n_j \left[(s_j - r_j x_j - R') p_j + (r_j x_j + R') p'_j - c_j x_j \right] \tag{4}$$

The $n_j \in \{0, 1\}$ variable indicates whether item *j* is chosen for display (1) or not (0). s_j , x_j , p_j , and p'_j represent daily sales, replenishment, pricing, and pricing with shipping losses for item *j*, respectively. r_j is the depreciation rate for item *j*, R is the aggregate historical inventory in the preceding seven days, and c_j is the cost price for item *j*. Known parameters are r_j , R , and c_j .

We find the overall profit function as:

$$\text{maximize } e_2 = \sum_{j=1}^{48} \text{profit}_j = \sum_{j=1}^{48} n_j \left[(s_j - r_j x_j - R') p_j + (r_j x_j + R') p'_j - c_j x_j \right] \tag{5}$$

2.2 Constraint Function

2.2.1 Dynamic Planning of Daily Replenishment and Pricing for Each Category of Vegetables

Pricing and replenishment need assessment based on supply, demand, and pricing impact. Chose state transfer equation to optimize objective function. Daily sales & replenishment affect inventory space, necessitating a temporal evolution equation. r_t defined as inventory space before replenishment, determined by previous day's inventory & replenishment volume.

$$r_{t+1} = r_t + \sum_{i=1}^6 (x_{ti} - s_{ti}) \tag{6}$$

Supermarkets face stock level constraints to prevent exceeding capacity. Frequent replenishment is crucial for vegetables' short shelf-life, limited by available stock. Replenishment volume must ≥ 0 kg per category, meeting daily sales requirements. Pricing forbids \$0/kg. Constraints stem from these factors.

$$\begin{cases} 0 \leq r_t \leq R_{max} \\ x_{ti} \geq 0 \\ \sum_{i=1}^6 x_{ti} \leq R_{max} - r_t \\ x_{ti} \geq s_{ti} \\ p_{ti} \geq 0 \end{cases} \tag{7}$$

$t = 1, 2, \dots, 7, i = 1, 2, \dots, 6$

R_{max} represents the superstore's inventory level, considering daily maximum.

Precise constraint values are crucial for daily replenishment quantities and pricing. Cost prices are usually fixed. Supermarkets often delay vegetable pricing adjustments by one day, relying on previous day's sales data for robust strategies. This delay aligns with anticipated daily replenishment adjustments. Key factor: STI sales volume. Expected sales are influenced by the previous day's sales discrepancy, adjusted by a specific percentage.

$$s_{it} - s_{t-1,i} = \theta (s_{it}^A - s_{t-1,i}) \quad (8)$$

where s_{it}^A denotes the expected sales volume of the i th category on the t th day, and θ is the sales volume adjustment factor, from which the sales volume can be calculated as:

$$s_{it} = (1 - \theta)s_{t-1,i} + s_{it}^A \quad (9)$$

Expected profits based on adaptive expectations theory, pricing influenced by previous day's disparity. Current pricing adjustable by a percentage. Eqn:

$$p_{it}^e - p_{t-1,i}^e = \beta (p_{t-1,i} - p_{t-1,i}^e) \quad (10)$$

Where p_{it}^e denotes the expected pricing for category i on day t , and β is the expected price adjustment factor, which leads to a pricing formula of:

$$p_{it} = p_{t-1,i}^A + (1 - \beta)p_{t-1,i} \quad (11)$$

2.2.2 Non-linear Programming Model of Daily Replenishment and Pricing for Each Individual Vegetable Item

To prevent excessive vegetable stock and escalating storage costs, the supermarket's cumulative daily replenishment must not exceed available inventory. Selecting 27-33 items from 48 saleable products ensures a diverse yet manageable selection. Popular items must have a minimum display quantity of 2.5

kg to avoid unsaleability. Different vegetable categories have varying discount rates, represented by p_j and p_j' 's proportional relationship. $\alpha_{i(j)}$ represents the j th category's discount rate. Pricing must be $\geq \$0/\text{kg}$ and guarantee at least one saleable item per category to meet market demand.

