

Research on the optimal relationship between race schedule and player pace based on dynamic model

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Abstract: This paper takes bicycle road racing as the research object, and establishes a P-X model considering power changes. Firstly, it analyzes the theoretical basis of the P-X model of Time trial specialist. On this basis, taking Sprinter as an example, a power model considering men's sprinter is established to help cyclists complete the race in the shortest time, and determine that this type of athlete can complete the race in the shortest time.

Keywords: Bike road race, P-X model, Time trial specialist, Sprinter

1. Introduction

Bicycle road race is a sport that challenges speed and endurance, in which there are many types of athletes that participate in individual time trials, such as Time Trial Specialist, Climber, Sprinter, Rouleur, Puncheur, and the athlete that can complete the cycling route in the shortest time is the winner. The length of riding time and the size of the athlete's power output is closely related, each athlete can produce different power output in different lengths of time, and as the race progresses, the athlete will become more and more fatigued. Therefore, the question of what power should be ridden at to be more likely to win during a race becomes a great concern for cyclists and team directors^[1].

In order to help cyclists to complete the race in the shortest time, this paper develops a mathematical model to inform the cyclists of the power value required for each position on the track, researches and analyzes the power situation of the sprinter during sprint, and determines that this type of athlete can complete the race in the shortest time. With this model, we can calculate at which specific position the high power delivered by the sprinter will result in the shortest riding time. We apply the above two models to the 2021 Tokyo Olympic cycling road race and the 2021 Belgian UCI and compare them with the winner's results. To further verify the feasibility of the model, we also designed a track with four turns and a hill^[2].

2. Model establishment and solution

2.1. Model considering power recovery P-X

2.1.1. P-X model from Time trial specialist

According to Newton's second law, the basic kinetic equations during road cycling can be established:

$$F - N - F_w - F_d - F_r = m \cdot a \quad (1)$$

Among them, m is the mass of the rider, and F is the forward force generated by the rider pedaling. The force is difficult to determine, so the forward F can be represented by the ratio of instant power and speed. In addition, the energy loss of the chain in the transmission process should also be considered. Some studies have shown that the transmission efficiency of the chain is positively correlated with the riding force^[3]. Under normal circumstances, the chain transmission efficiency loss of road bicycles is 98.5%.

Through the above dynamic analysis of the road bicycle, we can obtain the dynamic equation of the road bicycle when riding:

$$\frac{\eta P}{v} - \frac{1}{2} \mu_b S \rho (v + v_w)^2 - \frac{7}{8} \pi \mu_w r^2 \rho (v + v_w)^2 - \mu_r mg \cos \theta - mg \sin \theta = \left(m + \frac{2I}{r^2} \right) a \quad (2)$$

Time trial specialist's P-X model can clearly describe the relationship between riding power P and speed v, set the initial parameters of equation (1) as shown in Table 1.

Table 1: Initial parameter value table of kinetic equation

symbol	parameter	symbol	parameter
η	0.985	μ_r	0.004
ρ	1.266kg/m ³	r	0.35m
S	0.5m ²	I	0.08kg · m ²
μ_b	0.5	g	9.8kg / m ²
μ_w	0.0397		

Let's first consider the power situation of male athletes, taking the weight of 80kg as an example. From the weather-related data of road cycling competitions, we assume that the wind speed is $v_w = 3m/s$. From the data of large-scale international road cycling competitions, we can conclude that the average speed of world-class cyclists is about $10m/s \sim 15m/s$, and the average power is about $300w \sim 550w$. We extract data from the above two intervals with the steps of 1m/s and 1w respectively, and put 5 average speed values and 250 average power values into (1) one by one, in order to get the data when riding on a gentle road section, we set the slope angle to $\theta = 0$. At this time, we will get 1050 values of acceleration $a < 5 * 10^{-4}$. When a, we think that the acceleration at this time is approaching $a \rightarrow 0$, so we extract the values of P and v at a. The data can be obtained as shown in Table 2:

Table 2: P and v value distribution

\bar{P}	\bar{v}
370±30	10.46±0.6
420±15	11.2±0.6
520±22	12.13±0.12

From the above table, the median of P is 441w. In order to simplify the calculation, we take P as 450w.

It can be obtained from the data table of power-to-weight ratio that when a male world-class athlete weighing 80 kg rides with a power of 450w, the FTP value is 5.6, which corresponds to the riding level of an exceptional athlete, so we can assume that the world Level athlete riding with P=450w power on a gentle road is a fatigue-free riding.