In summary, we derive the following constraints:

$$\left\{ \begin{array}{l} \sum_{j=1}^{48} x_j \leq R_{max} - \sum_{j=1}^{48} R_j \\ x_j + R_j \geq 2.5 \\ 27 \leq \sum_{j=1}^{48} n_j \leq 33 \\ p_j' = \alpha_{i(j)} p_j \\ p_j, p_j' \geq 0 \\ R' = \sum_{j=1}^{48} R_j \\ n_j > 0 \\ j = 1, 2, \dots, 48, n_j \in \{0, 1\} \end{array} \right. \quad (12)$$

R_{max} , indicating the superstore's inventory level, is set by choosing the daily inventory's maximum value, which is 966 based on available data. R_j represents the j th item's inventory space, while n_i represents the i th category's total sellable items. Both R_j and n_i are analyzed as state variables.

The items are organized sequentially: foliage, cauliflower, aquatic roots and tubers, eggplant, chilli, and edible fungi, each with corresponding numbers. This reveals that the value of I is a segmented

function dependent on the value of j . In other words,

$$i = i(j) = \begin{cases} 1, 1 \leq j \leq 17 \\ 2, 18 \leq j \leq 19 \\ 3, 20 \leq j \leq 26 \\ 4, 27 \leq j \leq 31 \\ 5, 32 \leq j \leq 41 \\ 6, 42 \leq j \leq 48 \end{cases} \quad (13)$$

2.3 Algorithms Design

To maximize profits during 2023.7.1-7.7, a dynamic planning model is key for daily replenishment and pricing strategies for each veg category. Key variables: daily sales volume, cost price, and pricing for each category. Thorough analysis of influencing factors, like pricing-sales volume linearity and inventory space, is crucial. This model enables precise replenishment and pricing plans, ultimately maximizing profits.

To maximize supermarket veg revenue, we seek optimal daily replenishment and pricing using enhanced dynamic programming. We forecast next-day replenishment and pricing, factoring in fixed parameters like t , discounts, and wear & tear impacts.

3. Calculus analysis

According to the publicly available operational data of a vegetable super, the missing values are replaced by 0, and the outliers are replaced by the mean value.

To manage the abundance of vegetable items, some strongly correlated, the Boruta algorithm screens features. It selects representatives in each category based on shaded features and binomial distribution. Features are randomly disrupted, a model assesses importance, and a threshold is set. Features exceeding this are recorded, and hit counts accumulate. Hypothesis testing compares counts, iterations, significance levels, and indices to identify strongly correlated items. This repeats until all items are labeled or iterations reach a limit. ([7]) 65 items are typical for their categories. A correlation network diagram explores relationships, using the Spearman coefficient and three thresholds (0.1, 0.6, 0.8). Line thickness, color, etc., represent correlation strength, as shown in Figure 2.

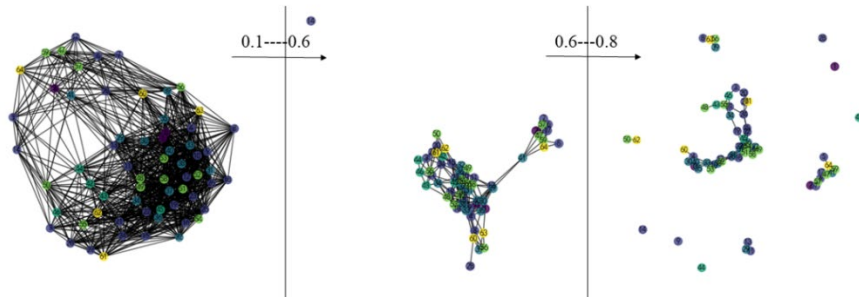


Figure 2: Network diagram of correlation of vegetable individual items

3.1 Objective function determination

Using multiple linear regression with SPSSPRO, we analyze the sales volume and cost-plus pricing relationship for each category, deriving a linear equation for eggplant as an example.

$$Q_t = -0.36P_t + 0.60Q_{t-1} + 11.52 \quad (14)$$

Mitigating heteroskedasticity via logarithmic transformation leads to the model:

$$\ln Q_t = -0.24 \ln P_t + 0.63 \ln Q_{t-1} + 1.62 \quad (15)$$

Other categories use the same approach as eggplant, leading to the model in Table 1.