Due to the influence of fatigue, when the riding power P>450w, with the increase of the duration, the larger the value of P-450, the higher the degree of fatigue, and it is easy to know that the power will decrease when the fatigue will occur. We set the duration of the original power. is Δt , the power P1 at the next moment can be obtained as:

$$P_1 = P - \frac{\lambda(P - 450)}{450} \Delta t \quad (3)$$

Considering that the human body will continue to do the same thing will also increase the degree of fatigue, we set the total riding time from the beginning of the ride to the current moment as t, for the sake of simplicity, the coefficient λ is taken as 0.01, and (2) is corrected to (3):

$$P_1 = P - \frac{0.01(P - 450)}{450} \Delta t * t \quad (4)$$

From Newton's laws of motion:

$$v = v_0 + at \quad (5)$$

$$x = v_0 t + \frac{1}{2} at^2 \quad (6)$$

$$P = mav \quad (7)$$

Assuming that the athlete starts with maximal power, we take 25w/kg in Table 2 as the instantaneous starting energy, and calculate the starting power of the 80kg athlete as $P_0 = 2000W$, and substitute it into (4), equation (5), and equation (6), we can Well, within 5 seconds of starting, the rider can increase the speed to about 14m/s, and the riding distance is about 50m. Compared with the power consumption of p=450w for 1 hour, the energy consumption in this starting stage can be ignored. remember. Therefore, we can ignore the effects of fatigue at this stage^[4].

2.1.2. Take Sprinter as an example

Based on the power model of the Time trial specialist, we consider the power model of the men's sprinter. Let's assume that the sprinter reaches 450w after the race starts at time k. The athlete needs extra time to recover the power level $P_{z(t-1)}$. Assuming that the athlete needs to recover the energy in the same time k, the power level of the recovery period can be obtained from the power level of the time-trial athlete. Considering that the sprinter can generate high power in a short period of time, we assume that the sudden power after the rider recovers is P, and t is the time it takes from the start of the race to the present, we can get:

$$P_{z(t)} = P_{z(t-1)} + 0.01\Delta t\sqrt{t - 2k} \tag{8}$$

The power of the rider during the recovery period can be given by the power model of the time-trial runners. We assume that at time Z, the rider releases the power of P_{r+1} at the instant of sprint, which is calculated by establishing an algorithm on the computer^[5].

2.1.3. Calculation results

Through the above algorithm, after a series of loop simulations, we can find the p-t graph that minimizes the whole time tmin, as shown in Figure 1.

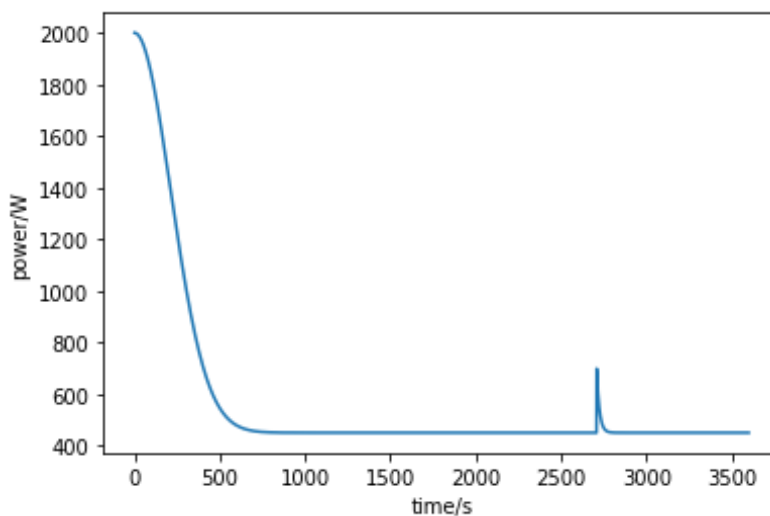


Figure 1: The p-t diagram with the shortest whole time tmin

2.2. Case Analysis

2.2.1. Take the Tokyo Olympics as an example

We simplified the cross-sectional diagram of the entire route of the Tokyo Olympics as shown in Figure 2.



Figure 2: Cross-sectional view of the entire route

The distance of a circle can be divided into 5 sections, and the parameters are shown in Table 3.

Table 3: The numerical value of the simplified model for the Tokyo Olympics

the sin value of the slope	Distance (km)	alternative model
0.0344	2.5	downhill
0.018	7	uphill
0.0216	6	downhill
0.0352	2.5	uphill
0	4.5	gentle

There are 10 stages in the whole Olympic Games. According to the model of the all-around rider, the predicted time can be calculated to be $T=3741s$, that is, 1:02:21, and the parameters we use are those of the average rider, that is, the average level of the rider. The difference between the rider and the champion (55:04) is only 7:17, so the model has a certain degree of feasibility. Figure 3 is a P-t diagram of the whole process, and Figure 4 is a P-t diagram of a circle.

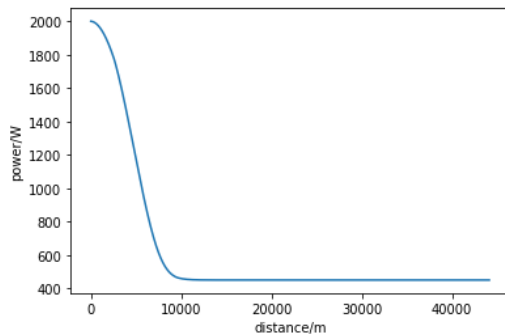


Figure 3: P-t diagram of the whole process

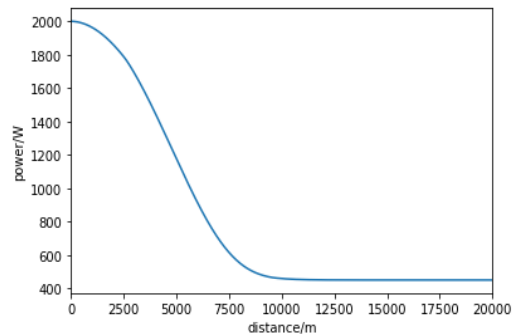


Figure 4: P-t diagram of one circle

According to the model of the sprinter, it can be calculated that his riding time to complete the entire Olympic track is $T=3733s$, that is, 1:02:13, which is not much different from the actual championship result. Figure 5 is a p-x graph of a sprinter who sprints in the final stage.

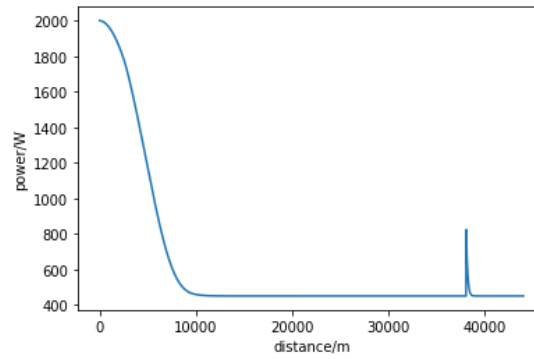


Figure 5: *P-x plot of the sprinter*

2.2.2. Take the World Championships as an example

The road map and cross-sectional view of the 2021 World Championships road bike obtained from the World Championships official website is shown in Figure 6.



Figure 6: *Roadmap and cross-section*

It can be seen from the figure that the whole process of the competition is relatively flat, the slope θ can be regarded as 0, and the known champion score is 47:47.83.

According to the model of the all-round driver, we can calculate the predicted score $T=3075s$, which is 51:15, which is 3 minutes and 28 seconds away from the championship's score. Figure 7 is the $p-x$ graph of the all-around driver at the World Championships.

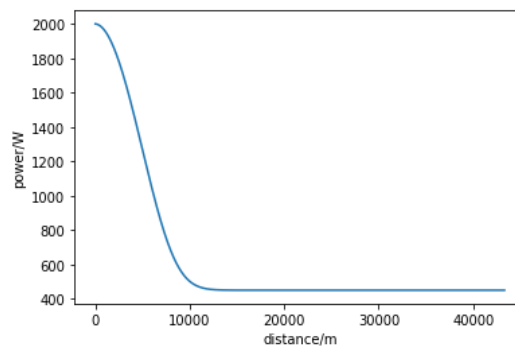


Figure 7: *The p-x graph of the all-round driver at the World Championships*

According to the sprinter's model, we can calculate that the sprinter's score is $T=3071$, that is, 51:11. Figure 8 shows the sprinter's p-x graph of the World Championships:

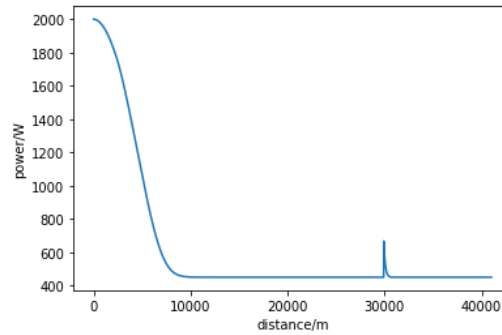


Figure 8: World Championship p-x chart of sprinters

2.2.3. Track design

In order to better verify our model, we designed a road bicycle riding route map with one gradient. The whole race needs to ride on this track for 2 laps, with a total distance of 41km. The parameter list is shown in Table 4.