Table 1: Linear model equations for each category

Kinds	Constructor	R^2
philodendron	$\ln Q_t = 0.18 \ln P_t + 0.53 \ln Q_{t-1} + 2.13$	0.51
cauliflower	$\ln Q_t = -0.4 \ln P_t + 0.67 \ln Q_{t-1} + 2.01$	0.71
aquatic rhizomes	$\ln Q_t = -0.24 \ln P_t + 0.63 \ln Q_{t-1} + 1.62$	0.62
eggplant	$\ln Q_t = -0.24 \ln P_t + 0.63 \ln Q_{t-1} + 1.62$	0.64
capsicum	$\ln Q_t = 0.21 \ln P_t + 0.58 \ln Q_{t-1} + 1.55$	0.57
edible mushroom	$\ln Q_t = 0.65 \ln P_t - 0.33 \ln Q_{t-1} + 0.78$	0.64

3.2 Model solution for daily replenishment

The logarithmic coefficients are solved for the dynamic programming model to obtain Table 2.

Table 2: Table of model coefficients

	Non-standardized coefficient		Standardized coefficient	T	P
	B	standard deviation	Beta		
constant	2.60	0.64		4.08	0.00
$\ln St_1$	0.81	0.08	0.81	10.79	0.00
$\ln St_2$	-0.21	0.07	-0.21	-2.84	0.01
$\ln Pt_1$	-0.72	0.31	-0.18	-2.31	0.02
$\ln Pt_2$	0.64	0.30	0.16	2.12	0.04

It can be shown that $b_1 = 0.81$, and from $b_1 = 1 - \theta$, we can find $\theta = 0.19$; $b_2 = -0.21$, and by the same token $\beta = 1.21$.

Table 6 reveals a strong negative link between category pricing and sales volume at a 1% significance level. A 1% change in pricing leads to a relative decrease of 0.72% in sales volume, confirming that higher pricing hurts sales. Balancing sales volume and pricing is crucial to avoid profitability losses due to reduced sales volume from overpricing. Following logarithmic processing, the relationship between expected sales volume and cost-plus pricing is derived as:

$$\ln s_{it}^A = a_0 + a_1 \ln p_{it}^A + a_2 \ln s_{t-1} \quad (16)$$

p_{it}^A represents the expected pricing of a category on a given day. Expected pricing and actual pricing are proportionally related. On average, fruits and vegetables have a discount rate of 0.71 ([8]). Pricing is selected based on market pricing, derived from expected pricing. However, due to demand changes and shorter freshness periods, actual pricing often differs from expected pricing. In most cases, the ratio between expected pricing and lagged actual pricing equals the discount rate. Therefore, it can be derived that:

$$p_{it} = k p_{t-2,i} + (1 - \beta) p_{t-1,i}, k = 0.71 \quad (17)$$

Finally, the dynamic programming model is identified as:

$$\begin{aligned}
 & maximize_1 = \sum_{t=1}^7 \sum_{i=1}^6 (s_{ti} \cdot p_{ti} - c_{ti} \cdot x_{ti}) \\
 & s.t. \left\{ \begin{aligned}
 & p_{ti} = kp_{t-2,i} + (1 - \beta)p_{t-1,i} \\
 & s_{ti} = (1 - \theta)s_{t-1,i} + exp(a_0 + a_1 \ln(kp_{ti}) + a_2 \ln s_{t-1,i}) \\
 & r_{t+1} = r_t + \sum_{i=1}^6 (x_{ti} - s_{ti}) \\
 & 0 \leq r_t \leq R_{max} \\
 & x_{ti} \geq 0 \\
 & \sum_{i=1}^6 x_{ti} \leq R_{max} - r_t \\
 & x_{ti} \geq s_{ti} \\
 & p_{ti} \geq 0
 \end{aligned} \right. \\
 & \beta = 1.21, \theta = 0.19, k = 0.71, \beta = 1.21, t = 1, 2, \dots, 7, i = 1, 2, \dots, 6
 \end{aligned} \tag{18}$$

3.3 Substituting data to solve

The dynamic programming model calculates daily replenishment and pricing for each vegetable category during the week of July 1-7, 2023. These results are summarized in Table 3.