Table 4: Simplified table of self-built track model

slope sin	Distance (km)	alternative model
0	2.5	flat ground
290/8000	8	uphill
280/6500	6.5	downhill
0	3.5	flat ground

The track route map is shown in Figure 9:

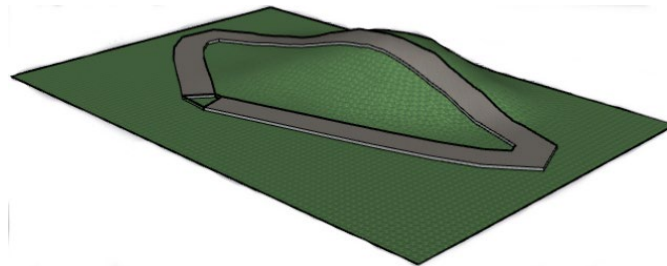


Figure 9: Track route map

The simplified diagram of the whole model is shown in Figure 10:

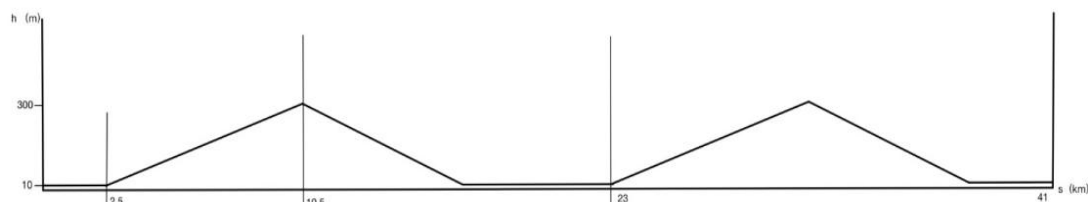


Figure 10: Simplified diagram of Cheng model

According to the above analysis, we concluded that on this track, the shortest time required by the all-around rider is 3520s, and the shortest time required by the sprint rider is 3514s. The P-x diagram is shown in Figure 11.

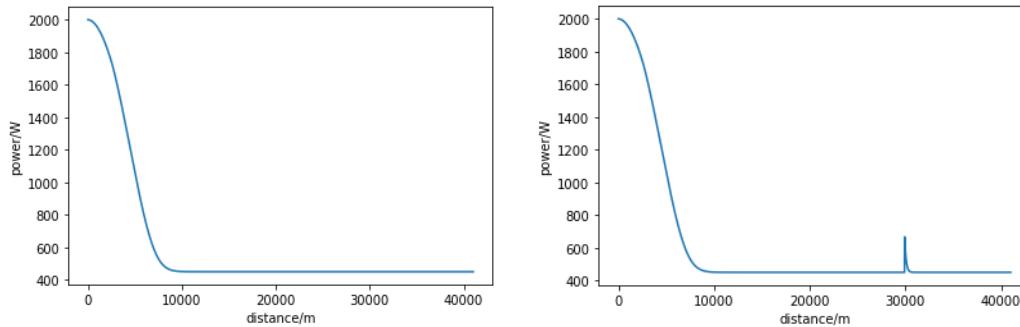


Figure 11: P-x diagram of calculation results

3. Model establishment and solution

Cycling competition is a sport that challenges speed and endurance. Generally, power is used to describe the riding ability of a rider. In order to explore where the rider should ride and at what power can make it reach the end in the shortest time, we use the power to describe the riding ability of the rider. The force analysis of riding, the P-x model is studied, and on this basis, the P-x model considering power recovery, the team's P-x model, etc. are established. By comparing the simulated test with the real value, and then applying the model to different competitions, the design track verifies the accuracy of the model. During the modeling process, we set reasonable assumptions to simplify the model and facilitate the construction of the model.

References

- [1] Cai Rui, Chen Liang, He Shenjie, Jiang Chongmin, Li Ran, Liu Zhi. *Dynamic Analysis and Simulation Research of Road Bicycle* [J]. *China Sports Science and Technology*, 2014, 50(01): 125-128.
- [2] Xu Wenxi, Xu Wencan. *Viscous Fluid Mechanics* [M]. Beijing: Beijing Institute of Technology Press, 1989.
- [3] Gong Shengxia, Huang Chengli. *Bayesian method inversion of the internal structure of terrestrial planets* [C]// *China Geophysics 2013 - Proceedings of the 25th Special Issue*, 2013: 24.
- [4] Wan Haibo, Han Yongqi. *Technical characteristics and countermeasures of bicycle road time trials* [J]. *Contemporary Sports Science and Technology*, 2021, 11(29): 245-248.
- [5] Zhang Rong. *Design and Dynamic Analysis of New Mountain Bike* [D]. Xi'an University of Technology, 2017.