Table 3: Daily replenishment and pricing of vegetables by category

dates	philodendron		cauliflower		Aquatic rhizomes		eggplant		capsicum		edible mushroom	
	replenishment	prices	replenishment	prices	replenishment	prices	replenishment	prices	replenishment	prices	replenishment	prices
7.1	148.04	5.02	19.86	13.90	18.32	16.08	17.10	8.12	83.46	7.17	37.85	10.78
7.2	142.26	4.99	17.45	13.64	22.53	15.90	19.58	8.22	92.09	7.18	40.70	10.68
7.3	139.25	5.18	16.61	13.77	18.15	15.68	22.01	8.12	83.56	7.50	41.81	10.67
7.4	158.90	5.17	16.57	13.70	20.22	15.42	26.39	9.21	95.67	7.27	42.25	10.48
7.5	140.05	5.24	16.77	13.61	17.38	16.22	25.08	8.20	92.13	7.29	42.42	10.66
7.6	137.76	5.14	16.94	13.49	18.53	16.31	22.30	8.46	89.29	7.28	42.48	10.88
7.7	130.85	5.10	17.02	13.48	14.09	15.85	17.62	8.39	83.64	7.29	42.51	10.72

Using the nonlinear planning model for daily replenishment and pricing of 30 sellable vegetable items, a maximum benefit of \$776.95 was achieved. Tables 4, 5, and 6 present the daily replenishment and pricing calculations for each item on July 1, 2023.

Table 4: Flowering and Leafy Vegetables

kind	philodendron						
item	broccoli	sweet potato tip	snow fungus	cabbage	Shanghai Youth	amaranth greens	
serial number	1	4	5	7	8	11	
replenishment	5.64	6.10	6.33	6.17	7.92	5.24	
pricing strategy	normalcy	12.00	5.33	3.64	4.90	8.00	4.00
	discount	6.01	2.29	1.56	2.10	3.43	1.72

Table 5: Flowering and Leafy Vegetables

kind	philodendron						
item	Small bok choy (1)	Yunnan lettuce	Yunnan lettuce	Yunnan oilseed rape	Yunnan oilseed rape	Brussels sprouts	
serial number	12	13	14	15	16	17	
Replenishment of individual products	3.12	16.12	15.71	10.96	10.70	7.56	
pricing strategy	normalcy	5.23	9.20	4.46	7.20	4.16	3.80
	discount	2.25	3.95	1.92	3.09	1.78	1.63

Table 6: Cauliflower, Aquatic Roots and Tubers, and Solanaceous Vegetables

Category	cauliflower	Aquatic rhizomes		eggplant				
item	broccoli	Zi jiang green stem	High melon (1)	Net root (1)	Green eggplant (1)	long term eggplant	Purple eggplant (2)	
serial number	18	19	20	24	27	29	31	
Individual product replenishment	19.94	12.37	3.24	21.24	3.28	2.76	17.92	
pricing strategy	normalcy	12.81	13.19	14.40	6.00	12.10	6.00	12.41
	give discount	3.81	3.86	4.21	2.61	5.26	2.61	3.69

3.4 Results and discussion

The model generates daily replenishment and pricing strategies for each category during July 1-7, 2023. For July 3, 2023, the replenishment of aquatic roots and tubers is 552.34 kg at \$18.15/kg. Figure 3 displays the calculated data as a line graph.

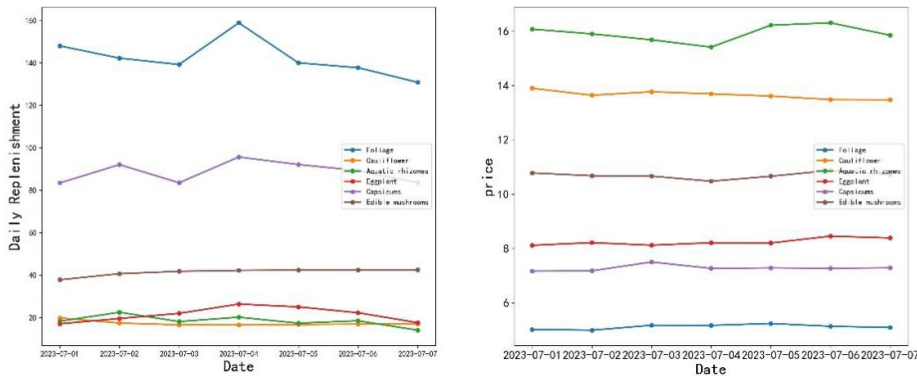


Figure 3: Daily replenishment and pricing line graphs

Fig. 3 outlines replenishment volume & pricing trends for veg categories next week. Daily replenishment volume dynamically relates to the previous day's. Foliage peaks on Day 1, then decreases, reflecting lower daily sales. Aubergine replenishment decreases before increasing, while mushrooms remain stable. Pricing is influenced by demand but limited by veg's necessity. Aquatic roots & rhizomes show more fluctuations, while other veg prices fluctuate minimally, indicating limited pricing variability.

Taking broccoli in the cauliflower category as an example, its daily replenishment in 2023.7.1 was 19.94 kg, with normal pricing of 12.41 yuan/kg and discounted pricing of 3.69 yuan/kg. The pricing model for the category in 2023.7.1 yields a maximum profit of \$701.57, and the pricing model for the single item in 2023.7.1 yields a maximum profit of \$776.95. It is found that the maximum profit obtained from pricing planning for the single item is higher than that obtained from pricing planning for the category, and therefore the pricing planning obtained for the single item yields a better result[9-10].

4. Conclusions

This paper explores daily replenishment and pricing strategies via planning, introducing dynamic planning and a nonlinear model with a lagged first order. Key takeaways:

(1) It is recommended to replenish a large amount of leafy vegetables on the first day, and gradually reduce the replenishment amount of leafy vegetables in the next few days, as well as the replenishment amount of eggplant decreases and then increases, and the replenishment amount of edible mushrooms tends to be the same, and other replenishment strategies;

(2) Pricing is affected by demand, but because vegetables are a necessity of life, pricing basically needs to be maintained around a threshold, in addition to aquatic roots and rhizomes have more pronounced fluctuations, the rest of the vegetables pricing are shown to rise first and then fall in a very small wavelets floating, in the replenishment volume and pricing strategy, the maximum profit in the

next seven days [701.57, 829.44, 789.51, 684.72, 733.43, 856.27, 697.00].

(3) Based on the Nerlove model, pricing strategies are considered to arrive at strategies such as reducing the replenishment quantity for items such as cauliflower that have a large difference between the discounted and normal selling price.

References

- [1] Mao Lisha. *Research on Pricing Strategy and Production and Marketing Mode of Vegetable Wholesale Market under the Perspective of Supply Chain*[D]. Central South Forestry University of Science and Technology, 2023.
- [2] Ning Ma. *Research on channel optimization of fresh produce for small farmers*[J]. *Journal of System Science*, 2021, 29(02):116-120+126.
- [3] Run Tang, Qianqian Li, Yangyang Peng. *Research on dual-channel market exit strategy for fresh produce considering quality loss* [J]. *Systems Engineering Theory and Practice*, 2018, 38(10):2542-2555.
- [4] Guohui Zhong, Jinhai Yao. *Response analysis of farmers' double-season late rice sowing area to rice price changes - based on provincial panel Nerlove model* [J]. *Price Monthly*, 2021(07):61-66.
- [5] Xiangfei Xin, Zuli Wang, Jimin Wang. *An empirical study on the supply response of broilers in China - Based on Nerlove model and provincial dynamic panel data* [J]. *Journal of Agricultural and Forestry Economics and Management*, 2017, 16(01):120-126.
- [6] Min Lang, Shuai Liu, He Xu. *Study on soybean supply response in Jilin province under the background of soybean revitalization--an empirical analysis based on Nerlove model* [J]. *Chinese Journal of Agricultural Mechanical Chemistry*, 2021, 42(08).
- [7] Xiaofei Ma, Pingfeng Xu. *Application of Improved Boruta Algorithm in Music Emotion Research* [J]. *Journal of Changchun University of Technology*, 2020, 23(03):104-107+40.
- [8] Jufang Ye. *Mathematical Modeling of Supermarket's "Thin Profit and High Sales" Strategy* [J]. *Journal of Hubei Institute of Vocational Technology*, 2020, 23(03).
- [9] Ge Tian, Kuang Wu, Hongrui Tian. *Sales prediction of retail goods during traditional holidays by matplotlib-based K-means-ARIMA model* [J]. *Computer Knowledge and Technology*, 2022, 18(18): 5960+63.
- [10] Zonghong Cao, Ju Zhao, Chengtang Zhang. *Joint inventory model with variable inventory cost and price* [J]. *A joint inventory model with variable inventory cost and price* [J]. *Journal*, 2013, 27(04):66-70.